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PHYSICAL REVIEW.

ON THE GROWTH AND DECAY OF COLOR SENSATIONS IN FLICKER PHOTOMETRY.

By M. Luckiesh.

Introduction.

NOTWITHSTANDING the extensive investigations in heterochromatic photometry the photometrician is still unable to equate brightnesses widely different in color with surety. This is due chiefly to the fact that the two practical methods of photometry—the flicker and the direct comparison methods—do not give concordant results. It is therefore necessary besides establishing an "average eye" to determine which method shall be the standard.

Since the flicker photometer was first proposed as a means of overcoming the difficulties attending a large color difference in photometry, it has won many supporters. But in the light of the work that has been done recently it is plain that those who had accepted the flicker photometer had done so before it had been thoroughly investigated. H. E. Ives[1] has perhaps investigated the two practical methods of photomery in parallel more extensively than any other investigator. Among other interesting data gathered in that investigation it was shown that the flicker method was not subject to the Purkinje effect but to a reversed effect. The writer[2] verified this and incidentally showed that the ratio of a red light to that of a blue-green light as measured was far different depending upon the method. In the present work it was found after balancing a red light and a blue-green light individually against total tungsten light by the direct comparison method, that the intensity of the red light must be reduced to 55 per cent. of its foregoing value in order to balance by the flicker method the foregoing intensity of the blue-green light. This result agrees closely with that obtained in the

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[1] Phil. Mag., 1912.

[2] Electrical World, March 22, 1913; London Illuminating Engineer, Vol. 6, p. 119.

previous work. Ives's data show this difference in the results by the two methods although it was not discussed or pointed out in the original paper. This difference and the fact that the flicker photometer is subject to a reversed Purkinje effect are sufficient to show the importance of investigating the possible causes for the discordance in the results by the two methods. It is apparent at once that the physiological phenomena occurring in the two methods are not wholly the same, for only in the direct comparison method is simultaneous contrast effective. Further, the flicker method is probably complicated by after images and the different rates of growth and decay of the various color-sensations.

APPARATUS.

An apparatus was constructed so as to include both a flicker and a direct comparison photometer in such a manner that by intercomparison certain data could easily be obtained without introducing uncertainties which might arise in using two instruments. The apparatus is shown diagrammatically in Fig. 1. The photometer consisted of a two-part field (a circular field about 10°, bisected vertically) one side of which could be converted into a flicker photometer field. Direct comparison could therefore be made of a flickering field against a steady field or the instrument could be used either as a flicker photometer or a direct comparison photometer. *G* and *R* are lamps movable on a two-compartment track whose cross section is shown at *C*. *D* is an end view of the compartments near the sector disk *A*. At *G* and *R* in *D* are placed respectively a blue-green and a red glass over ground opal glasses. These colored glasses whose transmission curves are shown in Fig. 2 were purposely selected nearly complementary to each other in order that little color difference would remain when the lights were mixed and compared with the tungsten standard light *W*. The sector disk *A* rotating about the center *E* causes one side of the photometer field to be a flicker photometer

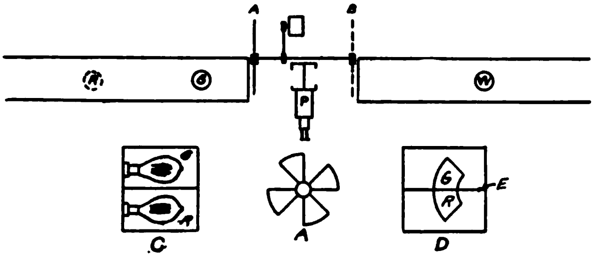

Fig. 1.

Combined Flicker and Direct Comparison Photometers.

field. The sector disk (50 per cent. opening) was covered with black velvet to eliminate reflected light. Tungsten lamps at *W* were used as standards. The lamp *G* was kept in one position throughout the measurements, the balances being obtained by moving either *R* or *W*.

Much of the data obtained by means of this apparatus was for purposes of verification of the outstanding difference in the results by the two methods and a study of the fundamental axioms which must underlie any correct method of photometry. Not all of these data are reported in the present work, some of it having been reserved until other observations can be made. However in order to illustrate more fully the adaptability of this apparatus to the general problem a regular series of observations will be described. First *G* was set at a certain position and with the sector disk removed a series of settings were made in which *G* was balanced against the standard lamp *W* by the direct comparison method. *W* was set at the mean value thus obtained and *R* was balanced against it. *R* was then set at the mean value just obtained and with *G* also lighted and with the 50 per cent. sector *A* now rapidly rotating this mixture was balanced against *W* by varying the position of *W*. Next *R* was balanced against *G* by the flicker method by varying the position of *R*. Finally *R* was balanced against *W* with a red glass interposed between the latter and the photometer, the balance being made by varying the position of *W*. It was then possible to match *G* against *R* by the direct comparison method. The mean values of a number of such series yielded interesting information regarding the two methods.

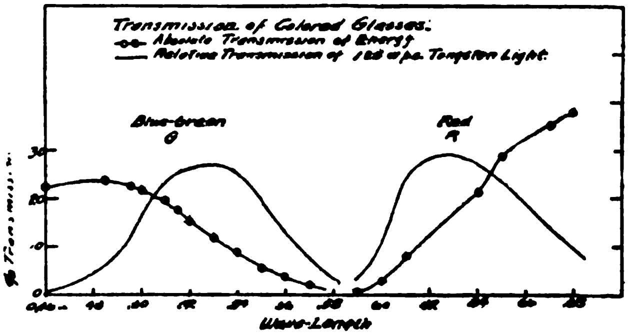

Fig. 2.

The measurements were made in all cases unless otherwise noted, at an illumination of about five meter-candles upon the photometer screen computed from the position of *W*. In the former work[1] with the same photometer it was found that at an illumination above one meter-candle

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[1] Electrical World, March 22, 1913; London Illuminating Engineer, Vol. 6, p. 119.

no complications arise from the Purkinje or reversed effect; that is both methods give the same results as at higher illuminations. However at an illumination somewhat lower than one meter-candle on the photometer screen results varied considerably from the values at higher illumination.

ON MIXING COLORED LIGHTS BY THE FLICKER METHOD AND BY SUPERPOSING STEADY LIGHTS.

An experiment was performed to ascertain if there was any difference in brightness on mixing the two colored lights by the flicker method and by superposing the steady lights. The colors being nearly complementary to each other the color difference in these measurements was small. When the colored lights were mixed by means of a sector disk the speeds of the latter were slightly above and below that at which flicker vanished. The sector disk was also rotated at a very high speed and finally was taken out and the lights permitted to mix by superposition. In all cases the comparison was made against *W* with the same slight color difference. Practically identical results were obtained in all cases showing that there was no difference in brightness when *R* and *G* were mixed by superposing the two colored lights or as in the flicker method providing the flicker was not more than barely apparent. This supports the validity of Talbot's law for colored lights.

ON THE GROWTH AND DECAY OF COLOR SENSATIONS.

It has long been known that the retina responds at different rates to lights of different color. Broca and Sulzer[1] have investigated the growth of luminous sensation for lights of various colors. Some initially far overshoot their final value while others barely exceed their final steady value. It was found that red, white, and blue overshoot considerably while green overshoots scarcely at all indicating that with green light there is either a very slight lag of fatigue behind impression or very slight retinal fatigue. A successful attempt was made to determine the maximum values of flickering lights at various flicker frequencies. A flickering field illuminated by *R* (Fig. 1) was compared with a steady red field and the maximum brightness of the flickering field was measured throughout a wide range of sector speeds. The same was done with the blue-green light *G*. In both cases the intensities of *R* and *G* were those resulting from a balance against *W* by the direct comparison method. These were the mean values of a large number of observations. The curves in Fig. 3 show the values of the maximum brightnesses of the

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[1] Comptes Rendus, 1903, p. 137, 977, 1046.

two flickering colored lights (each flickering against darkness) at various flicker frequencies. It is to be noted from these curves that flicker disappears at a lower speed for the blue-green light *G* than for the red light *R*. With the intensity of *G* unchanged from its foregoing value *R* was balanced against it by the flicker method. The speed of the sector disk necessary in this case was 12 cycles per second as indicated in Fig. 3. It will be noted that the maximum brightness of *R* at this speed is greater than that of *G*. Of course the flicker in each case shown in the curves was *R* or *G* against black while with the flicker photometer it was *R* against *G*. It must not necessarily be supposed that when the intensities of *R* and *G* are such that their flickers against black disappear at the same speed, that they will balance each other by the flicker method. However it appears from Ives's data and also from the writer's data that this is nearly if not absolutely true for the conditions under which these experiments were made. The results of this experiment combined with those which follow indicate quite strongly why the intensity of *R* when balanced against *G* by the flicker method must be decreased from its value as determined by a direct comparison measurement. The results shown in Fig. 3 were verified by many tests and while the eye seemed to vary in its sensibility to flicker very consistent results were obtained considering the difficulty in making such observations. An attempt was made to obtain the minimum brightness of the flickering lights but it was found that the maximum brightness was always so distracting in its effect that attention could not be riveted upon the minimum value. The curve representing minimum values would obviously start at zero

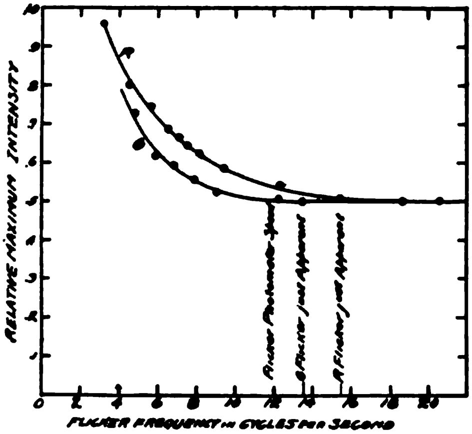

Fig. 3.

Maximum Brightness of a White Surface Illuminated by a Flickering Colored Light.

and finally reach a steady value which would be, as in the case of maximum values, one half of the steady maximum value. The latter statement assumes the validity of Talbot's law for colored lights which was verified in this work. The effect of change of intensity is shown in Fig. 4. Here the intensities of the red and blue-green lights are approximately those determined by the mean of the results obtained by the two methods. It is seen that at the higher illumination there is an overshooting of the steady value which is taken as unity with the sector stationary and open. This overshooting is greater for the red light than for the blue-green light. As the illumination decreases the over-shooting becomes less marked. This is to be expected from the work of Broca and Sulzer which work however was done apparently in a different manner and without special consideration from the standpoint of flicker photometry.

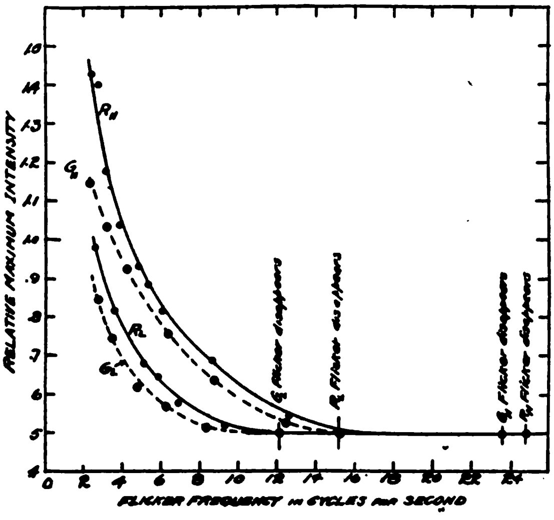

Fig. 4.

Showing the Effect of Intensity on the Maxima of a Flickering Light. Full Line Represents Red Light. Broken Line Represents Blue-Green Light. Subscripts Indicate High and Low Intensities.

EFFECT OF AFTER-IMAGES.

An arrangement, which is shown diagrammatically in Fig. 5, was devised to study the effect of after-images. Steady sources G_1 and R_2, respectively blue-green and red in color, as used throughout the work, were arranged to illuminate opposite sides of the photometer screen. Illuminations due to R_1 and G_2 of the same colors as the foregoing lights were arranged to be flickered by means of identical 50 per cent. sector disks, S_1 and S_2, rotating on the same shaft. By this means it was possible to flicker red light from R_1 on a steady blue-green field due to G_1 and compare this total brightness with that resulting from flickering blue-green light from G_2 on a steady red field due to R_2.

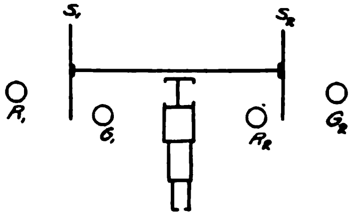

Fig. 5.

Apparatus for Superimposing a Flickering Colored Light Upon a Steady Light of Different Color.

All observations were made without color differences. The procedure was as follows: R_1 and G_2 were taken the same as previously determined by the direct comparison method, then R_1 was balanced against R_2 with G_1 and G_2 extinguished, the sectors being open and stationary. Next R_1 and R_2 were extinguished and G_1 was balanced against G_2. Finally with all four lights turned on and with the sector disks rotating at a speed such that flicker was barely apparent the photometer field remained balanced as might be expected. However as the speed was reduced the side of the photometer field upon which R_1 was flickering apparently became much brighter; at least the maximum value of the brightness was considerably greater. Flicker upon this side of the photometer was very violent while upon the other barely apparent. A great reduction in the speed was necessary before the flicker upon the other side became very marked. In fact the illumination on the side where the blue-green light flickered appeared quite uniform as the speed was reduced long after the side on which the red light flickered, became violently agitated. This same phenomenon was noticeable to a much less degree after the steady lights were extinguished.

Next, by combining certain features of the apparatus shown in Figs. 1 and 5, actual measurements were made of the maximum brightness attained by the red light flickering upon the steady green field and vice versa, the comparison source being a clear tungsten lamp W. The results are shown in Fig. 6. The intensities of the steady lights were twice the mean values of the flickering lights. The flicker photometer speed at this illumination would be about thirteen cycles per second in comparing the particular red and blue-green lights. When R and G were of such values as determined by the flicker method the above effects would have been greatly decreased in magnitude. In fact one could not be certain that under these conditions there was any difference in the flicker of the two fields. This experiment gives a strong clue to the reason for decreasing the intensity of the red light when it is balanced by the flicker method against blue-green light. The much greater agitation of the field consisting of a flickering red light and a steady blue-green light points very strongly to the conclusion that the red light is weighted more by the flicker method than by the direct comparison method because in order to obtain a balance the intensity of the red light must be diminished so that the flicker of *R* and *G* disappear at about the same speed. These data shown in Figs. 3 and 6 represent extreme experiments between which will be found the operation of the flicker photometer.

The difference in the rate of growth of sensation stimulated by lights of different color suggests the possibility that flicker photometers differing in design might not yield concordant results. The flicker photometer shown in Fig. 1 is of such nature that when balancing red light against blue-green light the color of the photometer field varies gradually from red through white to blue-green then to white and so on. In most flicker photometers the stimulus changes abruptly from one color to another. In the previous work[1] a Schmidt and Haensch flicker photometer was used. With that instrument the results obtained were not exactly the same as those obtained by the flicker photometer used in this

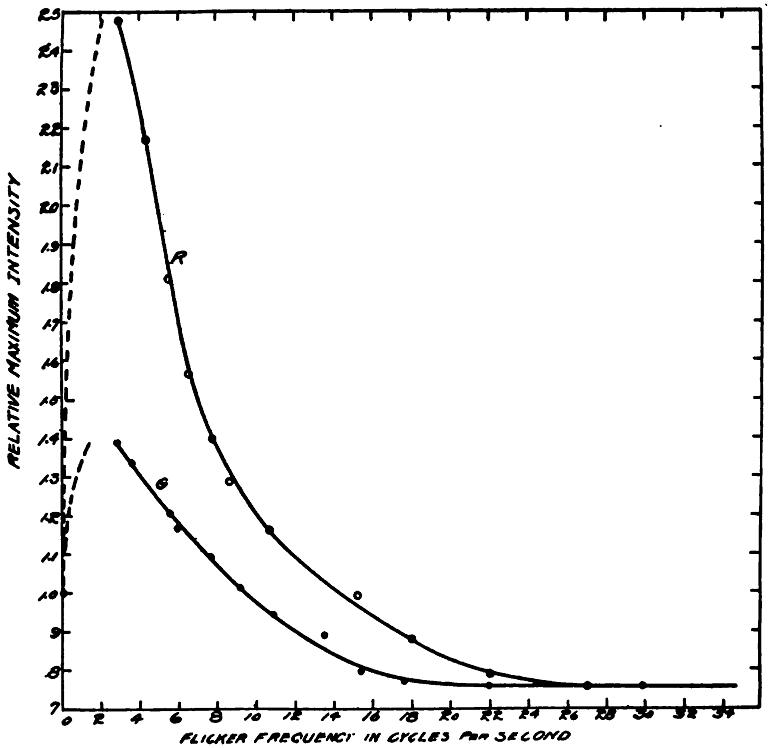

Fig. 6.

Maximum Brightness of a White Surface Illuminated by Superimposed Red and Blue-Green Lights, One of Which is Flickered. *R* Indicates Red Flickering on Steady Blue-Green Field. *G* Indicates Blue-Green Flickering on Steady Red Field.

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[1] Electrical World, March 22, 1913; London Illuminating Engineer.

work and reported in the first part of this paper. The writer cannot say that the difference in the results (which was not great) is due to the difference in the design of the photometers. However the effect of contour or wave-form of flicker may be of importance in flicker photometry.

On Vanishing-flicker Frequency.

The vanishing-flicker frequency (often referred to as critical frequency) is that frequency at which flicker vanishes when the light is rapidly alternated with darkness. This has long been known to vary with the intensity of the flickering light or brightness of the target. Kennelly and Whiting[1] state that "the vanishing-flicker frequency does not depend upon the mean illumination on the target: or at least only to a relatively small degree. It depends on the maximum and minimum cyclic illumination." They also conclude that "the vanishing-flicker frequency does not depend appreciably upon the wave-shape of flicker, that is, upon the manner in which the illumination varies in passing between the maximum and minimum cyclic values." As the apparatus diagrammatically shown in Fig. 1 readily lent itself to a study of this problem some experiments were carried out in order to determine whether wave-form of flicker is sufficiently important to bear investigation in regard to its effect in flicker photometry. The experiments were made with clear tungsten light. It has been established by Porter[2] that $f = k \log I + p$ represents the relation between critical frequency, f, and the illumination, I, where k and p are constants. In Fig. 7 the writer's results obtained with flickering lights of different contours of flicker. In cases *a*, *b*, and *c* the maximum, minimum, and mean cyclic values of illumination were respectively the same. In case *d*, for a given mean cyclic illumination the maximum illumination was 4.5 times that in cases *a*, *b* and *c*. The mean illumination on the photometer screen in meter candles is plotted logarithmically. Similar to what was found by Porter one straight line relation is seen to hold down to a low illumination where another straight line relation seems to begin. The location of this point of course depends upon the absorption of light in the photometer used and in these cases seems to depend somewhat upon the contour of flicker although the points where the lines of lesser slope begin are not well defined. The data shows that critical frequency varies even with equal values of maximum and minimum illumination. Further on comparing *a*, *b* and *c* it is seen that the contour of flicker determines the critical frequency. On comparing *d* with the other cases it is seen that the critical frequency at a given mean illumination is much higher when the period of darkness is greater or

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[1] Proceedings of the National Elec. Light Assn., 1907.

[2] Proc. Roy. Soc., LXXIX., 1902, p. 313.

where the maximum value is greater. Further it is seen that the shorter the period of darkness the lower is the critical frequency at a given mean illumination when the maximum and minimum values of the flickering lights are the same. In other words the more abruptly does the stimulus rise in value the greater is the critical frequency. It is thus seen that the contour of flicker has a highly important bearing upon the critical frequency. These results contradict the statements quoted from Kenelly and Whiting. From these data it is seen that critical frequency is influenced by the contour of flicker, and the maximum, minimum and mean values of illumination during the cycle.

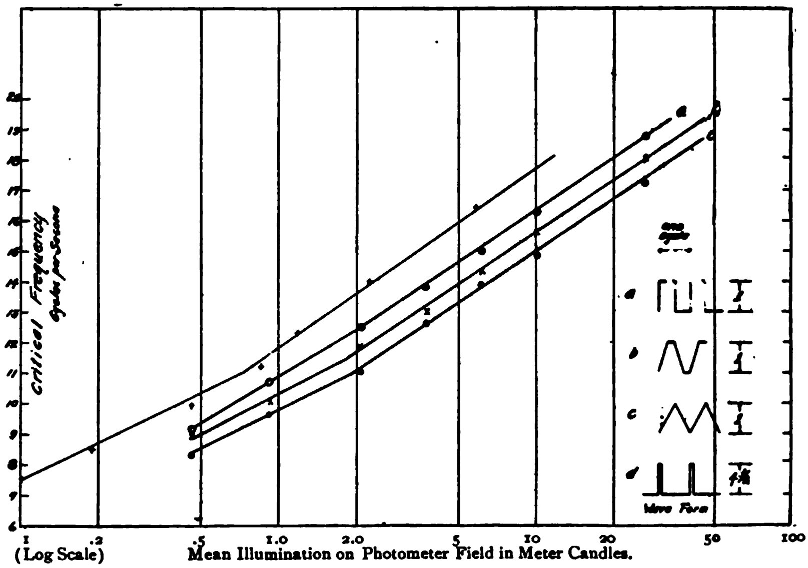

Fig. 7.

Effect of Wave-Form of Flickering Lights on Critical Frequency.

SUMMARY.

1. Red and blue-green lights add in the same manner whether by direct superposition or by alternately flickering them as in the flicker photometer when the speed is not lower than that at which flicker is barely apparent.

2. The maximum value of a flickering red light is always greater than that of a blue-green light flickering in the same manner and at the same frequency, when the steady values of the intensities of these two lights are those determined by a direct comparison balance. This is very much more marked when a red light is flickered upon a steady blue-green field than when a blue-green light is flickered upon a steady red field. In the latter case the agitation of the field which consists of a red light flickering

upon a steady blue-green field is much more marked than in the opposite case at practically all speeds below the critical-frequency speed. This seems to account for weighting red light more than blue-green light by the flicker method than by the direct comparison method.

3. The speed at which flicker disappears is much higher for the red light in the foregoing cases just noted than for blue-green light of the same intensity as determined by comparing the two lights by the direct comparison method.

4. It is shown that the contour of flicker has considerable influence upon the critical frequency which is the frequency at which flicker vanishes at a given illumination when the light is alternated with darkness.

The writer is indebted to Mr. Leonard Krill and Mr. Abe Shapero for assistance in the experimental work and Mr. H. MacMullen for the preparation for the drawings.

APPENDIX.

An interesting experiment the results of which point with favor to the flicker photometer is found in comparing lights of the same color but differing in spectral character. By means of dyes two yellow lights identical in color were produced. One was a pure yellow while the other was a subjective yellow composed of green and red light. These were balanced by the direct comparison method very readily because there was no color difference. When balanced by the flicker method it was found that the results were the same as obtained by the direct comparison balance within at least one per cent. The same experiment was performed with "white" lights, one being the continuous spectrum light from a tungsten lamp, the other being made by mixing red and blue-green lights. Here again the results were practically the same by the two methods. These experiments indicate quite strongly that the flicker photometer is not effected by the difference in the rates of growth and decay of color sensations. Of course other combinations of colored lights to produce two lights of the same color but differing in spectral character might give different results by the two methods. If no differences are found it might be fair to assume that the flicker photometer measures true brightness and that the differences in the results by two methods are entirely attributable to the effect of simultaneous contrast and other phenomena involved in the direct comparison method. The latter method does not satisfy the fundamental axioms already mentioned which is a serious cause for looking upon this method with disfavor when seeking a reliable method of photometry for the standardization laboratory. In summing up the evidence it appears that for a laboratory method the balance is in favor of the flicker photometer.

THE MAINTENANCE OF VIBRATIONS.

By C. V. Raman.

Part I.

In my paper on "Some Remarkable Cases of Resonance" published in the Physical Review for December, 1912, I described a new class of forced vibrations which form apparent exceptions to the general principle of resonance, i. e. in respect of which we find systems exhibiting marked resonance under the action of forces whose periods do not necessarily stand to their own in a relation of approximate equality. Of this class, the first type is the well-known case of double frequency exemplified by the form of Melde's experiment in which the vibrations of a stretched string are sustained by the action of a varying tension imposed by a fork having double the frequency of the string. My experiments showed that the vibrations could be maintained in the following cases:

(1) When the frequency of the fork is 2 times that of the string;
(2) When the frequency of the fork is $\frac{2}{2}$ times that of the string;
(3) When the frequency of the fork is $\frac{2}{3}$ times that of the string;
(4) When the frequency of the fork is $\frac{2}{4}$ times that of the string;
(5) When the frequency of the fork is $\frac{2}{5}$ times that of the string;
And so on.

Photographs illustrating the modes of vibration in each of these cases were published with the paper.

Some very interesting phenomena are noticed when a stroboscopic disk is used in observing these types of maintained motion. A Rayleigh synchronous motor on which is mounted a blackened disk with narrow radial slits cut in it is very suitable for this purpose. One of the disks which I use has thirty slits in it, the armature-wheel of the motor having the same number of teeth. The electric current from the self-interrupter fork which maintains the string in vibration also runs the synchronous motor. In making the observations, the stroboscopic disk is held vertically and the string which is set horizontal and parallel to the disk is viewed through the top row of slits, *i. e.*, those which are vertical or nearly so and move in a direction parallel or practically parallel to the string as the disk revolves. It is advantageous to have the whole length of the string brilliantly illuminated and to let as little stray light as possible fall upon the reverse of the disk at some distance from which the observer takes his stand. A brilliant view is then obtained. Under these circumstances, we see the string in successive cycles of phase along its length, and the peculiar character of the maintained motion in these cases is brought out in a very remarkable way. *The string is seen in the form of a vibration-curve,* which would be identical with those obtained by other methods but for the fact that the amplitude of motion is not the same at all points of the string, being a maximum at the ventral segments and zero at the nodes.

If we use a fork with a frequency of 60 per second, the *free* oscillations of the string should have a frequency of 30 in the case of the first type, 60 in the case of the second, 90 with the third, 120 with the fourth, 150 with the fifth, and so on. With the disk having 30 slits on it we get 60 views per second of any one point on the string, and with the even types of motion, *i. e.*, the second, fourth, etc. the 'vibration-curve' seen through the stroboscopic disk appears single. Figs. 3 and 7 show the curves for the second and fourth types respectively. With the odd types on the other hand, *i. e.*, with the first, third, fifth, etc., *two* vibration-curves are seen, one of which is as nearly as can be seen the mirror-image of the other, intersecting at points which lie or should lie upon the equilibrium position of the string. Figs. 1 and 5 show the curves for the case of the first and third types. The reason why with the odd type we see the vibration-curve double is fairly obvious and forms an excellent illustration of the principles of stroboscopic observation. The double pattern in the case of the third and higher types brings home to the eye in a very vivid and convincing manner the fact that under the action of the variable spring the "amplitude" and "period" of the motion periodically increase and decrease after the manner of "beats."

An interesting variation on the experiment is made by using a disk with 60 slits. We then get 120 views per second and with the even types we get the vibration-curves double, but one of the curves is not the mirror image of the other, the motion not being symmetrical with respect to the position of equilibrium. On the other hand, with the odd types we see the vibration-curves in quadruple pattern, the third and fifth types in particular giving noteworthy effects. Figs 2, 4, 6, and 8 exhibit the patterns obtained in this manner with the first, second, third and fourth types respectively.

Before leaving this subject, it is worth while to remark that the special method of stroboscopic observation described above can be very effectively applied to the study of the vibrations of stretched strings produced or maintained in any other manner, *e. g.*, by bowing or striking.

The vibrational forms for various points over the entire length of the string can all be observed together simultaneously with the apparatus, and this is often a very great convenience. For example, the widely-differing ratios between the forward and backward velocities at various points on a bowed string can all be seen together and compared in one experiment, and the effect of applying the bow at different positions, of reversing its direction of motion, or of removing it altogether and allowing the motion to die away gradually can be very conveniently studied. It is necessary in such work that the string should be tuned to the correct frequency in the first instance (by observation through the stroboscopic apparatus) so that a stationary figure may be obtained.

PART II.

A stroboscopic disk with radial slits mounted on a synchronous motor can be very effectively used in observing the peculiar properties of the small motion at the nodes of a vibrating string described by me in a previous publication in this REVIEW.[1] In this case an arrangement somewhat different from that described in Part I. of the present paper should be used. An electrically-maintained fork maintains the string in oscillation in any convenient number of loops by imposing a *transverse* obligatory motion at one point on it. The current passing through the fork also drives the synchronous motor on which is mounted a stroboscopic disk having just double as many apertures as the armature-wheel has teeth. The disk therefore gives two views of the fork and of the string maintained by it, which are practically stationary provided the point of observation is fixed and the motor is running satisfactorily. In the present case it is essential that the position taken up by the observer or the camera should be quite close to the stroboscopic disk and should be so chosen that the slits through which the vibration is observed are as nearly as possible parallel to the string and move in a direction at right angles to its length. The most convenient plan is to have the string horizontal and the plane of its vibration vertical. The disk should then be set vertically and the observations made through the region of the disk in which the radial slits are horizontal or practically so. By changing the point of observation, successive phases of the motion and the periodic "travel" of the "nodes" over a large horizontal range seen under the intermittent illumination can all be observed at leisure.

For photographic work, the camera employed is brought up close behind that one of the slits on the disk which is horizontal. The lens is stopped down by a plate which has a rectangular slit cut in it so as to

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[1] "The Small Motion at the Nodes of a Vibrating String," PHYS. REV., March, 1911.

correspond with those on the disk. By racking up the lens-front of the camera by successive small distances till it has moved through a length equal to that between contiguous apertures on the disk, a complete set of photographs can be obtained on one plate showing successive stages of the motion of the string. Fig. 9 reproduces a photograph obtained in this manner and showing the cycle of changes in 13 stages. It will be seen that the point of intersection or "node" which is first in the center moves off to one side of the field, first slowly and then more quickly, till after the lapse of a time which can be seen to be exactly half the period of the cycle, it has gone well off the plate and the positions of the string seen in the photograph are sensibly parallel. Direct observation shows that the point has moved off to a great distance, in fact to a distance equal to half the length of a ventral segment. It simultaneously appears at an equal distance on the other side and moves in from that direction first quickly and then more slowly till it reaches the center again and the cycle is complete.

From Fig. 9, it is quite obvious that the phase of the small motion at the node differs by quarter of an oscillation from that of the large motion on either side of it at a distance.

Part III.

A very neat modification of Melde's experiments can be arranged by attaching one extremity of a fine cotton or silk string to a prong of an electrically maintained fork held so that the string lies in a plane perpendicular to the prongs but in a direction inclined to their line of vibration.[1] In view of the ease with which the experiment is performed and the great beauty of the form of oscillation maintained (this must be seen to be fully appreciated), the following brief discussion of the results may be of interest.

The obligatory motion imposed at one extremity of the string may be resolved into two components, one parallel and the other perpendicular to the string. The two components may be put respectively equal to $\gamma \cos pt \cos \theta$ and $\gamma \cos pt \sin \theta$. The transverse component maintains an oscillation having the same frequency as that of the fork and having an even number of ventral segments when the tension of the string is suitably adjusted. The longitudinal component will then generally be found to maintain an oscillation having half the frequency of that of the fork. The success of the experiment lies in isolating the two vibrations, the frequency of one of which is double that of the other, into perpendicular planes. This is easily secured by a simple little device. The

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[1] See Phys. Rev., Vol. XXXII., page 311.

end of the string is attached to a loop of thread which is passed over a prong instead of directly to the prong itself. The result of this mode of attachment is that the frequencies of vibration in the two planes at right angles differs slightly and this has the desired effect of keeping the component vibrations confined to their respective planes, if the tension of the string lies anywhere within a definite range.

If the distance of any point from the fixed end when at rest is x, the transverse components of the maintained motion may be written as under

$$Y = \gamma \sin \frac{\theta R_x}{R_b} \cos (pt + E_x - E_b), \tag{1}$$

vide Lord Rayleigh's Theory of Sound, Art. 134.

$$Z = B \cos \left(\frac{pt}{2} + E\right) \sin \frac{\pi x}{b}, \tag{2}$$

the values of B and E being ascertained from the investigation given in the PHYSICAL REVIEW, Vol. XXXV., page 451. If we exclude any consideration of the motion at points near the nodes of the maintained oscillation, equation (1) may be written in the simple form

$$Y = \gamma \sin \theta \sin \frac{px}{a} \cos (pt + E').$$

If $p/a = 2\pi/b$, (1) and (2) may be written in the form

$$Y = A \sin \frac{2\pi x}{b} \cos (pt + E'), \tag{3}$$

$$Z = B \sin \frac{\pi x}{b} \cos \left(\frac{pt}{2} + E\right). \tag{4}$$

It should be understood that in these equations Y and Z do *not* refer to the coördinates of any point fixed relatively to the string but to the points at which a plane transverse to its equilibrium position cuts the surface generated by the moving string. The distinction is of importance in view of the fact that each point on the string possesses a small longitudinal motion derived from that imposed by the fork and the x coördinate of any point fixed relatively to the string is therefore not itself constant.

In particular cases equations (3) and (4) may be reduced to very simple forms. Thus if $E' = 2E$ and also in the special case when both E and E' are equal to zero

$$\frac{Y}{A} \operatorname{cosec} \frac{2\pi x}{b} = \frac{2Z^2}{B^2} \operatorname{cosec}^2 \frac{\pi x}{b} - 1, \tag{5}$$

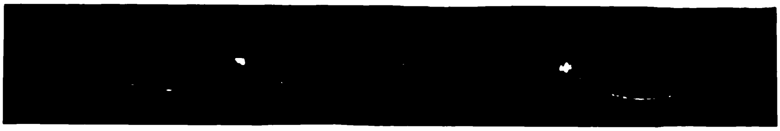

FIG. 1.

FIG. 2.

FIG. 3.

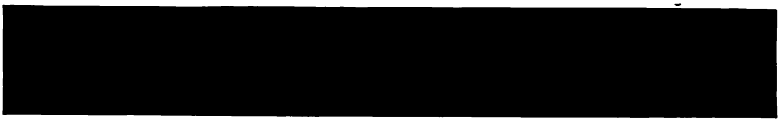

FIG. 4.

FIG. 5.

FIG. 6.

FIG. 7.

FIG. 8.

FIG. 9.

which is the equation of the surface generated by the moving string, the sections of which by planes perpendicular to the axis of x are parabolic arcs. The curvatures of the arcs are in opposite direction in the two halves of the string. When $E' = 2E + \pi$, a parabolic type of motion should also be obtained. This case can be realized approximately by having the tension of the string as low as possible consistent with the maintenance of the motion. In the intermediate case when $E' = 2E + \pi/2$, the sections of the surface described by the moving string elsewhere than at its center are 8 curves. At the center, the curve is a very flat parabolic arc, as the phase of the small motion at the node differs by quarter of an oscillation from that of the rest of the string.

Indian Association for the Cultivation of Science,
Calcutta, India.

THE LUMINESCENCE OF KUNZITE.

BY E. L. NICHOLS AND H. L. HOWES.

THAT the variety of spodumene known as kunzite is phosphorescent has been known almost from the day of its discovery. In August, 1903, Geo. F. Kunz[1] described its occurrence and properties and in September of that year Baskerville[2] who was making an exploration of the optical properties of the minerals in the American Museum of Natural History proposed the name of kunzite for this substance and in the course of his description stated that although it showed no sign of fluorescence or phosphorescence when exposed to ultra-violet light, X-rays excited it to a prolonged whitish phosphorescence.

Kunz and Baskerville[3] subsequently tested the luminescence of numerous minerals and reported that kunzite was wonderfully phosphorescent under the action of radium; that ultra-violet light, contrary to Baskerville's first observation, excited certain specimens to prolonged luminescence and that X-rays affected all the crystals of that mineral which were examined.

A writer in Nature, presumably Mr. F. Soddy,[4] speaks of the excitation of kunzite by radium as due chiefly to the β rays and mentions also the powerful excitation obtained by kathodo-bombardment.

Pochettino[5] in his extended investigation of the luminescence of minerals included the spodumenes and in particular certain samples of kunzite. He found this substance feebly fluorescent under the action of light with the emitted rays polarized.

Under the kathode rays he was able to observe a spectrum consisting of a very strong broad band extending from λ .690 to λ .515 and a weak one from .48 to .42. The band of longer wave-length was of protracted phosphorescence while that in the blue disappeared "instantly" upon cessation of the excitation. Pochettino noted further the phenomenon of thermo-luminescence at 235° C.; in which case the rays were not polarized. The power of emission ceased at about 400° C.

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[1] Kunz, Science, 18, p. 280 (1903).

[2] Baskerville, Science, 18, p. 303 (1903).

[3] Kunz and Baskerville, Science, 18, p. 769 (1903).

[4] "F. S.," in Nature, 69. p. 523 (1904).

[5] Pochettino, Il Nuovo Cimento (V), 18, p. 245 (1909), and (VI), 1, p. 21, 1911.

It was the purpose of our experiments to study more in detail the location and character of these bands, to seek for the absorption bands which are generally if not universally associated with fluorescence and to determine the law of decay.

FLUORESCENCE.

The crystals at our disposal appeared to be of two varieties: (1) Those with the pink or rose-lilac hue generally considered to be characteristic of kunzite and (2) crystals nearly colorless but with a suspicion of a green- or aqua-marine tint.

Under the action of kathode rays both were strongly fluorescent but the colors differed. The pink crystals glowed with the powerful yellow-red light described by previous observers, which resembles nothing more closely than the light from some transparent body, as quartz or glass at a temperature of about 900° C. The colorless specimens were of a rose-violet hue.

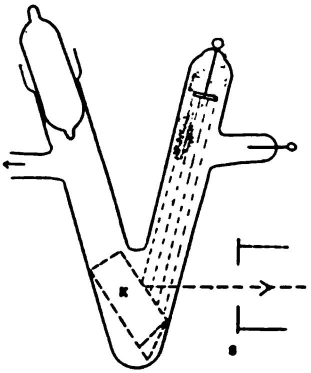

Fig. 1.

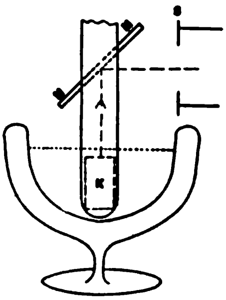

Fig. 2.

Observed with a hand spectroscope this difference is seen to be clearly due to the relative strength of the fluorescence bands. In the case of the colorless crystals the blue band is much stronger and its intensity is such as to notably modify the color of the fluorescent light.

The specimen (*K*) to be studied was placed in a V-shaped vacuum tube as shown in Fig. 1 and the face, bombarded from above by kathode rays, was observed with a spectrophotometer, the slit of which is shown at *S*.

The strong broad band lying between .7 μ and .5 μ was readily measured spectrophotometrically and was found to have the well known and typical form observed in numerous other fluorescent substances and depicted in previous papers.[1]

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[1] Nichols and Merritt, Phys. Rev. (1), Vols. XVIII., XIX. and XXI.

The curve of distribution of intensities (*B*, Fig. 3) is as usual nearly symmetrical but somewhat steeper towards the violet. The well-defined crest is at .590 μ. There is nothing to suggest that it is a composite of overlapping bands.

To determine the effect of cooling which is known in many cases to modify the character of the fluorescence spectrum, the tube was immersed in liquid air to a level above that of the crystal as shown in Fig. 2. In this case observations were made by placing a mirror *M* between the arms of the vacuum tube so as to reflect the fluorescent light, emanating vertically from the crystal, into the collimator slit at *S*.

The appearance of the specimen when thus cooled is distinctly redder to the eye and this impression is verified by the spectrophotometric measurements. The curve (*A*, Fig. 3) which is somewhat narrower, has its crest shifted towards the red to .598 μ and indicates accession of the longer wave-lengths together with marked diminution on the violet side. There is no indication of even a partial resolution into a group of narrow bands.

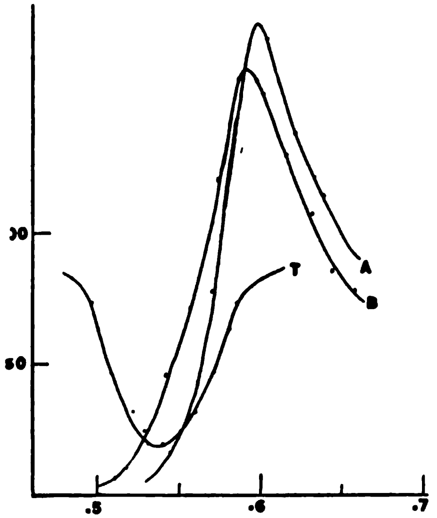

Fig. 3.

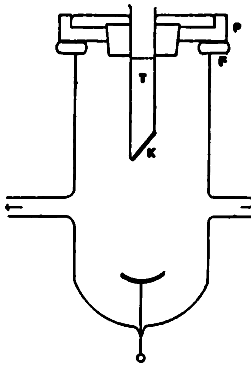

Fig. 4.

That a crystal, thus situated in vacuo within a glass tube and subjected to the heating effect of even a very moderate kathodic bombardment, will be in thermal equilibrium at some temperature considerably above that of the liquid air surrounding the tube is obvious. That this difference of temperature may be more than 100° C. is indicated by previous experience with substances subjected to somewhat similar conditions.

In a case which was described in a paper presented before the American Physical Society, the character of the spectrum afforded a sufficient criterion on which to base this estimate.

A modification of the apparatus enabled us to determine the fact that even at a temperature really approaching that of liquid air no further marked modification in the position or appearance of the band occurred. For this purpose a small piece of the kunzite was powdered and cemented to the outer surface of a test-tube the bottom of which had been softened by heat and obliquely flattened so as to form a convenient target for kathode rays. This test tube was mounted within a vacuum tube of the form shown in Fig. 4 in which *P* is a cast-iron plate ground to the flange *F* of an open-mouthed glass tube having a kathode at the bottom. The test tube *T* was inserted through a rubber stopper fitted to a central opening in the iron plate. When a proper vacuum had been attained liquid air was poured into the test tube cooling the coating of powdered kunzite at *K*. The fluorescence of *K* could be observed in the manner previously described and any change in the character of the spectrum noted. The measurements while less satisfactory than those indicated in Fig. 3 were in essential agreement with them.

The fluorescence band of shorter wave-length which begins at .49 μ and extends into the violet, is less favorably situated for study with the spectrophotometer and we contented ourselves with an estimation of its extent and general character by a photographic method. Spectrograms were taken, using both the pink and white crystals, upon two different plates so as to lessen errors due to the selective sensitiveness of the photographic film. The negatives indicated that the same bands were present in both cases but, as had already been noted by visual observations, were of very different relative brightness. The blue band was found to lie between .49 μ and .40 μ with a crest at approximately .432 μ. Since the location of the crest by visual inspection was rather uncertain one of the negatives was measured as to density, step by step at intervals of 1 mm. by means of a photoelectric cell. This determination which was kindly made for us by Mr. W. G. Mallory gave boundaries and location of the crest in excellent agreement with our visual estimates. The form of the curve indicated, as in the case of the red band, that if complex this band must also consist of close and completely overlapping components. This band is shown graphically in Curve *B* of Fig. 5, in which however the relative intensities of the two bands are only approximately indicated.

To determine the strength of the two bands observations were made with the spectrophotometer; the brightness of the crests of the red and blue bands of the white and the pink kunzite under kathodo-excitation being compared respectively with the corresponding regions in the spectrum of the acetylene flame. It was found that when the crest of the band at .690 μ in the case of the pink kunzite was made equal in brightness to the same region in the acetylene spectrum, the intensity of the crest of the blue band at .432 μ was about 43 per cent. of that of the corresponding region in the acetylene spectrum whereas the white crystal under like conditions had a blue band about 2 per cent. brighter than the acetylene spectrum. In other words when the red bands of the two varieties are equally bright the blue band of the white kunzite is about 2.33 times as intense as that of the pink variety.

Absorption.

By analogy with numerous substances previously studied one would expect to find absorption bands in kunzite associated with the two fluorescence bands, of approximately the same width and displaced towards the violet. Looking through a thickness of several millimeters of the substance, however, with a hand spectroscope of small dispersion nothing of the kind is perceptible. The crystal appears to be of about the transparency of ordinary white glass and as free from selective absorption. Looking lengthwise through a crystal of considerable size which afforded an absorbing layer of about 5 cm. the color of the transmitted light is distinctly purple and the spectroscope reveals a rather vague band which is by no means opaque even in the center. Spectrographs bring it out clearly but are not suitable for measurement because of the selective absorption of the film, which in all the plates at our disposal has a region of more or less pronounced weakness which overlaps this band on the side towards the violet. By spectrophotometric meas-

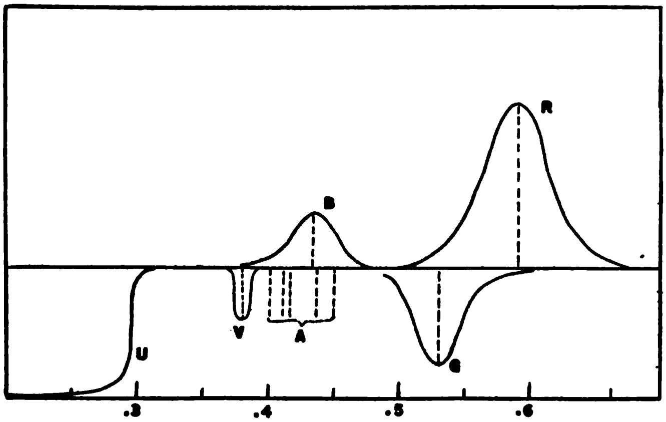

Fig. 5.

urements this band was easily mapped. It extends from .64 μ to .49 μ with a well-defined crest at .538 μ and, as may be seen from Curve *T*, Fig. 3, which depicts approximately the transmission of the crystal, it overlaps the violet edge of the fluorescence band *B*.

Photographs show a series of narrow absorption bands (*A*, Fig. 5) at .4019, .4117, .4170, .4375 and .4501 μ. These have a width of only about 70 Ångström units. Beyond .4 μ this thick crystal layer became rapidly opaque making the photographic exploration difficult. By placing the crystal transversely across half of the slit of a quartz spectrograph; the transmitting layer being about 4 mm., it was possible to obtain photographs extending to .3 μ, at which point even this thin layer became abruptly opaque as is indicated by Curve *U* in Fig. 5. Such photographs however failed to show the absorption bands already detected and located.

To determine if possible whether a broad band associated with the blue fluorescence existed, a series of spectrograms through crystals of such intermediate thickness as were available were made. One of these photographs, taken through a layer of 15 mm. revealed such a band lying between .3750 and .3875 μ with a crest at .380 μ. This band which lies in the usual relation to the blue band is shown at *V* (Fig. 5).

Fig. 5, to which reference has been made, is intended to depict in a rough way the fluorescence and absorption of kunzite in so far as is possible from the present study. In the diagram the fluorescence bands (*R* and *B*) are shown above the base-line and the absorption bands *A* and *V* below. Whether a group of narrow fluorescence bands associated with the group *A* exists we have not thus far been able to determine. If so they would probably be comprised within the band *B*, and unless very brilliant, would be indistinguishable.

PHOSPHORESCENCE.

The phosphorescence of kunzite has certain interesting characteristics. While it is of comparatively long duration being easily discernible for several minutes after adequate excitation, decay is at first of extraordinary rapidity so that measurable brightness remains only in cases where the initial intensity of excitation is very great.

To determine the law of decay a simple photometer was improvised. It consisted of a small Lummer-Brodhun cube *L*, Fig. 6, mounted at the junction of two short tubes *C* and *D*. Light from the phosphorescent body *K* passed through a yellow screen *S* and through the central transmitting field of the cube to the eye which was focused upon the interface by means of an eyepiece of low power at *E*. The reflecting field was illuminated by an acetylene flame with diaphragm (*F*) which was mounted upon a photometer bar (*BB*) two meters in length. The brightness of this comparison field was reduced by the interposition of a dense milk-glass screen at *M* and a sufficient color match was secured by means of an amber screen at *Y*.

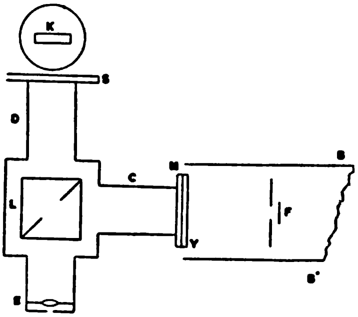

Fig. 6.

This arrangement had a range of about 180 to 1 and by substituting for the dense milk glass a ground glass at *M*, this could be increased to 18,000 to 1.

With the milk glass screen that part of the curve of decay from about 0.4 second after close of excitation to 60 seconds could be determined. The times of beginning and close of excitation were recorded on a chronograph and by shifting the comparison flame to successive predetermined positions on the bar and recording the times when a photometric balance was passed for each position ten or twelve points on the curve were obtained after a single excitation of the crystal.

The tube containing the kunzite crystal was connected to a Gaede mercury pump in continuous operation and under these conditions a reasonable constancy of intensity of excitation was secured.

Curve *D* in Fig. 7 is a typical example of the numerous curves obtained with this apparatus. It shows only a small portion of the period of rapid decay immediately following the end of excitation. To include the record of the first half second of time it would be necessary to extend the ordinate of intensities about three hundred times the height of the existing diagram. Using the reciprocal of the square root of intensity as ordinates the data give the linear branches 2 and 3 in the same figure.

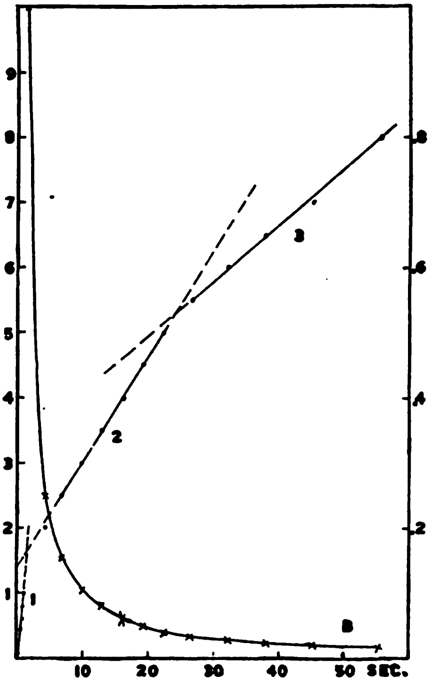

Fig. 7.

As is well known it has been customary to describe the decay curves of Sidot blende, willemite and numerous other substances, thus plotted, as composed of two such linear processes.[1] Lenard[2] and others have contested the linear character of the initial process although the determinations of Waggoner[3] and of Zeller[4] whose measurements covered the first few hundredths of a second, an interval impossible to observe by the usual methods, showed a linear relation between $I^{-1/2}$ and time.

In the case of kunzite the line marked "2" in the diagram clearly does not represent the initial process of the decay of phosphorescence for its intercept corresponds to an intensity far below the brightness of the substance at the close of excitation. Observations made six tenths of a second after the beginning of decay consistently gave measured intensities much greater than this intercept would indicate.

Since the excitation was by kathodo-bombardment it was easy to measure the initial brightness directly by substituting the ground glass screen and thus increasing the brightness of the comparison light one hundred times. It was also possible with this screen to determine the brightness for times ranging between 3/10 sec. and 6/10 sec.

These values plotted for $I^{-1/2}$ and time lie in a straight line which passes through the intercept for the initial brightness as measured and constitute the process marked (1) in Fig. 7. Since this process gives the true value of the intensity at the close of excitation it may be termed the initial process. It is shown on a larger scale in Fig. 8.

This result is quite in accordance with the experiments of Waggoner and of Zeller whose initial process was also linear and of a steeper slope than what had been taken for the first process in previous studies of decay of long duration.

Kunzite differs from many phosphorescent substances in its comparative inactivity to photo-excitation. A spark discharge between iron or zinc terminals sufficient to leave willemite, Sidot blende, and numerous other substances brilliantly phosphorescent has so feeble an effect on kunzite that special precautions are necessary to enable any after effect whatever to be discerned. Baskerville and Kunz[5] noticed photo-phosphorescence in the case of the white variety only and not at all in pink crystals. The latter, however, are capable of excitation as may be determined by passing a bundle of rays from the carbon arc or mercury lamp through a light

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[1] Nichols and Merritt, Phys. Rev., XXV., p. 362; C. A. Pierce, Phys. Rev., XXVI., pp. 312 and 454.

[2] Lenard, Annalen der Physik, XXI., p. 641 (1910).

[3] C. W. Waggoner, Phys. Rev. (1), XXVII., p. 209.

[4] Carl Zeller, Phys. Rev. (1), XXXI., p. 367.

[5] Baskerville and Kunz, l. c.

filter transparent only to blue and the shorter waves, and then through the crystal to be examined. The ruddy path of the beam is clearly discernible particularly when viewed through an amber or yellow glass. The exciting effect of the individual lines of a mercury arc spectrum from .4359 μ to .3663 μ can also be detected with proper precautions for the exclusion of stray light.

The appearance of dichroism in the pink variety is probably due, in part at least, to its fluorescence. The disappearance of color when the crystal is viewed through a Nicol prism occurs at an angle for which the fluorescent light which as Pochettino[1] has shown is polarized, would be cut off. The pink color disappears, moreover, when the crystal is illuminated by rays deficient in exciting power.

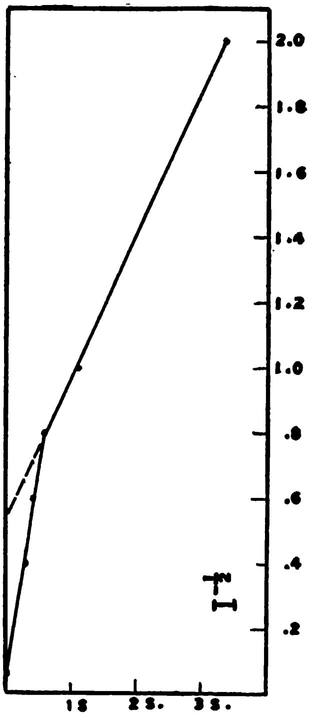

Fig. 8.

May it not be that the feeble photo-excitation of kunzite, certain calcites, etc., is due to their transparency even in the absorption band? Kunzite is almost impervious to X-rays and responds to their excitation while the kathode rays, which are completely absorbed, produce the most powerful effect.

Like most, if not all, substances whose phosphorescence is of long duration, kunzite requires an appreciable time to reach its maximum of excitation. Under kathodo-bombardment saturation is reached after a few seconds. The length of time required for the phosphorescence to decrease to 1/18,000 of the initial brightness was no greater after an excitation of five minutes than after ten seconds. Excitation of less than one second however produced phosphorescence of comparatively short duration, the time of decay to 1/18,000 being about one half that for the longer exposures. Under the feebler excitation by X-rays or when the sparks from a Holtz machine are caused to pass across the surface of the crystal it is possible to follow with the eye the gradual growth of the fluorescence.

We did not attempt to make a quantitative study of these phenomena because kunzite suffers fatigue under continued bombardment from which its complete recovery is slow. Indeed there is undoubtedly a permanent

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[1] Pochettino, l. c.

effect which shows itself, as in many other substances, in a dullness and discoloration of the surface. Red and infra-red rays have no appreciable effect upon the degree of excitation or the rate of decay. In this respect kunzite differs from Sidot blende.

In all the measurements on phosphorescence the substance was viewed through a yellow screen (*S*, Fig. 6) in order to prevent blending of the light from the two bands. This was however an unnecessary precaution since the blue band is of exceedingly rapid decay as could easily be determined by observation through a blue screen. No after effect lasting as much as a tenth of a second could be detected.

In this respect kunzite is similar to the phosphorescent sulphides of Lenard and Klatt and it resembles them also in becoming inactive, according to Pochettino, at about 400°.

Throughout these experiments the important feature of polarization was ignored. That a detailed study of this aspect of the luminescence of kunzite would yield interesting results there is little doubt. Indeed the general subject of the polarization phenomena in luminescence, concerning which we have as yet only the suggestive data of Pochettino, J. Becquerel and a few other observers deserves further consideration.

Physical Laboratory of Cornell University,
April 18, 1914.

NOTE ON THE RELATION BETWEEN THE EMISSION SPECTRUM OF A COMPOUND AND ITS ABSORPTION SPECTRUM IN SOLUTION.

BY ALBERT K. CHAPMAN.

IT is the intention of this note to call attention to a certain relation which appears to exist between the spectrum of a compound, excited by the ordinary vacuum tube discharge, and the absorption band which the same compound exhibits when in solution. In every case the compound was introduced into a spectrum tube of the usual form, fitted with aluminum electrodes, exhausted, sealed off and excited by the discharge from an induction coil. The tube was heated in order to vaporize the compound; the compounds used having been selected simply on account of their low melting points.

In the case of mercuric iodide the pressure was reduced to about 0.08 mm. when sealed off. Before heating, when the discharge was allowed to pass, the spectrum showed the lines of mercury and iodine, beside those due to the residual air. After the tube had been sufficiently heated to vaporize the mercuric iodide to a considerable extent, the character of the discharge was radically changed, becoming an intense violet in color. In addition to the lines of mercury and iodine there appeared a brilliant band, shading off toward both the red and the violet, extending from about λ 4420 to λ 4450 as seen visually: on the photographic plate it is shown broader. All measurements were taken by means of a Hilger constant deviation spectrometer. For making the photographs the telescope was replaced by the usual camera of 21-inch focal length. In observing the absorption spectrum the following solution was used:

KI	11.18 gr.
HgI_2	10.60 gr.
Water	100.0 gr.
Thickness of absorbing layer	12.32 cm.

The observations were made at room temperature with a Nernst glower as the source of continuous spectrum. It was necessary to use the potassium iodide in order to get the mercuric iodide into solution: a separate solution of potassium iodide was examined for absorption but none was found.

Fig. 1 shows: (*a*) Continuous spectrum of Nernst glower. (*b*) Absorption spectrum of mercuric iodide solution. (*c*) Mercuric iodide spectrum. (*d*) Mercuric iodide spectrum (shorter exposure). (*e*) Iodine spectrum. (*f*) Mercury arc spectrum. (*g*) Oxygen spectrum.

It will be observed that the emission and absorption bands correspond, although it is to be remembered, of course, that the width of the absorption band may be changed by varying the concentration, thickness of absorbing layer, etc. As no photographic scale was available for comparison, a few of the prominent lines are marked.

A similar set of observations was made on stannic iodide, the tube having been exhausted to 0.07 mm. In this case it was not necessary to heat the tube as it became hot enough from the discharge to vaporize the compound. In addition to the lines for tin, iodine, oxygen and nitrogen, the spectrum showed a band extending from about λ 5160 to the violet limit of visibility. The following is the absorption solution used:

SnI_4	0.1790 gr.
Ethyl alcohol	100 c.c.
Thickness of absorbing layer	1.95 cm.

Fig. 2 is as follows: (*a*) Continuous spectrum. (*b*) Absorption spectrum of solution. (*c*) SnI_4 in vacuum tube, 25 min. exposure. (*d*) SnI_4 in vacuum tube, 15 min. exposure. (*e*) SnI_4 in vacuum tube, 7 min. exposure. (*f*) Tin spark with capacity. (*g*) Iodine spectrum. (*h*) Oxygen spectrum.

The absorption covers the region from the extreme violet to about λ 5330, while the emission band extends from the violet to λ 5160, giving fair agreement between the emission and absorption bands.

In the case of ferrous iodide the spectrum was very faint and difficult to photograph; pressure 0.01 mm.

SOLUTION.

FeI_2	1.075 gr.
Sugar	10.00 gr.
Water	600 c.c.
Thickness of absorbing layer	1.93 cm.

Fig. 3 is as follows: (*a*) Continuous spectrum. (*b*) Absorption spectrum of solution. (*c*) FeI in vacuum tube. Here again the emission band corresponds to the absorption band but does not extend so far into the violet. The main emission band extends from about λ 4550 to λ 4850, but there is also a faint emission band at λ 4150. Longer exposure might have shown this to be wider.

It is interesting to note, in this connection, that mercurous chloride gives a very strong emission band from λ 5350 to λ 5750, while the salt itself is white and insoluble except in very small quantities. The emission spectrum is shown in Fig. 4.

It is seen that a wide emission band, shading off toward both the violet and red, is characteristic of the compounds investigated. When the absorption spectrum of the compound is available, there seems to be a definite relation existing between the absorption and emission bands. It would seem, therefore, that the vibrating system responsible for the emission is also effective in producing absorption in solution.

These experiments were carried on at the Physical Laboratory of the Ohio State University under the direction of Dr. A. W. Smith, to whom my thanks are due for his help and interest at all times.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY.

PHYSICAL REVIEW, VOL. IV., SECOND SERIES
July, 1914.

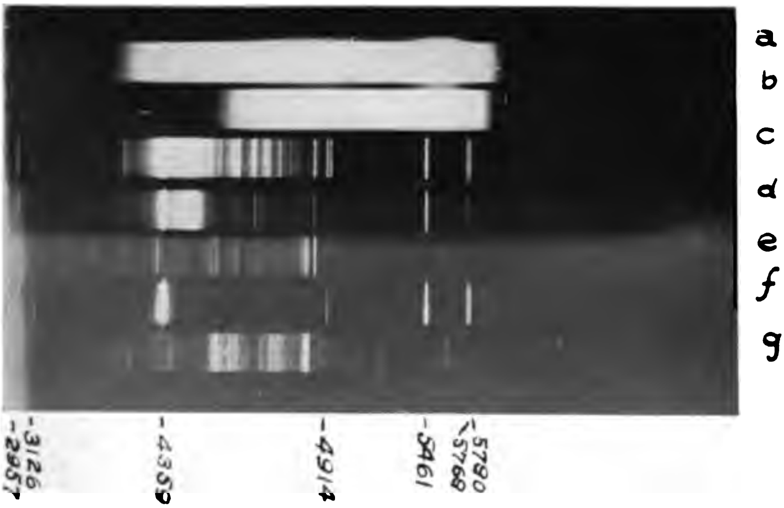

Fig. 1

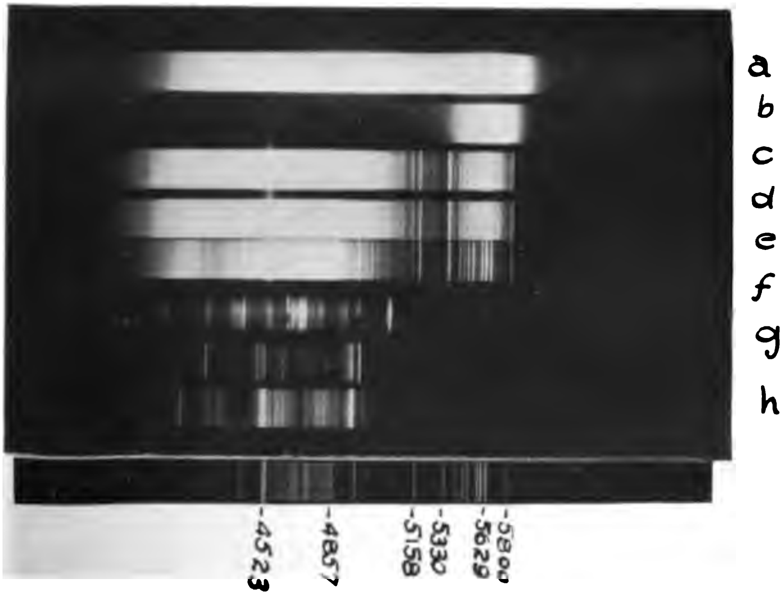

Fig. 2.

ALBERT K. CHAPMAN.

SOME BRUSH DISCHARGE PHENOMENA PRODUCED BY CONTINUOUS POTENTIALS.

By Stanley P. Farwell.

Introduction.

When there exists a large difference of potential between a wire and neighboring conductors such as a similar and parallel wire, or a coaxial cylinder, the discharge phenomenon known as corona is likely to occur. For alternating differences of potential, this phenomenon has been extensively studied by Peek, Whitehead, Russell and others. The alternating corona takes the form of a more or less continuous and uniform bluish glow along the wire. The corona produced by continuous potentials has not received so much attention, presumably on account of the fact that its practical bearing on engineering problems is not so great. Watson[1] and Schaffers[2] have carried out experiments on the corona thus produced. Watson has experimented on wires as small as 0.7 mm. and at pressures as low as 360 mm. Schaffers has worked with cylindrical fields, using wires as fine as 0.006 mm. and various sizes of tube and has determined the critical voltage for visual corona at atmospheric pressure.

Description of Experiments.

The writer has been studying the corona as produced by continuous potentials for wires from 0.037 mm. to 1.285 mm. diameter and tubes 3.50 cm. and 4.45 cm. diameter. The relation between difference of potential and current between wire and tube has been studied for atmospheric pressure for the different sizes of wire; the critical voltage for visual corona has been obtained for pressures from somewhat above atmospheric down to 2.0 mm. of mercury and the character of the discharge noted; and the effect of variation of voltage for a constant low pressure has been investigated.

The object of this paper is to present especially some of the phenomena observed at these lower pressures, the influence of a short arc in series with the apparatus upon the character of the discharge, and the increase of pressure in the tube due to the ionization.

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[1] Watson, Electrician, Sept. 3, 1909, Feb. 11, 1910, Feb. 18, 1910.

[2] Schaffers, Comptes Rendus, July, 1913, p. 203.

INFLUENCE OF PRESSURE UPON CHARACTER OF DISCHARGE.

The series of photographs in Fig. 1 represents the change in the discharge when the wire is negative to the tube and the pressure in the tube is varied from a low value up to atmospheric. The wire was No. 30 B. & S. (0.26 mm. diameter) bare copper taken from a coil obtained from the manufacturer. A glass tube 25 cm. long was lined with a brass sheath, except for a slit 6 mm. wide extending from end to end, and having an internal diameter of 3.5 cm. The wire was stretched tightly along the axis, passing through glass plates at the ends. The tube was rendered air tight by sealing wax and it could be exhausted through a branch tube attached to the side of the cylinder.

For the lowest pressures, the discharge takes the form of brilliant beads encircling the wire. Each has a bright cylindrical core, outside of which is a dark space which in turn is surrounded by a purplish glow extending out for some distance from the wire. As the pressure is increased, the difference of potential required to produce a discharge increases, and the number of beads increases. The central nucleus contracts and the bead becomes more and more like a brush until finally there is a line of brushes along the wire. Each brush consists of a bright spot on one side of the wire, with a fan-like purple glow spreading out from it, the plane of the fan being perpendicular to the axis. With the slit tube, the brushes point in different directions but if a similar test be run on a tube without a slit and one looks along the wire, the bright nuclei are seen to lie all in a plane, with alternate brushes on opposite sides of the wire, as a general rule.

As the pressure is raised toward atmospheric the isolated brush type of discharge gives place to such a discharge as is pictured in Fig. 1 for 357.0 mm. pressure. An occasional brush is left, mixed up with a more or less continuous glow which is very irregular. For atmospheric pressure, the discharge looks like the upper picture. The isolated brushes are very few, the rest of the wire presents an extremely "messy" appearance, the glow is bright and purplish and the discharge seems in constant movement.

For the lower pressures, a slight increase of voltage above that required to produce beads is sufficient to produce a violet arc-like discharge across the gap between wire and tube at one or two points and if this discharge be allowed to continue, the wire will be burned in two.

The photographs for 261.8 mm. pressure in Fig. 2 show the transition from one form of discharge to another, as it takes place for somewhat higher pressures. At the critical voltage, a continuous glow appears. Then as the voltage is raised slightly, the glow becomes spotted, followed, at a higher voltage, by the gradual breaking up of the glow into the isolated brush form of discharge. Sometimes this process is not so gradual as here indicated. Suppose a difference of potential be impressed of a value above that required to just produce a glow. At the instant of closing the circuit, one sees a continuous glow which dissolves into the brush discharge, the brushes emerging one by one, until the entire wire is strung with them. The upper picture illustrates the regularity of spacing of the brushes, which will be taken up later.

The characteristic appearance of the discharge with the wire positive is that of continuous, uniform, bluish glow of diameter little greater than that of the wire. Its appearance is not noticeably changed by changes in pressure, but it gets brighter with increasing difference of potential.

Effect upon Discharge of an Arc in Series.

The effect upon the discharge of a short arc in series with the apparatus is shown in Fig. 2 for a pressure of 112.6 mm. When the wire is positive, the introduction of an arc causes the glow to brighten, increase in diameter, become more purple, and more ill defined as to boundary. The currents recorded on the photographs are obtained from the deflections of a D'Arsonval galvanometer. When the arc is introduced, the current so obtained is much less than one would expect from the small increase in resistance of the circuit caused by the arc. Evidently the discharge with arc in series is made up of two forms of discharge superimposed; the effect due to the continuous potential and an alternating effect caused by the oscillations set up in the circuit by the arc.

This superposition of effects is clearly illustrated when the wire is negative. The arc here causes a marked change in the discharge. The result is a continuous glow with a few isolated brushes strewn along it.

To test out the effect produced by the arc in apparently producing oscillations in the circuit, a condenser was connected across the cylindrical field. The introduction of the condenser caused the discharge to take the same form it had before the arc was introduced, except for there being a few less brushes. When there is a condenser thus in the circuit and the switch is closed, the transition from a continuous negative glow to the brush form of discharge is prolonged. With the condenser still in the circuit, the disconnection of the impressed difference of potential gives an opportunity for a discharge of the condenser across the cylindrical field. At the instant the line circuit is opened, no change in the appearance of the brushes is noticeable. Then as the condenser discharges and its potential falls, there is presented a "moving picture" of the stages of the discharge down to darkness. This discharge was a matter of several seconds. As the voltage fell, the brush type of discharge was maintained: each regular arrangement of brushes giving place to another regular arrangement of fewer brushes. Since the resistance of the field is large, the condenser discharge must be of the continuous type.

DISCHARGE BETWEEN PARALLEL WIRES.

Two No. 34 copper wires were placed parallel and 2 cm. apart inside a tube of glass 25 cm. long and the photographs of Fig. 3 were taken for pressures less than atmospheric. The tendency of the negative wire to show an isolated brush discharge and the positive to give a continuous glow is evident here. There is evidently a tendency for the positive sections of continuous glow to break up into spots or streamers. For constant pressure, the increase of the number of sections of the discharge with increase of voltage will be noted. The spacing of the sections is approximately regular and would undoubtedly be more so if the wires were more exactly parallel and stretched more tightly to make them straighter.

The two upper photographs show the effect produced by an arc in series. There is no longer the great difference between the appearance of the two wires. The negative wire, however, still shows a tendency to discontinuity of discharge. When the current is sufficiently great, violet streamers cross between the wires as shown in the upper picture. The current indicated is, again, only the component given by the galvanometer.

SPACING OF BRUSHES AS A FUNCTION OF THE VOLTAGE.

The slit tube previously described was fitted with an arrangement for stretching the wire tighter and a series of photographs was taken of the discharge under constant pressure, with the wire negative and varying difference of potential. This series is shown in Fig. 4. The lowest picture shows the appearance of the discharge at a voltage little higher than that required to produce visual corona. It will be noticed that there are many tiny brushes and no regularity of spacing. For a little higher voltage, the number of small brushes has decreased and there are a number of large brushes disposed at quite regular intervals. The succeeding photographs show the effect of increasing the voltage still further. The number of brushes continually increases and the spacing is very regular. For the lower voltages, the brushes are fixed in position for a given voltage and will always show up in the same position as the circuit is interrupted and then closed again. When the voltage approaches the value at which there will be an arc between wire and tube, each brush is in constant movement back and forth in a short path, but the number of brushes is constant for a given voltage.

Fig. 5 shows the relation between difference of potential and current. In connection with this graph it might be noted that the critical voltage for visual corona was 2,440.

It would appear from a close observation of the character and spacing of the brushes that there are only certain voltages for which there appears a regular distribution of full-sized brushes. For intermediate voltages, there is more or less irregularity in the size of the brushes and their spacing. For the pictures of Fig. 4 an effort was made to pick out those points at which the distribution was the most regular. Fig. 6 shows the variation with the difference of potential of the number of full-sized brushes and the current per brush. When there was a marked variation in the size of the brushes an estimate was made of the equivalent number of full-sized brushes. These graphs clearly indicate that a definite relation exists between the voltage and the number of brushes, for a given pressure, and that the current per brush is not a constant but also varies with the voltage.

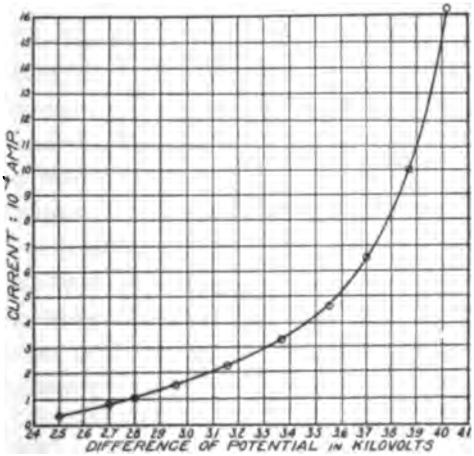

Fig. 5.

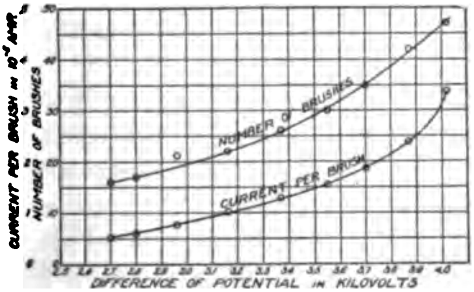

Fig. 6.

The question may be raised as to whether the isolated brush form of discharge may not be due to oscillations in the circuit. In order to make it clear that this is essentially a direct current phenomenon, there is given below a description of the generating apparatus used in producing the continuous potentials and some experiments and arguments which support this view.

The source of electromotive force was a battery of small direct-current generators, self-excited and connected in series. The machines were rated at 500 volts and 250 watts and there were thirty in all, giving 15,000 volts at normal voltage. The machines were arranged in two sets, ten in one and twenty in the other, each set being driven by its own direct current motor. Variation of voltage was obtained by field control of the generators and of one of the motors. To prevent damage to the machines through short-circuits, a water resistance of a rather large value was connected between the generating apparatus and the tube. The negative terminal of the machines was grounded. Electrostatic voltmeters were used to measure the difference of potential and a D'Arsonval galvanometer in connection with an Ayrton shunt box to give the current.

The appearance of the brushes and the current indicated by the galvanometer are constant for a given voltage, no matter what combination of machines are used as the source of potential. One of the sets may be used and the appearance of the spots and the voltage and current noted. Then if the other set be used to give the same voltage with a different number and speed of machines, the same results are obtained. If there were oscillations set up perhaps by sparking at the brushes, we would not expect this agreement.

Mention has been made before of the effect of the introduction of a condenser in parallel with the tube. To test whether the current sent through the tube by the condenser in discharging was direct or oscillatory, another experiment was performed. The condenser was connected across the positive and negative bus-bars to which the generating apparatus was connected through the water resistance. Then a switch connecting the machines to the bus-bars was closed as was also a switch leading to the tube. The deflection of the galvanometer was noted and the appearance of the brushes. Then the generator switch was opened and the condenser discharged through the tube and the galvanometer. After the switch was opened, the galvanometer deflection gradually decreased, the rate of decrease of the deflection being slower and slower as the discharge proceeded. The opening of the switch caused no immediate change in the brushes, only the gradual change already noted.

That the discharge of the condenser must be continuous is shown by the deflection of the galvanometer and it can be further proved by a rough calculation. Assuming the resistance of the cylindrical field as given by E/I and taking a set of values of E and I for the comparatively low pressures at which the brushes are best formed, we obtain $R = 1.83 \times 10$ ohms. Assuming the very large value of 0.1 henry for the inductance of the circuit, and the approximate value of 2 mf. for the capacity, we find the R is about 4.1×10^4 times as great as $\sqrt{4L/C}$ and hence it is clear that the condenser discharge must be of the continuous type.

By running wires from the terminals of an induction coil to the central wire and the tube and then adjusting the discharge points on the coil to such a distance that a silent discharge took place between them, it was possible to obtain an almost uniform hazy glow along the wire. But no effect could be obtained like the uniformly spaced brush discharge.

It is well known that an arc is the source of electrical oscillations and it has been shown by a previous figure that a short arc in series with the tube disturbs the brushes due to the direct current by the superposition of an alternating current effect so that the glow becomes more or less uniform and the difference in the appearance of the glow for different polarities becomes much less. So the introduction of an oscillatory current acts to suppress the isolated brush form of discharge and not to cause it.

The difference between positive and negative electricity is hardly better demonstrated by any other phenomenon. It should be stated here, however, that Peek[1] by a stroboscopic method has also observed "more or less evenly spaced beads" on the negative wire when there was corona between parallel wires caused by an alternating difference of potential of 80,000 volts at atmospheric pressure. The wires used by Peek were 0.168 cm. in diameter, spaced 12.7 cm. apart.

EFFECT OF MAGNETIC FIELD.

A strong horseshoe electromagnet was placed in various positions with its poles against the tube and the effect upon the various forms of discharge of making and breaking the magnet circuit was observed. No change could be noted in the appearance of the discharge or the current flowing.

VARIATION OF PRESSURE IN TUBE WITH VOLTAGE.

A No. 36 B. & S. copper wire, 0.135 mm. in diameter, was stretched tightly along the axis of a brass tube 4.45 cm. in diameter and closed at

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[1] Proc. A. I. E. E., Vol. 31, No. 6, p. 1123 and Plate LXV.

the ends by glass plates through which the wire passed. A small branch tube was soldered to the side of the main tube and from it connection was made to an air-pump. An open manometer of small bore containing a light oil was connected to the side of the branch tube. Everything was rendered airtight after disconnecting the manometer and the tube was exhausted. Then dry air was gradually admitted through a tube containing soda-lime and a wash-bottle containing concentrated sulfuric acid, until the pressure was again atmospheric, 744.0 mm. in this case. The manometer was again connected and various differences of potential impressed.

As soon as the voltage reached the critical value to cause an appreciable current to flow, a jump in the columns of the manometer was apparent. This jump occurred lower for the wire negative and it was difficult to tell just the voltage at which it began. When the wire is negative, any little dust particle on it will be sufficient to start a discharge at a lower voltage than would be required to cause a general glow along the whole wire. But for the wire positive, the critical point is very marked and the jump occurs, as closely as one can judge, at the same time that a faint bluish glow is seen along the wire. Fig. 7 shows the increase in the

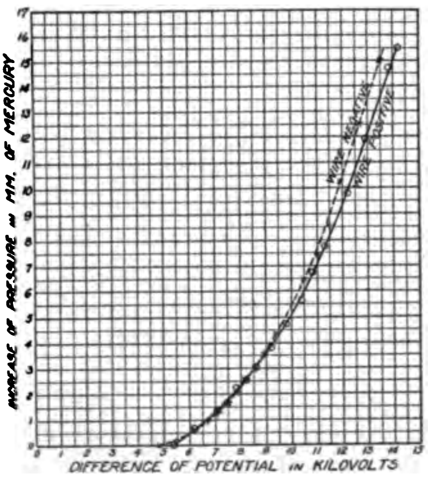


Fig. 7.

pressure as the voltage is raised. This graph has exactly the same appearance as the graph plotted between voltage and current, as one might expect from the theory of the conduction of electricity through gases. It will be noted how the curves for the two polarities cross at low voltages and that the increase of pressure for a given voltage is greatest for negative

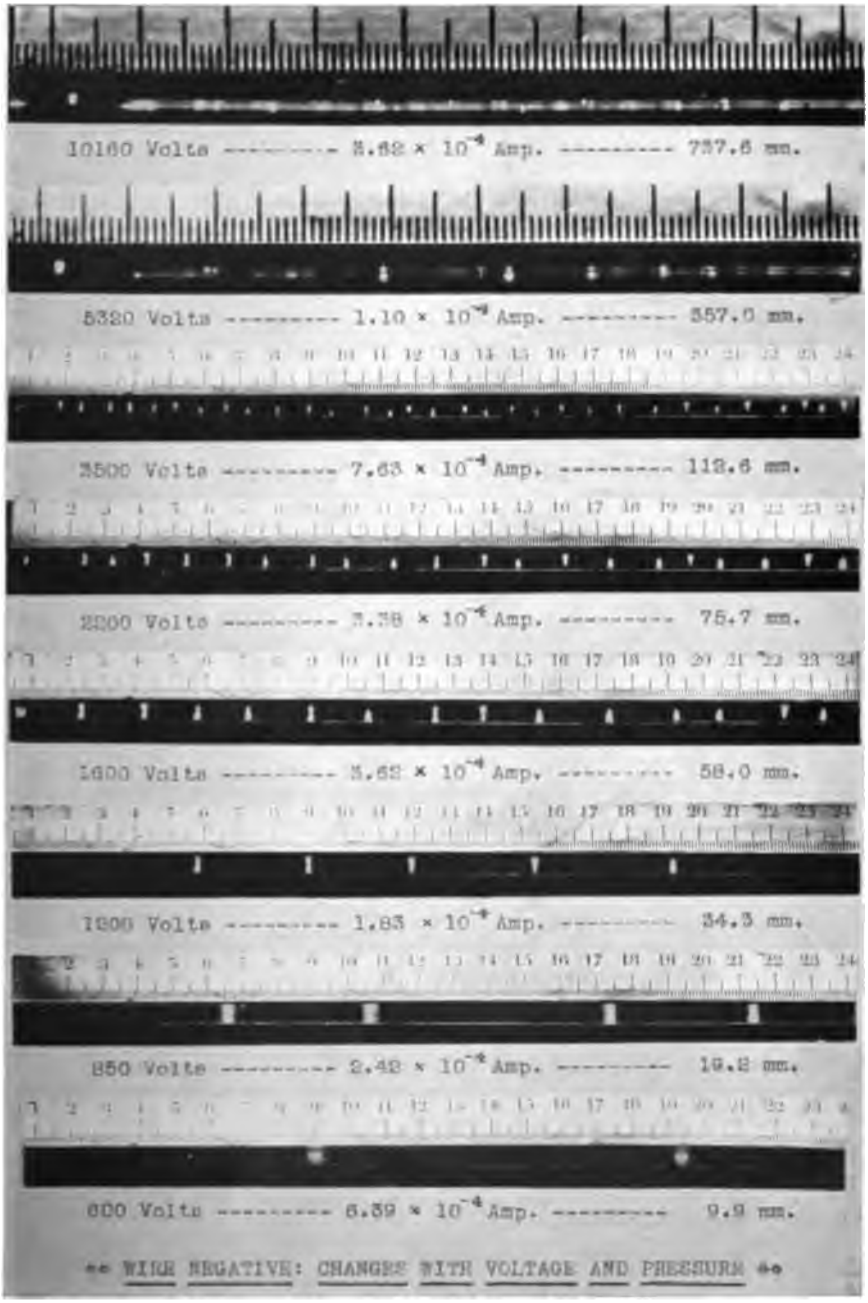
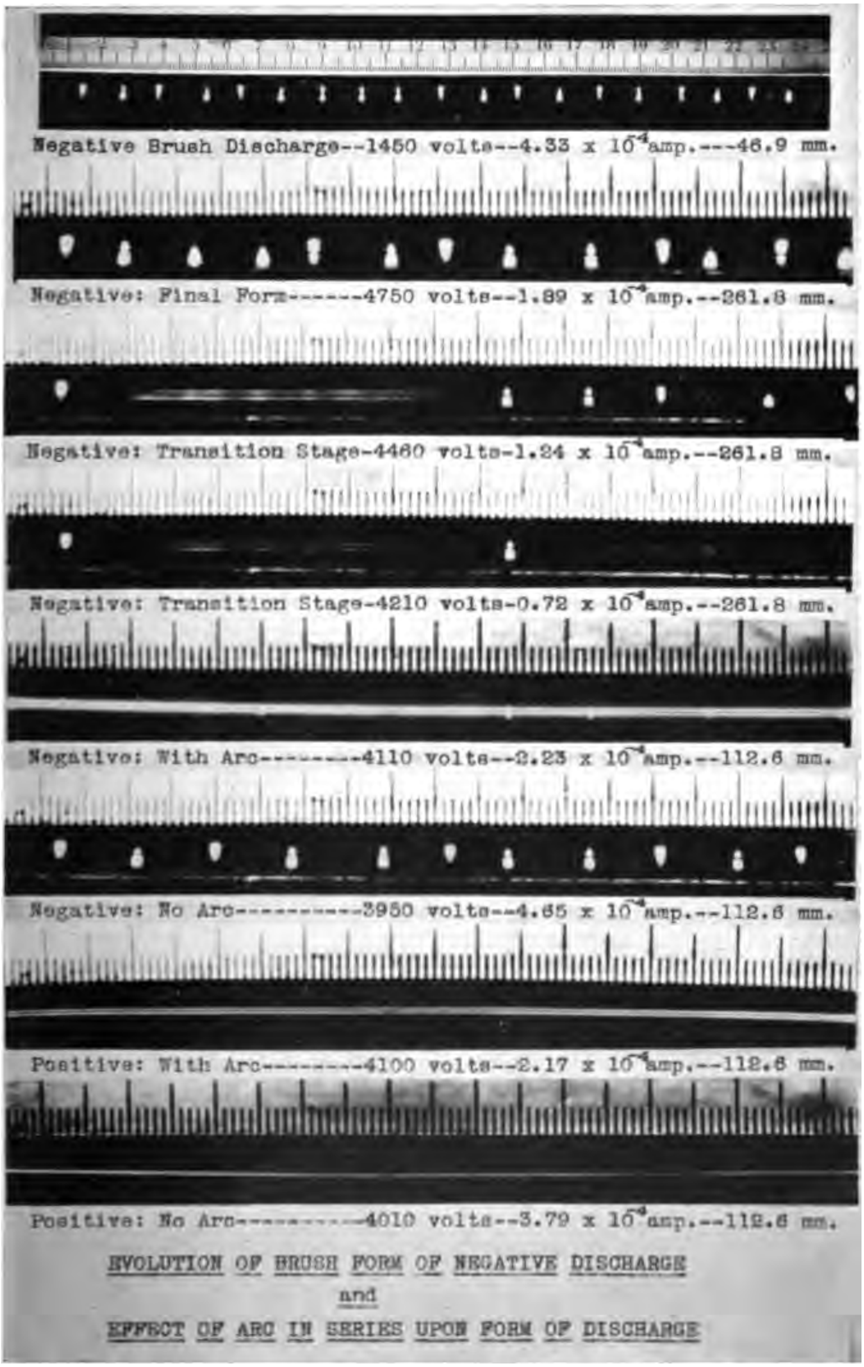


Fig. 1.

STANLEY P. FARWELL.



Negative Brush Discharge--1450 volts--4.33 x 10^{-4}amp.---46.9 mm.

Negative: Final Form------4750 volts--1.89 x 10^{-4}amp.--261.8 mm.

Negative: Transition Stage-4460 volts-1.24 x 10^{-4}amp.--261.8 mm.

Negative: Transition Stage-4210 volts-0.72 x 10^{-4}amp.--261.8 mm.

Negative: With Arc---------4110 volts--2.23 x 10^{-4}amp.--112.6 mm.

Negative: No Arc-----------3950 volts--4.65 x 10^{-4}amp.--112.6 mm.

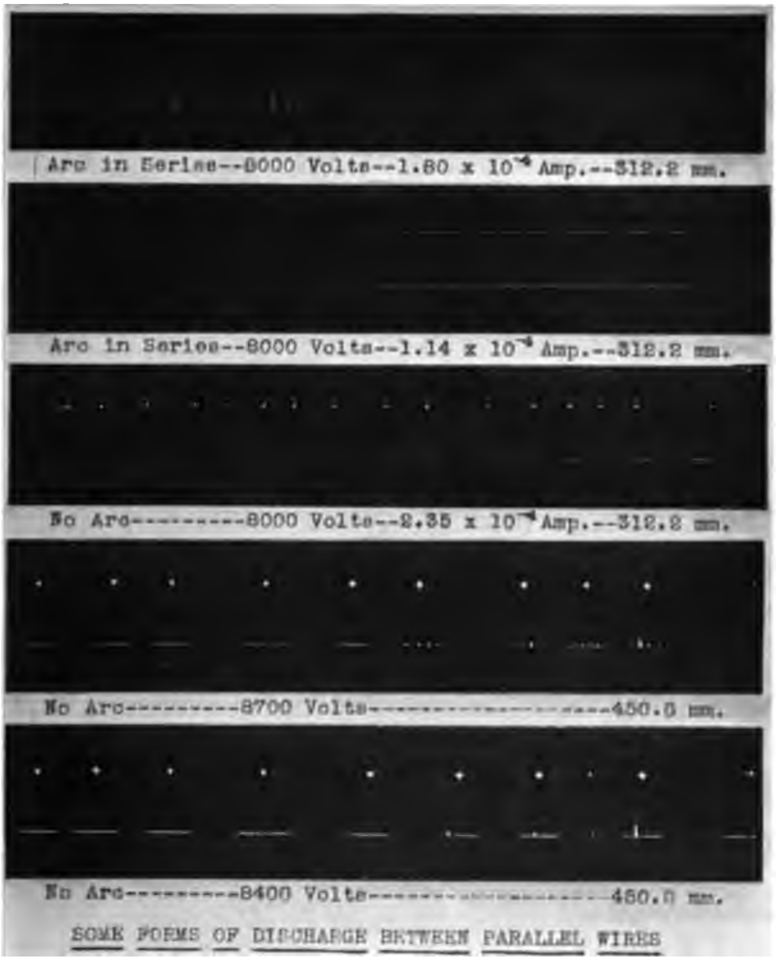
Positive: With Arc---------4100 volts--2.17 x 10^{-4}amp.--112.6 mm.

Positive: No Arc-----------4010 volts--3.79 x 10^{-4}amp.--112.6 mm.

EVOLUTION OF BRUSH FORM OF NEGATIVE DISCHARGE
and
EFFECT OF ARC IN SERIES UPON FORM OF DISCHARGE

Fig. 2.

STANLEY P. FARWELL.



#34 Wires-0.165 mm.-2 cm. apart-25 cm. long
Upper Wire is Negative

Fig. 3.

STANLEY P. FARWELL.

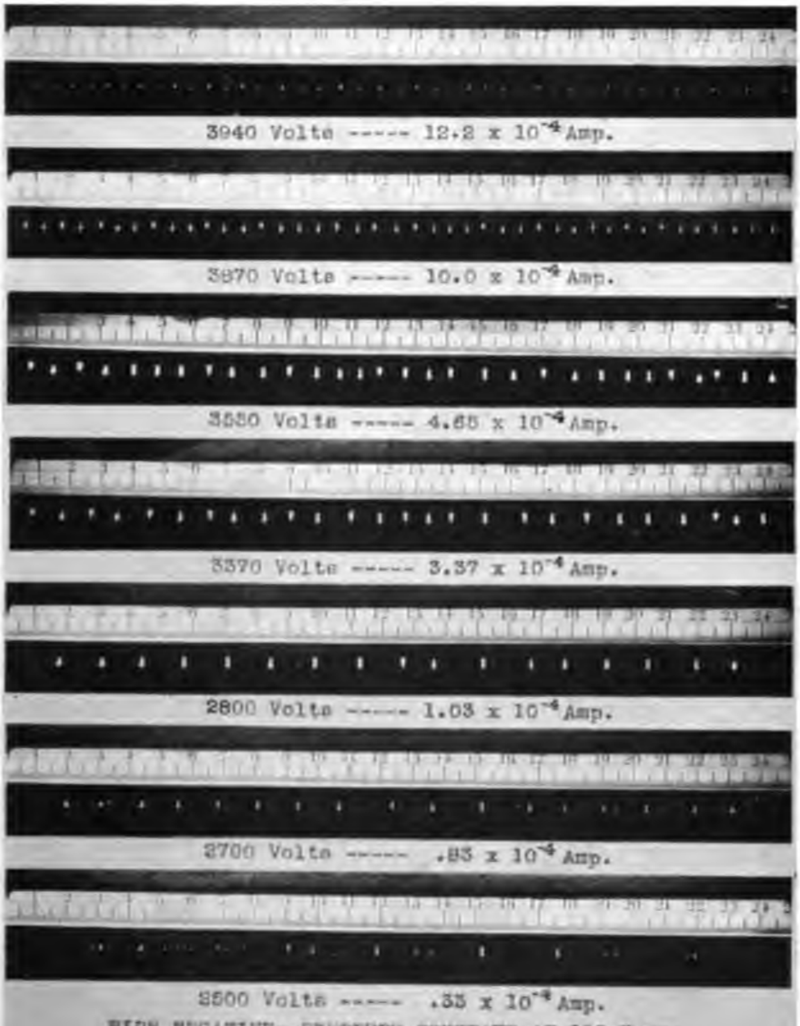


Fig. 4.

STANLEY P. FARWELL.

polarity of the wire during the greater part of the range. The crossing of the curves is typical of the voltage-current graphs.

In addition to the sudden jump at closure of the circuit there is a gradual increase of pressure due to the heating effect of the current and hence care was taken to read the heights of the columns of liquid as soon as possible after the circuit was closed.

The work upon which this paper is based was performed in the laboratory of physics at the University of Illinois under the direction of Dr. Jacob Kunz, asst. prof. of physics. To him and to Prof. E. B. Paine, of the electrical engineering department, the writer wishes to acknowledge his indebtedness for many helpful suggestions as to the conduct of this work.

Laboratory of Physics,
University of Illinois,
April, 1914.

THE REPRODUCTION AND MEASUREMENT OF VERY SHORT INTERVALS OF TIME.

BY JOHN COULSON.

IT is often found desirable to have at one's disposal some means by which two neighboring electric circuits can be broken in quick succession in such a way that the interval between these two operations is accurately predictable and under the control of the experimenter. This paper contains a brief description of a simple apparatus which can be made to give time intervals of very short duration.

This work was suggested to the writer by Professor B. O. Peirce, to whom it was of much interest, since he required in his investigation of the dielectric constants of certain materials, a shorter time interval than was attainable, with accuracy, by any known apparatus. The many devices then in use either gave intervals much longer than would serve, or intervals which could not be repeated with a sufficient degree of accuracy.

The measurement of very short intervals of time is a matter of but little difficulty. More than sixty years ago Helmholtz[1] used in his investigation "*Sur la vitesse de propagation de l'agent nerveux dans les nerfs rachidiens,*" a horizontal magnetic pendulum for determining the duration of an electric impulse. The apparatus was of the form of a Thompson galvanometer, and the angular deviation of the magnet from its position of rest was taken as a measure of the duration of the electric impulse sent through the galvanometer. The behavior of such swinging magnets had been studied by Gauss and W. Weber[2] while making their magnetic measurements at Göttingen.

Of the many methods used by later investigators for measuring such intervals, perhaps the best is by determining either the partial discharge from a loaded condenser,[3] when short-circuited during the interval in question through a given non-inductive resistance, or the fractional part

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[1] Helmholtz, Comptes Rendus, 30, 1850; 33, 1851.

[2] Gauss, and W. Weber, Resultate des Magnetischen Vereins, 1837.

[3] H. R. Kempe, Handbook of Electrical Testing, 1887, 4th ed.; Mascart and Joubert, Electricity and Magnetism, 1888, Vol. 2; Devaux-Charbonnel, Comptes Rendus, 142, 1906; L'Electricien, 31, 1906; H. W. Morse, and C. L. B. Shuddemagen, Proc. Am. Acad., 44, 1908; C. L. B. Shuddemagen, Proc. Am. Acad., 44, 1908.

of a full charge acquired by an originally empty condenser through a given non-inductive resistance when a constant-electromotive force was applied throughout the charging time interval. It has long been known that by a proper choice of apparatus this method of procedure yields very accurate measurements, and is capable of being applied in a variety of ways. It furnishes a satisfactory means of determining the time of contact of a hammer and an anvil,[1] or the velocity of a projectile, or the "specific duration"[2] of impinging spheres. For many years Professor Peirce had as regular experiments in his course on experimental electricity and magnetism, the determination of cable insulation resistance, and the time of contact of two impinging steel balls, as applications of this method. He also applied it in his investigation on the duration of a "quick tap on a telegraph key."[3] In every case, when the apparatus was properly set up, this method has proven to be a good one, yielding results which are limited in accuracy only by the unsteadiness of the ballistic galvanometer used to measure the condenser charge.

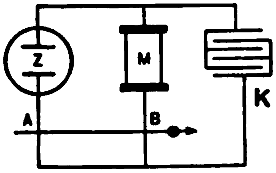

Fig. 1.

Apparatus arranged for the determination of the unloading time.

The problem of finding the fractional part of a full charge left on a condenser after unloading through a non-inductive resistance for an interval of t seconds is a simple one, and the solution is very familiar. Let the apparatus be arranged in some such form as indicated in Fig. 1, where Z is a battery of constant-electromotive force, M a variable non-inductive resistance, and K a standard condenser. Suppose the condenser has acquired its full load Q_0; then, if the battery lead be severed at A by a rifle ball, the condenser will discharge through the resistance r, made up of M and the lead wires, until the ball cuts the second wire at B; and the charge left on the condenser will be, $Q_t = Q_0 e^{-t/kr}$ where t is the time taken by the bullet to travel from A to B, and k the capacity of the condenser in farads. If t is the quantity sought, as in the present investigation,

$$t = kr \log_e \frac{Q_0}{Q_t}.$$

With the aid of a suitable ballistic galvanometer and key, both Q_0 and Q_t can be easily determined.

The design of an apparatus which will repeatedly give the same ex-

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[1] R. Sabine, Telegraphic Journal, 4, 1876; Phil. Mag., 1, 1876.

[2] A. E. Kennelly, and E. F. Northrup, Journal of the Franklin Institute, 1911.

[3] B. O. Pierce, Proc. Am. Acad., 42, 1908.

tremely short interval of time within say one per cent. of itself, so that it can be controlled and used without measuring its duration in every experiment, is a matter of much greater difficulty. It has been customary to use a bullet travelling at a high velocity to sever two parallel lines placed close together, but the results from successive observations by this method often differ as much as 25 per cent. Fig. 2 is reproduced from the photograph of a bullet which was in the act of severing two wires while travelling at a velocity of 500 meters per second. The photograph was taken by Mr. W. A. Hyde, Bureau of Ordnance, Navy Department, to whom I am indebted for permitting its reproduction. It illustrates what has long been known, that even the most brittle material, placed in the path of a bullet under high velocity, will bend before it breaks, and very often drag on the sides of the bullet; which in either case may cause serious errors.

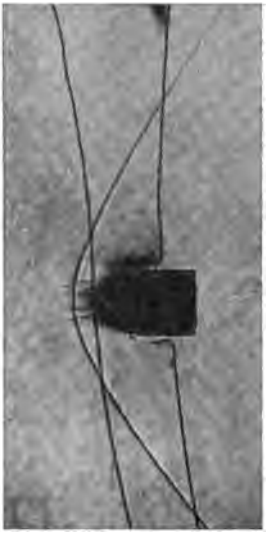

Fig. 2.

Bullet, travelling with a velocity of 500 meters per second, in the act of breaking two parallel wires placed in its path. It is evident that the ruptured wire bent very perceptibly before breaking.

The apparatus here described is based on a different principle. If a massive weight, falling freely under gravity, strikes a collar on a metal rod which is supported vertically, an elastic wave or impulse travels out along the rod in each direction from the collar with a finite velocity.[1] It is evident that if the impact takes place at the middle point of the rod, these waves will reach the ends of the rod at the same time. If, however, the point at which the impact occurs be not at the middle of the rod, the impulses thus started will reach the ends at times that differ by an interval which will depend on the path differences. In this way controllable time intervals extending over a considerable range may be secured, and can be measured easily with the aid of proper apparatus. I have experimented with numerous forms in order to be sure that the elastic wave excited in the comparatively long stout metal rods used was of sufficient magnitude to interrupt the passage of current in two related circuits, of which the rod in question formed a part, by breaking contacts at its extremities.

Fig. 3 shows diagrammatically the arrangement of the apparatus which has given the most consistent results. The framework, about 2 meters

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[1] A. E. H. Love, Mathematical Theory of Elasticity, 2d ed., 1906.

in height, and 40 centimeters wide, constructed from well seasoned pine studs 5 centimeters thick, and 15 centimeters wide, stands on a concrete foundation, and is bolted to the brick wall of the laboratory thus avoiding all possible jarring of the apparatus when in use. The central crosspiece, which is subjected to heavy blows when the apparatus is in operation, consists of a heavy block of oak set into the sides of the frame, and held fast by long screws. The rod *AB*, which serves as a circuit breaker is of Bessemer steel 1.3 centimeters in diameter, it terminates in a square metal block pierced by a rectangular hole, and is held vertical by passing through holes in the center of the cross pieces. It is supported in the position of closed circuit by a spiral steel spring *Y*, the lower terminal *B* resting against the fixed contact *D*, in the cast iron bar *H* which passes through the rectangular hole in the square block attached to the end of the rod, while the upper terminal *A* comes in contact with a piston which is inserted in the screw *P*. The cast iron bar is well insulated on both sides by strips of hard rubber, and is supported by stout pillars on a metal

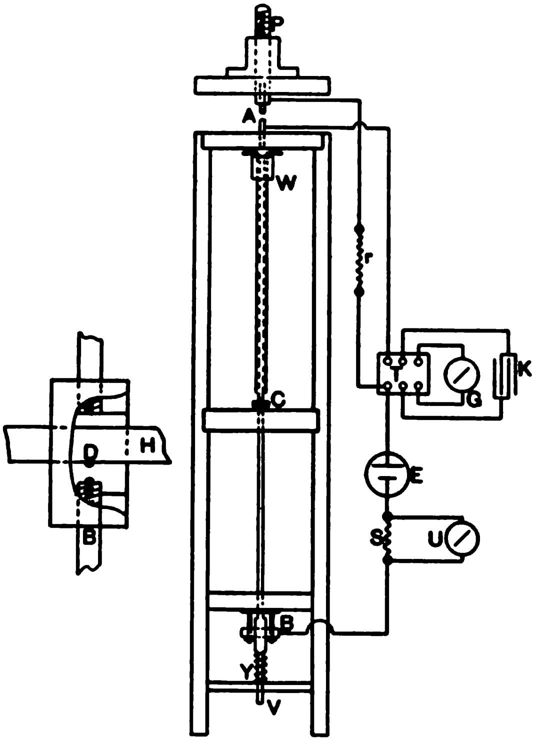

Fig. 3.

Diagrammatic arrangement of the apparatus used in making the observations described in this paper.

base which is attached rigidly to the frame. The pressure of the piston at the upper terminal of the rod is regulated by a weak spiral spring inserted in piston hole. To avoid the violent jar which results from the sudden stopping of the weight, this part of the apparatus is fixed on a separate shelf attached to the wall. The weight *W*, which is about 12 kilograms, slides on a brass tube supported independently by a bracket, thus protecting the rod from the downward motion of the weight until it has reached the collar *C*, the desired point of impact. The collar, or anvil consists of a thick disc of self-hardening steel, fastened rigidly to the rod by a steel set pin and silver solder, and is the fixed origin of the elastic waves set up in the rod by the sudden shock. The impact carries the rod down until the collar reaches the heavy wooden cross piece below. It is held from rebounding by means of a spring latch. The elastic waves on which the operation of the apparatus depends are propagated from *C* to *A* and *B* respectively.

The arrangement of the electrical apparatus is very simple, as will be seen from the diagram, but great care was taken to have all parts highly insulated. The connecting wires are all enclosed separately in small rubber tubing, and then placed side by side in order to avoid self inductance. The battery *E* consists of three bichromate cells in series, placed on a platform supported by glass insulators. A sensitive galvanometer *U*, shunted by a low resistance *S*, served as a check on the constancy of the current in the circuit. The double pole double throw switch *T*, consisted of hard rubber with platinum contacts, and is operated by means of a long brass rod attached to the armature of a strong electromagnet placed at some distance from the apparatus. A bank of storage cells supplied the current to work the electro-magnet, which by suitable keys and an automatic interrupter driven by a time clock, could be made to throw the switch *T* in either one direction, or the other at a given instant. In this way the poles of the condenser *K*, which is a standard microfarad condenser (Elliot Bros., No. 72) recently calibrated at the Bureau of Standards, was brought into the position of loading, or the reverse; that is across the terminals of the standard non-inductive resistance *r*, or in series with the galvanometer *G*, which is a ballistic d'Arsonval of high sensibility.

In taking the observations the manipulation of the apparatus, from the instant the condenser began to load, was entirely automatic; thus eliminating errors from variation in charging time, and otherwise.

The method of procedure was as follows: With the rod in closed circuit, and the weight *W* in its elevated position, the condenser was thrown into circuit with *r* at the signal from the time clock, which later, after a predetermined interval, released the weight. The elastic wave travelling downward then broke the contact *B*, leaving the condenser to discharge through *r* until the wave travelling upwards reached the second contact at *A*; and before either end of the rod had time to regain its closed position the impact carried the rod bodily downward into the position of open circuit. On reaching this position it closed a key at *V*, which operated the electromagnet, which in turn discharged the condenser through the galvanometer immediately. In this way, using different values for *k* and *r*, many sets of observations were taken in quick succession. The results from three different rods, divided in different ratios by the steel collar, are tabulated below, Table I.

Table I.

Rod.	r in Ohms.	k in Microfarads.	Log$_e$ Q_0/Q_t.	$t \times 10^6$ Seconds.
L	800	0.05106	0.3818	15.61
L	700	0.05106	0.4383	15.69
L	600	0.05106	0.5038	15.45
L	500	0.05106	0.6206	15.82
L	400	0.05106	0.7701	15.75
M	600	0.05101	0.0988	3.028
M	400	0.05101	0.1484	3.030
M	200	0.05101	0.2964	3.025
N	800	0.05101	0.1570	6.41
N	700	0.05101	0.1814	6.50
N	600	0.05101	0.2110	6.48
N	500	0.05101	0.2502	6.40
N	401	0.05101	0.3148	6.44
N	201	0.10228	0.3118	6.41

The apparatus described here is the result of many efforts to construct one that would function consistently for a given rod. The contacts at *A* and *B* were sources of much trouble; platinum, silver, hard steel, German silver, and platinum-iridium terminals were experimented with. The platinum and silver contacts were too soft; the German silver and steel offered contact resistance when used for any length of time, but the platinum-iridium terminals proved to be entirely free from these defects. In the early experiments the inductance in the battery circuit was also a source of disturbance, but this was alleviated by carrying the lead wire from *B* parallel to the rod, and close to it, through a hole in the collar, and out at *A*, where the well insulated wires all came together. The consistency with which the apparatus worked finally, is illustrated by a set of observations taken with rod *L*, Table II.

It will be seen from Table I. that the greatest deviation in time interval, which elapses between the interruption of the currents, in a given rod, at

TABLE II.

$r = 800$ Ohms.		$k = 0.05106$ M. F.	
Galvanometer Deflections, αQ_t.	Galvanometer Deflections, αQ_0.	$\log_e Q_0/Q_t$.	$t \times 10^6$ Seconds.
8.18 cms.	11.98 cms.	0.3818	15.61
8.20	12.05		
8.30	12.20		
8.31	12.12		
8.25	12.09		
8.30	12.10		

the points *B* and *A* when different combinations of resistance and capacity are used, is about 2.5 per cent., as compared with the 25 per cent. obtainable with a bullet. In Table II., which is a typical set of observations, the agreement is about 1.2 per cent. If the resistance and capacity be well chosen, for a given rod, a set of observations should not differ from each other by more than 1 per cent.

I wish to express my indebtedness to the late Professor B. O. Peirce for his many helpful suggestions throughout the work, and for the loan of the standard apparatus used in making the observations mentioned in this paper.

THE JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.,
April, 1914.

NOTE ON EVIDENCE FOR ELECTRON ATMOSPHERES.

By F. F. Householder.

IN the Philosophical Magazine for August, 1912, Professor R. W. Wood describes certain experiments in which the electrical resistance of silver films was but slightly changed by a number of transverse rulings, made with a diamond edge, provided the rulings were less than two sodium wave-lengths in width. These results were given, along with others, as evidence for the existence of an atmosphere of electrons extending to a distance of about one wave-length (0.6 μ) from the surface of metals. Though the rulings in question were examined microscopically and seemed continuous, nevertheless lack of continuity, *i. e.*, the presence of metallic bridges across the gaps appeared to be the probable explanation of the observed effects, and it seemed worth while to attempt to prove definitely that this was the case. In repeating the experiments I used film strips only 0.6 mm. wide (much narrower than Wood's), so that the chance of obtaining a continuous cut was greater, and the cut could be examined with greater ease and certainty. The cuts were examined with a Zeiss apochromat objective, n. a. 1.30, oil immersion, and compensating ocular No. 12. The measured magnifying power was 1,500 diameters, and the resolution was improved by using a green light filter. Many strips were ruled without greatly changing their resistance, but in all of these metallic "bridges" were found. Finally the following rulings were obtained:

Strip Number.	Initial Resistance Ω.	No. of Cuts.	Width of Cut.	Final Resistance Megohms.
1	43	1	.6 μ	> 100
2	20	7	.5 μ	> 100
3	6	1	.9 μ	> 100
4	50	1	.5 μ	> 100
5	6	1	.4 μ	> 100
6	50	1	.3 μ	> 100

The conclusion is that if there is an electron atmosphere it's depth is much less than the value assigned by Wood; it must be less than .15 μ.

Department of Physics,
University of Wisconsin,
April, 1914.

WAVE-LENGTH-SENSIBILITY CURVES FOR LIGHT SENSITIVE SELENIUM AND THEIR SIGNIFICANCE.

BY F. C. BROWN AND L. P. SIEG.

IN our last paper[1] we showed that the sensibility curves for one of Giltay's cells was distinct and different from any that had been obtained by other investigators. Since then we have investigated a number of light-sensitive varieties made by Giltay, Ruhmer, Dieterich, and ourselves, and have found the interesting result that selenium as such does not have a characteristic sensibility curve. Such results will be set forth in this paper. Further we shall allude to the fact that the magnitude and position of the maxima of sensibility in the spectrum can be controlled by the treatment of the selenium in the process of making. Incidentally it will be shown that there is an interesting correlation between the change of resistance and intensity of illumination for short exposures no matter what the characteristic curve may be. Lastly we shall indicate the bearing of our results upon the theory of light-action in selenium.

GENERAL CONSIDERATIONS.

The phenomena of light action in selenium are almost unique, and it is for this reason that we should expect any theory of value to incorporate all the diverse phenomena that appear in this element. Of course we shall ultimately wish to explain the conductivity as well as the structure of selenium on the basis of the electron theory. Thus far an analysis reveals that any electron theory that is based on a single type of curve of sensibility is doomed to failure. And as a corollary to this we may say that any electron theory that presupposes only one light sensitive constituent in the selenium is also doomed to failure. For an analysis of exposure and recovery curves for a number of specimens of selenium has revealed that there are at least three components in selenium, at least two of which are acted upon by the light. The results of this paper are quite in agreement with such a complex structure of selenium, although we can not devise the necessary assumptions to determine if the three components are sufficient.

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[1] PHYS. REV., N. S., Vol. 2, p. 487, 1903.

Mr. E. O. Dieterich[1] has been led by his researches in this laboratory to believe that the differences in the characteristics of light-sensitive selenium are purely the result of different crystal formations. His argument is very direct and will be presented as soon as his data are completed. Also it may be mentioned that we have produced two new crystal forms of very large size by subliming selenium at the same temperatures at which selenium cells are annealed. This makes it reasonable to assume that these same crystals with perhaps others also are in a more or less stable condition mixed together in light-sensitive selenium.

It is of little avail to select a single sample of selenium made by a particular treatment of the selenium, for the purpose of deducing the nature of light action, because all samples of light-sensitive selenium contain a mixture of certain components, some or all of which may bear direct relations to crystal formations. Any particular pressure, temperature, or time treatment only alters the relative amounts of the components present.

APPARATUS AND METHOD OF MEASUREMENT.

Apparatus.—The general method described in our former paper[2] was followed. The apparatus was improved by the substitution of a Hilger monochromatic illuminator in place of the one by Spindler and Hoyer. In the Hilger apparatus the lenses were 32 mm. or *f*/5.6 in diameter, as compared with the *f*/12.7 of the Spindler and Hoyer apparatus. We were thus able to augment our energy with a given slit width by approximately 5 times. As in the former work the source of light was a Nernst filament kept at practically constant intensity by supplying it with current from storage cells. The filament, while designed for use with 110 volts, was over-volted to about 120 volts, in order to increase the intrinsic brilliancy. The image of the glower was focused upon the first slit of the monochromatic illuminator. In front of this slit was placed a sectored disc, and an optical wedge, either of which, or both could be used to reduce the intensity of the light. The bundle of quasi-homogeneous light coming from the second slit of the illuminator (this slit was kept of the same width as the first one, usually about 0.7 mm.) fell upon a concave silvered mirror. This latter, together with the thermopile, and the selenium cell, was located in a light-tight box. This mirror could be rotated from without, so as to throw the focused image of the slit upon either the thermopile or the selenium. If it was desired to

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[1] This work has just been completed and will soon be published.

[2] Loc. cit.

cover the whole cell with diffused light, the cell had merely to be drawn out of focus. Thin glass plates of the same thickness covered the thermopile and the selenium cell.

The thermopile, of the linear Rubens type, was one of the new series-parallel instruments of Bi with an alloy of Bi and Sn, made by Dr. W. W. Coblentz.[1] Its resistance was 2.25 ohms. The galvanometer used with the thermopile was of the Thomson type, made by Siemens and Halske. With its coils in parallel its resistance was 1.35 ohms. The sensibility depended of course upon the period, and with a period of about 6 sec., which we usually employed, its sensibility was about 5×10^{-10} amp. With this thermopile-galvanometer arrangement, a deflection of 1 mm. was obtained on a scale at a distance of 125 cm. when an incident energy of 4.4×10^{-8} watts/mm.^{2} covered the thermopile strip. This latter determination was made with a calibrated carbon lamp at a distance of 2 meters, in accordance with the instructions in the certificate of the Bureau of Standards. In our experiment the area of the focused image of the slit was only 1/7 the area of the thermopile strip. Hence a deflection of 1 mm. in our actual experiments indicated an energy of 7×4.4 or 30.8×10^{-8} watts/mm.^{2} upon the thermopile, and hence upon the selenium cell.[2]

One other device should be described. This was the arrangement by which we could expose the selenium cell at a certain moment, and then throw in the galvanometer for a certain short time. The same device that opened the shutter from in front of the Nernst glower, operated a trigger that released a pendulum. As this pendulum started moving it removed a shutter, thus exposing the cell. As soon as desirable after this operation (usually at once) a key was thrown by this same pendulum thus connecting a galvanometer in circuit. At a certain time later a second key was tripped, disconnecting the galvanometer. The change of resistance could thus be determined by the galvanometer throw for a short interval and at any reasonable time after the exposure of the selenium. The selenium was placed in one arm of the Wheatstone bridge, in which various E.M.F.'s were used. In some cases long exposures were made, but unless otherwise indicated our curves are for mean exposures of 0.4 seconds. In either the long or short exposures we employed the same galvanometer described in our previous paper.

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[1] PHYS. REV., N. S., III., p. 59, 1914.

[2] A correction should be made to the footnote of our previous paper, Vol. II., p. 489. While a deflection of 1 mm. meant an energy of 9.3×10^{-8} watts/mm.^{2}, using the total area of the thermopile, the area of the image of the slit was, in this case, about 1/6 of that of the thermopile strip. Then in our previous experiments 1 mm. deflection corresponded to 6×9.3, or 55.8×10^{-8} watts/mm.^{2} upon the thermopile, and hence upon the selenium. So our present arrangement is approximately twice as sensitive as was the former.

As in our previous work, care was taken to adjust for equal energy throughout the observations for each curve. Possibly equal energy is not the proper basis upon which to compare light actions, but at any rate it is the most definite and fundamental basis upon which to work. While equal energies were obtained as determined by the thermopile, the question might nevertheless be raised as to whether equal values of the energy penetrated and acted upon the selenium. If the selenium were selectively reflecting this would be a serious source of uncertainty. However, in connection with a separate investigation we measured the amount of energy reflected from the surface of the selenium in a typical cell for which we determined the characteristic curve and we found that for each wave-length the total reflected energy was only a little more than one one hundredth of that incident of the selenium. Thus it appears that a selenium cell is a fairly good black body. We shall deal with the optical properties of selenium in a separate paper.

Effect of Varying Slit Width.—As is well known, the beam of light from a monochromatic illuminator is only a quasi-homogeneous beam, and its non-homogeneous character becomes more and more evident as the red end of the spectrum is approached. Any curves for such an apparatus can be corrected with considerable definiteness[1] for the slit width, although the calculation is rather a laborious one. In view of the labor involved in the calculation, it was thought best to try first an experiment to test the extent of the inaccuracy that arises from the use of a finite slit width.

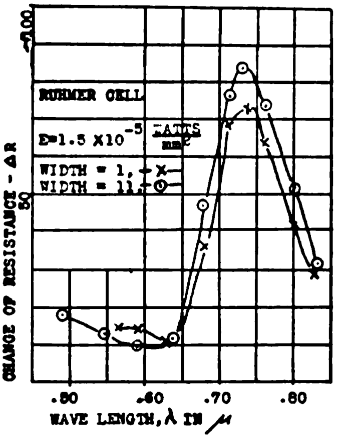

Fig. 1.

Fig. 1 represents the results of this experiment. The two curves are clearly indicated in the figure, the one for a very narrow slit with (about .14 mm. on both the collimating and focusing telescope) and the other for the unusual slit width of about 1.5 mm. The slit width was thus varied by about a factor of eleven. By means of the rotating sector the energy intensity was lessened to correspond to that of the narrow slit, thus giving the same change of conductivity in the selenium at wave-length $\lambda = .64\ \mu$. There is no special significance to be attached to the slight variation of the magnitude of the maximum. It merely indicates that there are small outstanding errors that nullify any necessity for

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[1] Paschen, Ann. d. Phys., 60, 712, 1897.

making slit width corrections. Having justified the use of such a wide slit width, we believe ourselves perfectly secure in our results which were usually obtained with slit widths of 7 mm. This gave us a maximum of energy with a negligible error from spectral impurity.

Effect of the Order of Exposure.—Thinking that the shape of the curves might vary slightly with the order of exposure to the light of varying periodicity, an experiment was tried the results of which are shown in Fig. 2. First a curve was taken for a Ruhmer cell by the usual procedure, *i. e.*, by measuring the sensibility at the shortest wave-length first and then proceeding step by step to the reading for the longest wave-length. Then this procedure was exactly reversed by taking the sensibility for the longest wave-length first and going backward. We concluded from these curves and a number of other cases where it was advisable to check readings that the order of exposure made no material difference. However, a little greater reliability was attached to the first order of exposure. The curves given in this paper are substantially correct. After readjusting our apparatus and replacing some of the cells, we were able to duplicate results with surprising accuracy.

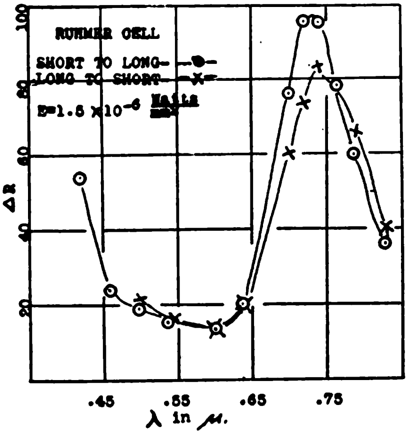

Fig. 2.

The Character of the Sensibility Curves.

General.—When this work was begun it was believed that sensibility-wave-length curves obtained by using short exposures should for all varieties of selenium show the same characteristics, providing that the rapid changes in selenium always arise from a common light-action on one component of the selenium. Or conversely it was believed that the sensibility curves should be complex, providing that the rapid changes in the selenium by light arise from action on two or more components of the selenium with widely varying rates. It is the converse view that naturally suggests itself following the analysis made by one of the authors.[1]

Fig. 3 shows the various types of sensibility curves which we have obtained with different samples of selenium, and we do not claim to have exhausted the list. The observed changes of resistance recorded on each

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[1] Phys. Rev., 33, p. 403, 1911.

curve were taken for exposures of equal intensity, wherever we found that the form of the curve was materially affected by change of energy. The scales for each curve are however arbitrarily chosen, so that the maximum ordinate of each curve lies at about the value 100. Thus the curves serve a purpose resembling a spectrum analysis of an element. The intensity of the lines can be compared with the lines of the same spectrum in order to establish the character of the unknown. In spectrum analysis the amount and the temperature of the element producing the spectrum are matters of secondary importance. Just so the curves in Fig. 3 are characteristic curves, but they neglect to bring out the absolute or even the relative sensibility of the different samples of selenium. The chart accompanying the curves gives the names of the types of cells used and also the energy intensity that resulted in the indicated curves. However it is rather unusual for the character of the curves to change with varying intensity such as had been previously observed both by Pfund[1] and ourselves.[2] Only two of the curves in Fig. 3 showed such a variation. In some instances tests showed the character to be only imperceptibly changed by intensity variations of the ratio of 50 to 1. Furthermore we found that the character of the curves was not necessarily much affected by the duration of exposure. Only two of the eight samples, whose curves we are discussing, showed such large variations. This also is new.

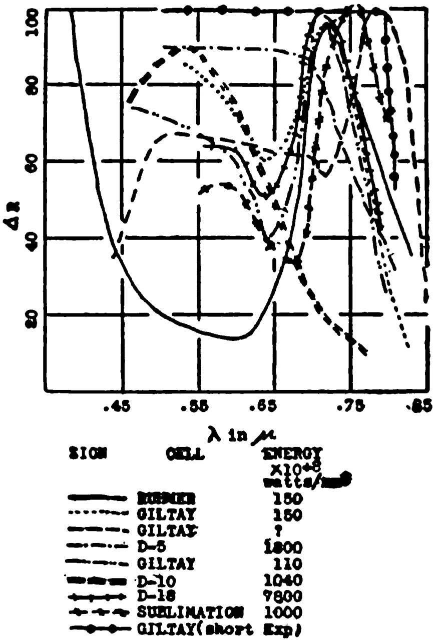

Fig. 3.

It is obvious that there is no characteristic curve to be found by the method of procedure that we followed. A Ruhmer cell in vacuum indicated a maximum in the ultra-violet too far out for us to locate, and another maximum at .72 μ. This selenium had a broad minimum extending from .55 μ to .62 μ. Recently Mr. Dieterich has succeeded in producing two samples of selenium with a maximum near the ultra-

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[1] Phys. Rev., 34, p. 370, 1912.

[2] Loc. cit.

violet. But with these exceptions all samples which have come to our notice decrease more or less precipitously in their sensibility back of .5μ. Many specimens have the indicated maxima at about .7 μ, but this maximum wanders all along to .8 μ, and in the typical Dietrich cell, as indicated in our paper by the letter D, this maximum dwindles or disappears altogether. Stated briefly the maximum that so frequently appears in the red, may be controlled both as to its location and its magnitude even to its disappearance. One cell has shown considerable sensibility out as far as .85 μ. Similarly as regards the minimum, which usually occurred about .64 μ. In the Ruhmer cell it occurred back as far as .6 μ and in the selenium whose crystals were deposited by sublimation in a high vacuum, it occurred at .68 μ and in one other cell at even longer wave-lengths. In one of Giltay's cells and also in one of Mr. Dieterich's cells, the fourth and ninth curves respectively in the chart, the minimum did not appear at all. At about .55 μ there was usually a broad maximum but it is observed that this region may contain a broad minimum as well.

Without any further work the conclusion seems obvious that light produces more than one action in the selenium, or in other words there is more than one component or kind of selenium if you please existent simultaneously in the selenium cell.

The Giltay and Dieterich Types.—While it is true that both Giltay and Dieterich do make cells of widely varying character, it is nevertheless true that all of Giltay's cells of which we have any knowledge have a pronounced maximum at .69 μ or beyond and a more or less well-defined one in the region of .55 μ. On the other hand Dieterich's samples usually have a very pronounced maximum about .55 μ, and a controllable maximum in the red which may be entirely eliminated by the method of making. This so called Dieterich type is shown by curve D, Fig. 4, where the energy intensity was 6.6×10^{-6} watts per mm.^{2} For greater or less illumination intensity, the character of the curve was as far as we could determine identical to this one. In the same figure, the curves G_1 and G_1' are for a typical Giltay cell, where the intensity of illumination varied by a factor of 20 to 1. It is common for the Giltay cells to maintain the same character unaltered for widely varying intensities, just as these curves indicate.

The Effect of Varying Electrical Intensity.—Since it has seemed advisable to regard the selenium as in a state of equilibrium under the action of a number of forces, it is reasonable to expect that an alteration of any one of these forces might alter the form of the sensibility curve. We have yet to test this point particularly for the varying temperature conditions. An examination of the voltage effect shows that the character of the curve is only slightly altered, while the sensibility is uniformly and materially altered in magnitude. The curve in Fig. 5 indicated by circles shows the characteristic curve of cell D7 for an intensity of 0.1 × 10^{-6} watts per mm.^{2}, when exposed for 30 seconds. There was a potential difference of 3.5 volts across the selenium. Next this was changed to 0.5 volts and the intensity altered till it produced the same change of resistance at .45 μ as was produced with less intensity and greater potential. With this intensity which was 18 times greater the curve indicated by solid dots was obtained. It is to be noted that altering the intensity by a factor of 18 to 1 had the same effect as altering the potential by a factor 7 to 1. However it must also be noted that while the deflections were nearly identical for the sake of comparison, that the absolute change of resistance was only about one seventh in the second case of what it was in the former.

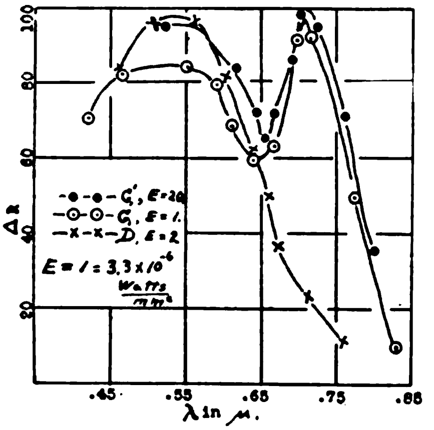

Fig. 4.

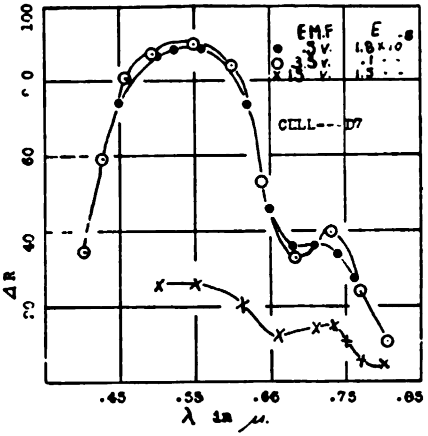

Fig. 5.

It may be estimated that a change of potential of the order of 50 to 1 will so balance the selenium in this cell that the same absolute change of resistance would be produced when the intensity of illumination is varied by a ratio of 18 to 1. The conclusion is that for this cell the alteration of the electric intensity alters the character of the curve only slightly, but that it makes a large difference in the absolute change of resistance by light. This last point has also been verified in former papers. The lower curve in the figure is for short exposures, and with 15 volts across the selenium. Thus changing the duration of exposure, the electrical intensity and the light intensity does not alter the character of the curves to any definite and marked degree in this particular cell.

The Maxima are Inherent in the Selenium.—Since the well-known experiments of Pfund and Berndt no one has questioned that at least a great part if not all the light sensitiveness of selenium is in the selenium structure itself and not because of some impurity or selenium compound. Yet when different samples show such widely varying characteristics, the question may well be raised again as to whether impurities do not play an important rôle in prescribing the characteristics. Mr. Dieterich is obtaining a large amount of evidence to prove that impurities are of very secondary importance if they enter at all. One point that may be mentioned is illustrated in Fig. 6. He took a Giltay cell marked "normal G" and subjected it to purely physical treatment, after which the characteristic curves were of the form indicated by curves E_1 and E_{10}, where the illumination intensity varied by a factor of ten. By this treatment which he will relate in detail, the Giltay cell was transformed into a Dieterich cell. It is also worthy of note that the character of the curves was unchanged by a variation of the intensity by a factor of ten. As has been previously pointed out, the selenium components seem to be put in a dynamic equilibrium in the process of making the cell under the action of a number of forces such as heat, and pressure treatment, and further, these components are maintained in equilibrium under a number of forces such as pressure, temperature, illumination, absorbed materials, electrical stress and the crystal formation itself.

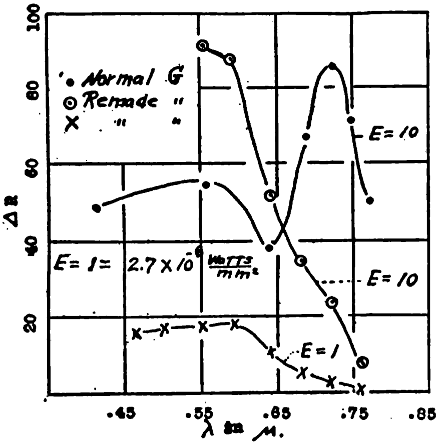

Fig. 6.

That the characteristics of any given specimen are inherent in the selenium itself is yet further verified by the fact that the maximum in the red can be put in or eliminated at will by the heat treatment. And it is expected that the exact conditions will soon be discovered which determine the presence or absence of the maxima in the shorter wavelengths of the visible spectrum and even in the ultra violet. The curves in Fig. 7 are taken from Mr. Dieterich's samples. The remarkable difference in these curves arose solely from the difference in heat treatment during the process of making. In fact he can predict almost without failure what will be the character of a given sample of light-sensitive selenium. It should be noted that the intensity of illumination was the same for all curves in Fig. 7.

The Variation of the Sensibility.—Generally the light sensibility of selenium is regarded as a function of the amount of selenium that has been transferred from the amorphous to the crystalline state. And yet it seems perfectly futile to attempt to explain the enormous variations in sensibility on this basis. The variations so greatly outnumber the possible combinations. Mr. Dietrich has been able to vary the sensibility over a very wide range without altering the character of the sensibility curve, but there seem to be a number of factors entering into the exact control, only a part of which he is aware of. Nevertheless he has produced a low resistance cell which shows an effective sensibility at .59 μ about ten times greater than that of any Giltay cell in our possession. Its character is shown by the curve D14, Fig. 8, for two intensities varying by ten to one. And again starting with a sample of the same amorphous selenium and proceeding differently with the treatment he produced a cell D11 which had about the same specific resistance in the dark and only a slightly different characteristic curve, but its sensibility as can be learned from the curves was only about one twentieth of that of cell D14. In making this deduction an allowance must be made for the lower voltage used with cell D14.

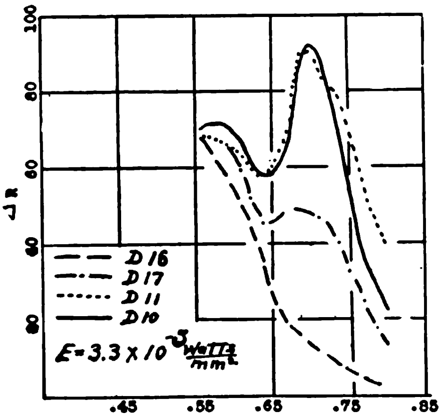

Fig. 7.

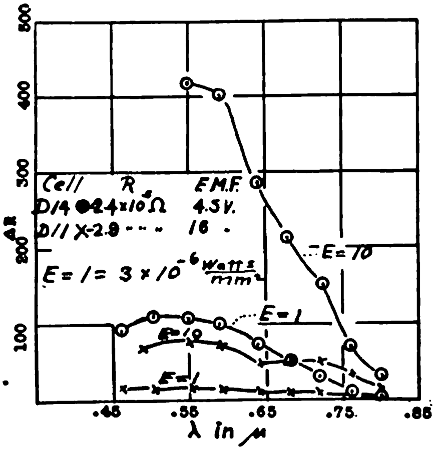

Fig. 8.

The Variation of Sensibility With Intensity of Illumination.

Quite recently Nicholson[1] has made a very able mathematical analysis of certain phenomena in selenium on the basis of one electron theory. Without discussing the reasonableness of his assumptions, it may be of interest to test some of his deduced equations over a wider range than

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[1] Phys. Rev., N. S., Vol. 3, p. 1, 1914.

was done in his work. He used only two varieties of selenium and he was only prepared to take exposures as short as 12.5 seconds. In applying his results to his theory he presumed that an exposure of such duration was equivalent in its effect to an instantaneous exposure. We are inclined to doubt this presumption. Our results should have the advantage that comes from a study of a larger number of varieties of selenium and also the advantage of exposures of 0.4 second duration, which more nearly approximate instantaneous effects.

The most critical test of Nicholson's theory perhaps lies in the derived relation between the change of conductivity and the intensity of illumination. This relation is expressed by the equation,

$$d = D \cdot I^{\beta},$$

where d is the change of conductivity, I is the intensity, and D and β are constants. According to Nicholson's theory β should have the value $\beta = 1/2$ for instantaneous exposures in the shorter wave lengths and $\beta = 1$ for the red and infra red, and $\beta = 1/2$ for unlimited exposures throughout the spectrum. The above equation was originally set up as an empirical equation by Pfund.[1] We find justification for the use of the equation when used as an empirical one and applied to any given cell, but we can not see that the simple assumptions regarding absorption, reflection and resonance as given in Nicholson's paper, are sufficient to explain the varying values of β in the different samples of selenium, nor the variations in a given sample. It must be remembered that the constant β occurs as an exponent, and consequently small deviations in its

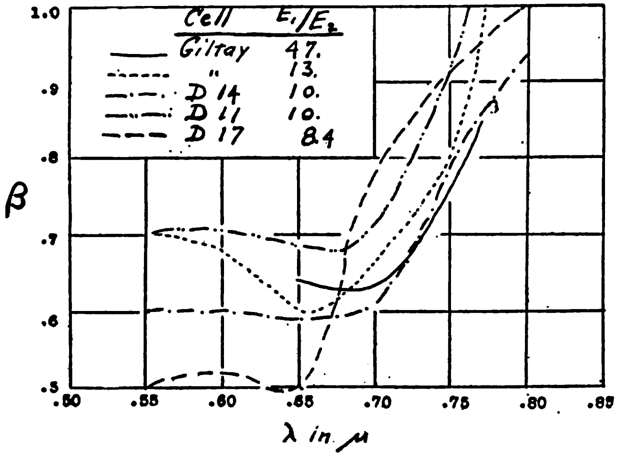

Fig. 9.

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[1] PHYS. REV., Vol. 34, p. 370, 1912.

value are of relatively greater importance than relative errors in a multiplying or additive constant.

The manner in which β varies for exposures of 0.4 second for a number of different samples of selenium is shown in Fig. 9. It is rather interesting that cells which gave such widely varying characteristic curves (see Figs. 3, 7, and 8) should show such a general likeness in the variation of the constant β, for the different periodicities of light. It is observed then that β varies in a general way between .5 and 1 for short exposures.

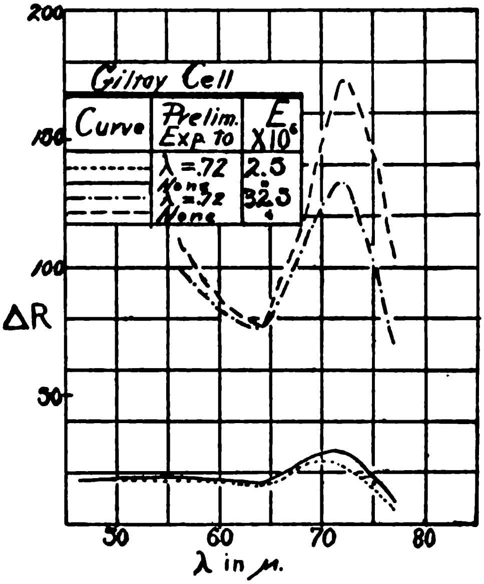

Fig. 10.

An attempt was made to discern whether any change in the light sensibility curve would result if before each wave length-deflection reading was taken, there was made a preliminary exposure to a fixed quantity of energy from a beam of light of a given wave length. Nicholson made exposure tests similar to this except that instead of making the additional exposure a preliminary one he left a steady exposure on the cell while he was

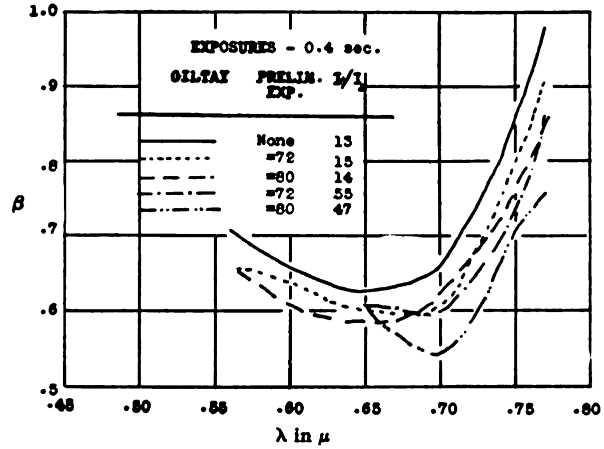

Fig. 11.

running a curve. In both cases the effect is much the same, and has the tendency to blot out the effects due to slow changes of resistance accompanying light-action. The effect of the preliminary exposure was slightly to iron out the maximum. However, the curves relating β and λ are not materially altered by the preliminary exposures. The first of these two results is shown in Fig. 10, connecting the change of resistance and wave-length; and the second point covering the variation of β and λ is shown in Fig. 11.

CONCLUSIONS.

The analysis made some time ago on the basis of the rates of change in the selenium, showed the structure of the selenium to be complex, consisting of three components in the specimens studied. Now we have made an analysis on an entirely different basis, that of the character of the sensibility curves, and our analysis reveals again a structure at least as complex as that revealed formerly by other samples of selenium. We are not yet able, however, in the latter analysis to set up a simple set of postulates whereby we can subject our analysis to a mathematical test as was done formerly. As far as can be seen our analysis requires at least two separate light actions in the selenium.

No doubt the action of light in any of the components is of such a nature that electrons are made free when one component changes to another, because the former analysis into several varieties of light sensitive selenium required that electrons be made free but in no case did it require the recombination of electrons. Knowing that the structure of light sensitive selenium is complex, and knowing what we do about the nature of this complexity we should now perhaps be in a position to apply the electron theory to an explanation of the phenomena.

We have isolated a number of new crystal forms of selenium by sublimation of selenium in a high vacuum. The conditions which determine what kind of crystals will form must be very definite, for we have had very large crystals of different forms grow up within a millimeter distance of each other where the temperature and pressure could not have varied greatly. In free unhampered space some of these crystals have a length of more than ten millimeters. We know of four formations of crystal selenium that are doubly refracting when formed in a vacuum. Whether the crystals in the selenium cell are doubly refracting we are not quite sure, and yet one cell made by the formation of crystals directly from the vapor state must have contained some doubly refracting crystals. The point that we wish to make here is that crystals which are directive in their structure towards light should also be directive in their electrical properties and if these crystals are instrumental in the conduction they should alter the conductivity differently depending upon their positions. If so we must in explaining the light sensitiveness of selenium allow for the formation of crystals either with axes pointed promiscuously or in consistently varying directions. Add to this a mixture of crystals in more or less stable equilibrium as to form and position and we have we believe a basis on which all the light electric properties of selenium may be worked out.

Previous results on the effect of abrasion[1] and on the pressure effect on the electrical conductivity indicated in a very marked degree that the slow changes accompanying light-action were the result of crystal changes. There was doubt concerning the seat of the rapid changes. It was conceded that perhaps light might expel electrons from the atoms in the transformation of one component to another. If in this present work we had found a single or slightly varying position of maximum spectral sensibility, we might have been justified in concluding with Pfund and others that the primary thing is the liberation of electrons from atoms of selenium. But if there is one significant conclusion that can be accepted as a result of the variation of the sensibility curves, it is that light does not act on the atoms as units. Rather a larger unit, no doubt of the order of a crystal unit, is the seat of the disturbance by light action. We are compelled to recognize crystal boundaries when dealing with the flow of electrons in selenium, possibly also in other elements. Otherwise there is no apparent basis for the unification of our knowledge.

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[1] Phys. Zeits., Nov. 15, 1912.

HIGH TEMPERATURE MEASUREMENTS WITH THE STEFAN-BOLTZMAN LAW.

By C. E. Mendenhall and W. E. Forsythe.

Introduction.

In view of the increasing use of extremely high temperatures (2000–3000° C.) the establishment of a reliable scale of measurement for this region becomes increasingly important. In almost all cases some form of radiation measurement is the most convenient if not the only possible method to use. At the hands of Day and Sosman[1] gas thermometry seems to have been pushed about to its limit in determining the melting point of palladium (1549° C.), and it is naturally desirable that the method used for extending the range should be something better than mere extrapolation of an empirical curve. There are four well-known radiation formulæ (for a perfect radiator) which might be available for this purpose:

(1)	$\int_0^\infty E_\lambda d\lambda = S = aT^4,$	Stefan-Boltzman law of total radiation.
(2)	$\lambda_m = bT^{-1},$	Deductions from Wien's displacement law.
(3)	$E_m = KT^5,$	
(4)	$\log E_\lambda = K_1 - \frac{C_2}{\lambda}\frac{1}{T},$	Wien's distribution law.

Of these, the last, (4) is usually the most convenient to use, being the basis of optical pyrometry, but its theoretical foundation is so uncertain that it amounts only to an empirical formula. The theoretical basis of (2) and (3) is certainly much better, but (2) evidently gives a very insensitive method, particularly at high temperatures, while the experimental use of either (2) or (3) involves complicated apparatus and extended observations. On the other hand (1) has the most simple and sound theoretical basis and can be very simply but accurately applied. There are in fact several well known "total radiation" pyrometers on the market, but these are usually calibrated empirically; and unless proper precautions are taken the method is liable to serious error. In connecting any one scale of measurement to another it is always desirable

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[1] American Journal of Science (4), 29, 93, 1910.

to compare them in the region in which they overlap, even though we are assuming that the two scales determine the same fundamental quantity, in this case ***absolute thermodynamic temperature.*** Hence the following work may be considered in three parts: (1) A comparison of the Stefan-Boltzman scale of temperature with the Day and Sosman gas scale between 1063° C. and 1549° C., the melting points of gold and of palladium. (2) A determination of the melting point of platinum in terms of the *S–B* scale. (3) A comparison of the optical temperature scale based on the application of Wien's law to a pyrometer of the Holborn-Kurlbaum type, with the Stefan-Boltzman scale up to 2820° C. Such a comparison has been carried out heretofore by Lummer and Pringsheim[1] up to about 2000° C., at which temperature their total-radiation temperature measurements involve discrepancies of nearly 20°, while their optical scale differs from the other by 10° or 15°. Gillette[2] has compared the two scales up to about 2500° C., and states that they "are in agreement," but does not state how good the agreement was, nor how accurately either of his two scales could be reproduced; from some figures he quotes it seems fair to infer that discrepancies of 10° in the neighborhood of 2200° C. were to be expected. In the work here described we have extended the range and considerably increased the accuracy.

Arrangement of Apparatus.—(*a*) Optical pyrometers. These have already been described,[3] though the spectroscopic eyepiece referred to in that article was not used in the present work; instead two eye screens of Schott No. 2,745 red glass each 3.37 mm. thick were used, giving a rather broad transmission band (200 A. U.) with center at $\lambda = 0.658\,\mu$ as determined by spectro-photometric measurements using radiation characteristic of about 2000° C. These instruments were calibrated by observations on black body furnaces held at the melting points of palladium and gold respectively. For the intermediate points and for extending the scale rotating sectors of known aperture were used, combined with equation (4) above, in which C_2 was taken as 14,500, and λ as $0.658\,\mu$. The optical pyrometer was calibrated once before beginning the work and once near the end; the two calibrations were in close agreement, but as there had been a long interruption in the work the first calibration is used for all points up to 2500° C., the last for the 2820° C. point. The pyrometer lamp filaments were of untreated carbon, of such size that a change of about 0.500 amperes in the lamp current corresponded to a change of 500° C. (from the melting point of gold to that of palladium). The lamps were never used above the apparent palladium temperature

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[1] Lummer & Pringsheim, Verh. Deut. Phys. Ges., 1, p. 3, 1903.

[2] Gillett, Journal of Physical Chemistry, 15, p. 213, 1911.

[3] Phys. Rev., July, 1911, p. 74, Vol. XXXIII., No. 1.

but were aged for 30 hours at a considerably higher temperature. The angular aperture of all the sectors used is known with sufficient accuracy for the present purpose, since the computed temperatures of the extended scale are but little affected by small variations in the aperture of the sectors. For example the most difficult sector to make, the "1/180" was found on measurement to have an opening of 2° 1′ 50″, but this departure from 2° made a difference of only 5° C. in the temperatures computed to extend the scale from 1549° C. to about 2800° C.

(*b*) With this method of calibration only one known fixed point is needed, to which all temperatures are referred; this is the melting point of palladium (1549° C.) as determined by Day and Sosman.[1] This temperature and the melting point of gold were reproduced in the laboratory by melting short strips of the metals held between platinum wires in electrically heated black-body furnaces provided with Pt-Pt Rh thermocouples. The thermocouples were merely for use in controlling the temperatures during the optical observations. Melting was indicated by an electrical signal. One black body was the standard form with diaphragms blackened inside; the other was made up of magnesia tubing and plate, with suitable diaphragms. No difference was found between the two. The gold used was the best from Eimer and Amend—the palladium the purest possible to obtain from Hereaus.

(*c*) The black body furnace and attachments for high temperature observations are shown in Fig. 1. Part of this apparatus has already been described.[2] The black body is a graphite tube (*T*) of 14 mm. inside diameter, 3 mm. walls and 30 cm. length, provided with a graphite diaphragm (*G*) placed 1 cm. away from the center, as shown. Hence one might expect better black body conditions in the longer segment of the tube, which was the only part used for primary observations. A glass window (*W*) opposite the short section permitted optical observations for purposes of control. A series of water cooled diaphragms D_1, D_3, D_4, D_5, served to protect the total radiation thermopile *P*, and a moveable water cooled diaphragm D_2, not only served as a shutter but contained the real limiting aperture *A*, which could be brought to a definite axial position against a stop. The geometric proportion was such that the pile *P* received radiation only from the graphite diaphragm (*G*) in the furnace.

A fundamentally important point is of course the accuracy with which the graphite tube furnace realizes black body conditions. Since graphite itself is a good radiator (having a reflecting power of only 21 per cent.

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[1] Loc. cit.

[2] Forsythe, Astrophysical Journal, 34, 353, 1911.

for red light at 2000° C.)[1] the importance of having a small ratio of aperture to surface of uniform temperature, is less than with a metal tube having a lower emissive power. The ratio of length to internal diameter of our tubes was greater than that frequently used with black body furnaces, which is an advantage. Our evidence for good black body conditions is two fold—first the satisfactory results of our comparison of the *S–B* scale with the gas scale (see below), and second a simple experimental test concerned with visible waves only. This consisted in drilling into the diaphragm *G* one or two fine holes, which would act as cavities within a cavity. The fact that these were not distinguishable from the plane face of the diaphragm is decided evidence in favor of good black body conditions.

The window W_2 above the thermopile allowed optical observations to be made on (*G*) when the aperture in D_2 was slightly displaced from its central position. Hence though optical and total radiation measurements of temperature could not be made simultaneously, they were at least made upon the same surface thus avoiding direct errors due to temperature differences in the furnace. All this careful screening not only eliminated stray radiation but also resulted in a very stable zero for the thermopile. The small motor *M*, mounted on a rod movable from the outside, carried a sector directly on its shaft. When the sector was swung up in front of the pile the radiation from the furnace was reduced in a known ratio depending upon the clear aperture of the sector. The sectors, made of two sheets of brass touching only at the center, to avoid the effects of absorption and re-radiation to the thermopile, were of such a size that two different apertures could be cut in each disc.

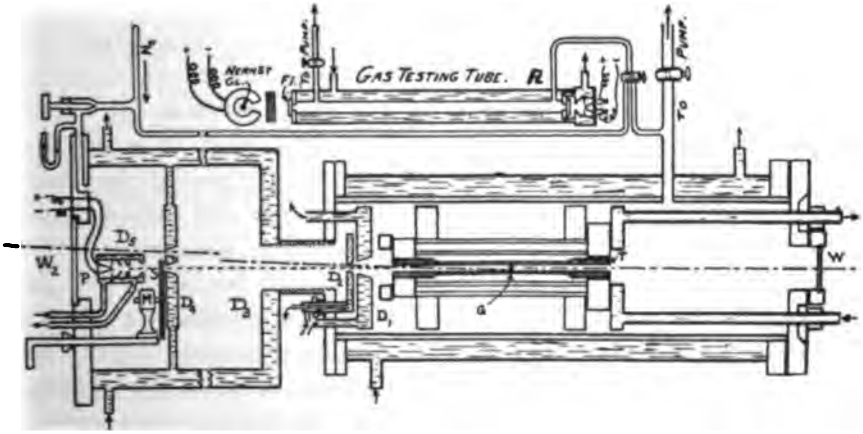

Fig. 1.

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[1] Mendenhall and Forsythe, Astrophysical Journal, June, 1913, p. 380.

The receiving thermopile consisted of a single junction of Bi-Sb and Sb-Cd alloys, the "hot" junction being soldered to a very light receiving disc of silver foil, 3 mm. in diameter, a similar disc being on each "cold" junction, where the alloys joined the copper lead wires. To still further reduce the drift of zero the water flowing immediately around the thermopile was passed through a large barrel of water, the thermal capacity of which effectively damped out small fluctuations. The receiving disc was blackened with acetelyne smoke as suggested by Coblentz, and mounted at the center of a hemispherical silver concave mirror according to Paschen. Under these circumstances the receiver has been assumed to be a perfect absorber, since only differential errors would be introduced if this assumption were not quite correct. The thermopile was connected in series with a low resistance Ayrton-Mather D'Arsonval galvanometer, and a resistance box for reducing the sensibility.

The necessary temperature correction to allow for the absorption and reflection of the glass window W_2, was carefully determined at one temperature, and computed for all others by using equation (4). Before the Stefan-Boltzman law can be legitimately applied it is necessary either to eliminate or determine the absorption of the gases in the furnace. The most dangerous constituents are water vapor and carbon dioxide. It was not desirable to work at very low pressure, since for one thing, it is difficult to maintain such pressures constant because of the gases given off by the hot parts of the furnace, and the sensibility of the thermopile is a rapid function of the pressure at low pressures. Furthermore the evaporation of the graphite tube is more rapid at low pressures. For these reasons a pressure of from 5 mm. to 15 mm. of mercury was used. In order to reduce as much as possible the danger from gas absorption, a steady stream of nitrogen was maintained at all times through the furnace. This was obtained by passing air over moist KOH, through a drier, over red hot copper turnings, and then again over KOH and P_2O_5. Exhaustion was maintained by a Fluess and a Gaede box pump. With steady pumping on the one hand and a graduated pin-valve controlling the inflow of nitrogen on the other, it was possible to maintain a pressure constant to within 0.2 mm. as indicated on a large mercury gauge observed microscopically. It was chiefly important that observations which were to be compared should be taken at the same pressure. The flow of the nitrogen was *from* the pile through the furnace tube. In order to test the absorbing power of the gas, the tube R (Fig. 1) was so arranged that it could be either exhausted or filled with gas from the main furnace tube. The ends of this testing tube were closed with fluorite plates, opposite one of which was a Nernst glower and opposite the other a thermopile. It was necessary that these should be outside the testing tube, in order that they might be unaffected as to temperature and sensiblity respectively, by changes in the gas content of the tube. The method of testing the gas was simply to compare the deflections produced by Nernst radiation transmitted through a vacuum (or low pressure nitrogen) and through the furnace gases, the current through the Nernst being maintained constant. In this way a very small loss by absorption could be detected, since the deflections were of the order of 30 cm. and could be read to 0.2 mm. It was always found that if the furnace was washed sufficiently at a high temperature by a stream of nitrogen, the absorption loss was negligible, not more than one tenth of one per cent. It may be objected that the gases in the furnace were hot while those tested were cold—but this would at least result in breaking down CO_2 to CO in the furnace, which would reduce the absorption.

Reduction of Observations.—Stefan-Boltzman Scale. If a certain deflection D of the thermopile galvanometer is obtained at a temperature T_1 abs. (always the gold or palladium melting point), with clear aperture, and the same deflection is obtained through a rotating sector, whose transmission ratio is $S(< 1)$, at a temperature T_2 (abs.) then

$$T_2 = \frac{T_1}{\sqrt[4]{S}}.$$

If it happened that the two deflections were not exactly the same (they never differed by more than a few parts in a thousand), it was assumed that the deflections were proportional to total radiant energy. The temperature of the shutter used to cut out the radiation from the furnace does not enter into the problem, because it and the rotating sector were at practically the same temperature.

Wien Scale.—If a current i in the pyrometer lamp gives photometric balance against a black body at temperature T_1 with clear aperture, and also at temperature T_2 through a sector of transmission ratio S, then

$$\log \frac{1}{S} = \frac{C_2 \log e}{\lambda}\left(\frac{1}{T_1} - \frac{1}{T_2}\right),$$

where T_1 and T_2 are the absolute temperatures in degrees Centigrade. To the value of T_2 determined from this equation must be added a correction for the absorption of the glass through which observations were made. This was independently determined and applied.

Results 1.—Comparison of Stefan-Boltzman and gas scale between 1063° C. and 1549° C. (Table I.). Twelve comparisons[1] were made.

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[1] Each comparison involved an independent set of observations and independent determination of the fundamental points.

Table I.

Summary of Total Radiation Data at 1063° C. and 1549° C. (Gas Scale) and 1755° C. on the Optical Scale.

1. Date.	2. Def. at 1063° C.	3. Def. at 1549° C.	4. Ratio 3/2.	5. Def. at 1755° C.	6. Ratio 5/2.
5–21	29.28	29.33	1.002	29.18	0.996
5–22	29.08	29.17	1.003	29.08	1.000
5–24	29.85	29.87	1.000+	30.03	1.006
5–26	29.77	29.97	1.007	30.04	1.008
5–29	29.37	29.32	0.998+	29.27	0.997
5–30	30.45	30.44	1.000–		
6– 1	29.89	29.92	1.001	30.11	1.007
6– 2	29.33	29.34	1.000	30.01	[1.025][1]
6– 3	29.33	29.33	1.000	29.47	1.005
6– 5	28.79	28.79	1.000	Mean......	1.003
6– 7	28.29	28.27	0.999		
6– 9	28.27	28.25	0.999		
		Mean......	1.001		

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[1] Rejected, because conditions became unsteady.

Table II.

Summary of Total Radiation Data at 2212° C., 2489° C. and 2500° C. on Optical Scale.

1. Date.	2. Approx. Temp., Opt. Scale, °C.	3. Def. at 1549 °C, Cm.	4. Def. at Temp. 2.	5. Ratio 4/3.
5–24	2212	32.26	32.15	0.996
5–26	2212	31.90	31.86	.998
5–29	2212	31.51	31.46	.998
5–29	2489	31.51	31.45	.998
6– 1	2212	32.16	31.98	.994
6– 2	2212	31.92	31.95	1.000
6– 3	2212	31.11	31.06	.998
6– 3	2489	31.11	31.08	.999
6– 5	2212	31.40	31.34	.998
		(At 1755° C.)		
6– 7	2500	32.67	32.63	.998
	Mean ratio for 2212..................................			.997
	Mean ratio for 2489 and 2500..........................			.998+

Table III.

Summary of Optical Pyrometer Readings (Current in Lamp) at 2820° C. on S.-B. Scale.

Date.	Opt. Pyr. Reading, Amps.	Date.	Opt. Pyr. Reading, Amps.
1–19	1.077	2– 7	1.080
2– 5	1.079	2–12	1.080 +
Mean..			1.079

The total-radiation sector having been cut in the ratio $[(1063+273)/(1549+273)]^4$, the ratio of the corresponding deflections should have been unity; the observed mean ratio was 1.001, the maximum ratio being 1.007 and the minimum 0.998. This indicates an agreement within the possible errors of experiment, which are about ± 0.5° C. at each point. The accuracy is limited by the optical rather than the total radiation measurements.

Results 2.—The melting point of platinum. Our value for this depends on three very concordant direct determinations of the melting point in a graphite tube black body furnace, carried out by Mr. C. G. Peters. The platinum was enclosed in porcelain and protected from the action of reducing gases by a flow of nitrogen. Mr. Peters's single transfer of this result to the *S–B* scale gives 1755° C.; our repeated transfer gives 1753° C. which should be given more weight. The indication is that the present accepted[1] value of 1753° C. is not far off. The individual determinations differed by less than a degree. This determination will be repeated, as it involves difficulties and uncertainties not present in the remainder of the work.

Results 3.—Comparison of the Stefan-Boltzman and Wien scales (Tables II. and III.). Comparisons were made at four temperatures as follows:

TABLE IV.

Number of Comparisons.	Temperature.	$T_W - T_{S.B.}$	Range of Observations.
9	1750° C.	less than ± 0.5° C.	
7	2200	less than + 2	2° C.
3	2500	about + 2	4
4	2820	about + 4	7

These differences may all be taken as not greater than the experimental error, but it is noticeable that they show a definite trend with temperature. To bring the two scales into agreement it is only necessary to assume a slightly different C_2 or λ; a change from 0.658 μ to 0.657 μ would accomplish the result, and this change is probably less than the uncertainty in the knowledge of λ. The uncertainty in the knowledge of λ prevents any final conclusion being drawn as to the value of C_2. The equipment is now being used in conjunction with a spectroscopic pyrometer especially for the accurate determination of C_2.

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[1] Day and Sosman, Journ. de Physique, 2, Ser. 5, 899, 1912.

SUMMARY.

The Stefan-Boltzman law has been used with greater accuracy than heretofore to extend the high temperature scale to 2820° C. and the optical scale based on the use of Wien's law with $C_2 = 14{,}500$ and $\lambda = 0.658\ \mu$ (for Jena glass 2,745) has been shown to be in very close agreement with the total-radiation scale. The total-radiation scale has been compared with the Day and Sosman scale between 1063° C. to 1549° C. and found to agree with it to at least 0.5°. Three determinations of the melting point of platinum gave the result 1753° C. on the *S–B* scale.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF WISCONSIN,
February, 1914.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

METHODS OF MEASURING TIME CONSTANTS OF LOW RESISTANCES.[1]

BY FRANK WENNER, ERNEST WEIBEL AND F. B. SILSBEE.

ALL values heretofore obtained for the residual inductance of "non-inductive" standards of low resistance have been based upon the calculated inductance of some conductor. This calculation of the inductance from dimensions involves serious assumptions as to current distribution, inductive effects in potential leads, etc.

The first method discussed below gives primarily the sum of the time constants of the two resistances used. By using three resistances three sums can be measured from which each time constant can be obtained, without calculation of the inductance of a resistance standard. In this method the resistances are each connected in series with a mutual inductance as indicated in the figure and excited by alternating currents substantially in quadrature. The constants of the circuits are then adjusted so that no current flows in either galvanometer. It can easily be shown that the following relations then hold:

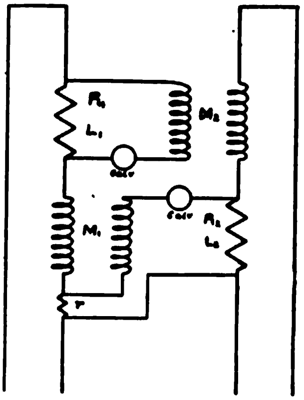

Fig. 1

$$R_1R_2 = p^2(M_1M_2 + L_1L_2) \tag{1}$$

$$\frac{pL_1}{R_1} + \frac{pL_2}{R_2} = \frac{prM_2}{R_1R_2} \quad \text{or} \quad \frac{r}{pM_1} \quad \text{very closely,} \tag{2}$$

where $p = 2\pi \times$ frequency. The first of these equations can be used in the absolute measurement of resistance and the second gives a measure of the sum of the phase angles of the two resistances.

This method requires (1) that the mutual inductances be pure, *i. e.*, give e.m.f.'s in exact quadrature with the primary current, (2) that the frequency be

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24–25, 1914.

held very constant during the measurement and (3) that the current be sinusoidal or that the detectors be sufficiently selective to the fundamental frequency.

The second method makes use of a standard whose resistance can be changed without changing its inductance. This can be done for example by using a copper standard whose temperature can be varied. Then if the observed decrease in phase angle be $\Delta\theta$ and the corresponding proportional increase in resistance be a we have

$$\frac{pL}{R} = \frac{1 + a}{a} \Delta\theta \tag{3}$$

This method requires merely an accurate measurement of the change in the phase angle.

In order to test the practicability of these methods the authors have made measurements on four low resistances. First, all of the six possible sums were measured directly by the first method. The phase angles of the individual standards at 60 cycles are given in Table I. These four values satisfy the six observations to within two minutes.

TABLE I.

Designation.	Resistance.	Phase Angle.
C	.000337 ohm	29.0′
K	.01 ohm	2.5′
R	.001 ohm	19.0′
S	.001 ohm	16.5′

Measurements by the second method on the copper standard (C), using a temperature rise of about 60° C., gave a phase angle of 30.5 minutes. This value agrees fairly well with the value given above.

To get further checks on the results several differences of the phase angles were determined by an independent method. The observed differences and those computed from the values in Table I. are given in Table II. below.

TABLE II.

Designation.	Differences Observed.	Computed.
$R-S$	1.9′	2.5′
$C-R$	10.1′	10.0′
$R-K$	14.8′	16.5′

It is interesting to note that accurate calculations assuming uniform current distribution give $C = 24.1$ minutes and $K = .8$ minute.

The above results indicate the methods to be entirely practicable and it is thought that by taking further precautions as to frequency control, purity of mutual inductance and sensitivity a precision of several tenths of a minute can be obtained.

EFFECT OF RESIDUAL GASES ON CONTACT E.M.F.'S AND PHOTO- CURRENTS.[1]

BY R. A. MILLIKAN AND W. H. SOUDER.

CURVES representing both the variation of contact E.M.F.'s and saturation photo currents with time have been taken on freshly cut sodium surfaces both in the best vacuum obtainable with a Gaede molecular pump and in vessels containing residual gases.

The most striking characteristics of the photo-current-time curves is that they are of altogether different form according as the frequency of the light with which they are taken is that of the extreme ultra-violet or that of the visible spectrum. Thus, when tested with light of wave-length 5,461 Å. the photo-sensitiveness of a *freshly cut* sodium surface is zero. It rises with time to a considerable value and finally falls again to zero When tested, however, with light of wave-length 2,535 Å., this same surface shows a large initial photo-sensitiveness which falls off steadily with time to zero.

These photo current-time curves taken in connection with the contact E.M.F.-time curves are interpreted as follows:

A freshly cut sodium surface is not sufficiently electro-positive to respond photo-electrically to light of wave-length 5,461 Å., but under the influence of an active gas, the sodium forms a new substance which is more electro-positive than the freshly cut sodium and hence is photo-sensitive to longer waves. The photo-curve taken with wave-length 5,461 Å. represents then merely the growth and decay of this substance. For sufficiently short waves, however, the freshly cut surface is itself so photo-electrically active that its own decay curve completely masks the rise and fall curve due to the growth and decay of the more electro-positive substance resulting from the action of gas upon the sodium.

In view of these results the authors raise a question as to the correctness of the conclusions of Wiedmann and Hallwachs[2] that photo currents are only obtainable in the presence of gas. We suggest that if the Wiedmann and Hallwachs experiments are repeated with light of sufficiently short wave-lengths instead of with visible light, it is not likely that the photo-sensitiveness will be found to vanish in the way in which it did in the experiments reported by these authors. Further experiments on potassium and lithium are in progress.

A DIRECT DETERMINATION OF "*h*."[1]

BY R. A. MILLIKAN.

THE experiments herewith reported were undertaken for the sake of subjecting to rigorous experimental test the three assertions contained in Einstein's photo-electric equation[3]

$$\tfrac{1}{2}mv^2 = PDe = h\nu - P.$$

These assertions are

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 24, 1914.

[2] Ber. d. D. Phys. Ges., 16, 107, January 30, 1914.

[3] A. Einstein, Ann. d. Phys. (4), 20, 199, 1905.

1. That there is a linear relation between the frequency of the impressed light and the maximum energy of emission of the electrons ejected by it.

2. That the slope of the line representing the linear relation between *PD* and ν is h/e, *i. e.*, that this slope times e is Planck's "h."

3. That the intercept of the *PD* line on the ν axis gives the frequency ν_0 at which the metal in question first begins to be photo-electrically active.

The second and third of these assertions have not heretofore been made the subject of accurate test nor can they be so made without simultaneous measurement *in vacuo* of both contact potentials and photo-potentials in the case of metals which are sensitive throughout a long range of frequencies. The spectral lines used in the test must further be strictly monochromatic and of frequency determinable with a high degree of precision.

The first of the assertions of Einstein's equation has previously been tested with widely varying results by Ladenburg,[1] Kunz,[2] Hughes,[3] Richardson and Compton,[4] and Cornelius.[5] The most satisfactory of these measurements are probably those of Richardson and Compton, though Pohl and Pringsheim[6] do not regard even these as sufficient to distinguish between $PD \propto \nu$ and $PD \propto \nu^2$, much less to test between any closer relations such as $PD \propto \nu$ and $PD \propto \nu^{\frac{3}{2}}$, the latter being the relation implied by Lindemann's theory. Very recent measurements made in the Ryerson Laboratory by Kadesch[8] between $\lambda = 3{,}900$ and $\lambda = 2{,}300$ seem however to speak positively in favor of a propoitionality between *PD* and ν. The present measurements made on sodium from $\nu = 6{,}800$ to $\lambda = 2{,}300$ fully confirm Kadesch's linear relationship and further, they fix the value of the slope, *i. e.*, of h/e at 4.123×10^{-15} volt/frequency with an error of not more than $\frac{1}{2}$ per cent.

Inserting the author's value of e, viz., 4.774×10^{-10} which is thought to be correct to within .2 per cent., there results from this value of h/e

$$h = 6.561 \times 10^{-27}.$$

This value of "h" combined with the author's value of e gives with the aid of Planck's equation the following value for the constant of total radiation

$$\sigma = 5.688 \times 10^{-5},$$

which is very close to the value obtained by Coblentz[9] from a study of all the data at present available.

When this value of σ is combined with the author's value of e and substitution made in Planck's equation, the Planck-Wien constant of spectral radiation

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[1] E. Ladenburg, Verh. d. D. Phys. Ges., 9, 504, 1907.

[2] J. Kunz, PHYS. REV., 29, 212, 1909; also 33, 208, 1911.

[3] A. L. L. Hughes, Phil. Trans., 212, 205, 1912.

[4] O. Richardson and R. T. Compton, Phil. Mag., 24, 575, 1912.

[5] D. W. Cornelius, PHYS. REV., 1, 16, 1913.

[6] R. Pohl u. P. Pringsheim, Verh. d. D. Phys. Ges., 15, p. 637, 1913.

[7] F. A. Linderman, Verh d. D. Phys. Ges., 13, 1107, 1911.

[8] W. H. Kadesch, PHYS. REV., May, 1914.

is found to be

$$c_2 = 1.434$$

which is within one part in 700 of the latest Reichsanstalt value of this constant.

The value of ν_0 obtained for sodium, the metal upon which all of these experiments have been made, is $.439 \times 10^{15}$, which corresponds to $\lambda = 6{,}800$.

Planck's constant "*h*" is thus found to stand out in connection with photoelectric measurements, perhaps more sharply, more exactly, and more certainly than in connection with any other type of measurements thus far made.

ERRATUM.

Page 486, Vol. III., No. 6, Second Series, lines 13 and 12 from the bottom of the page, in the abstract of a paper by G. W. Middlekauff and J. F. Skogland, entitled "Characteristic Curves of Tungsten Filament Incandescent Lamps and Their Application in Heterochromatic Precision Photometry," should read as follows: x = log per cent. voltage and y = log per cent. candlepower, log per cent. watts, log per cent. current, or log actual watts per candle."

Through typographical error, in the printed text, "cent." after "per" was omitted in four places.

NEW BOOKS.

Vorlesungen Über die Theorie der Warmestrahlung. By DR. MAX PLANCK. Leipzig: Johann Ambrosius Barth, 1913. Pp. xii + 206. Price, Mk. 7.

There is no doubt that the first edition of this volume of Planck's, and the articles which preceded it, have been the most stimulating contributions to physical literature of many decades, and since the second edition involves some radical changes in the point of view it is of fresh interest and importance.

The characteristic feature of the quantum hypothesis as used in the first edition to deduce the energy distribution law for a black body or perfect radiation was this, that the energy content of a resonator must always be an integral multiple of energy ϵ, whose magnitude depended on the frequency of the resonator, being $h\nu$, where ν is the frequency of the resonator and h is the quantum or unit of activity introduced by Planck. This idea of integral energy-content involved the further hypotheses of discontinuous absorption and emission of radiant energy on the part of the resonator. The commonly accepted idea of the early quantum theory seems to be that this discontinuity of absorption and emission necessarily involves the idea of attributing a granular *structure* to radiant energy in space. This is not necessary however, and the discontinuous behavior might better be attributed to a peculiarity of resonator (atomic) structure. In the second form of the quantum theory of radiation as presented in this new edition, and the articles on which the revision is based, the hypothesis of discontinuous changes in energy content is given up, absorption is supposed to be a continuous process taking place according to the ordinary laws of electro-dynamics, while emission is still considered as discontinuous. It is supposed that a resonator can only emit when its energy content is some integral multiple of $\epsilon = h\nu$. When emission occurs the resonator loses all of its energy, and the probability of emission actually occurring at the possible times is determined by a probability function which increases as the energy content increases. This change in the point of view, as well as an article of Planck's which has appeared subsequent to the publication of this second edition of his lectures, makes still more evident that the quantum theory is really a theory of resonator structure, or atomic structure, if it is to be extended to anything beyond Planck's ideal resonators. The fundamental hypotheses as to the relation between entropy and probability, and the definition of probability remain as in the first edition, but the new point of view necessitates an entire rediscussion of the relation between the mean energy and entropy of a group of resonators. The restriction that energy interchange occurs only between resonators of the same frequency, so that the resonators as a whole would not of themselves assume a particular equilibrium distribution of energy, still holds as in the first treatment, though in the later article above referred to Planck suggests one way of removing this restriction. The outcome of the whole discussion is the well-known Planck law of energy distribution.

Planck was led to the change in hypotheses which we have just outlined because the hypothesis of discontinuous absorption seemed unreasonable. A skeptic might question whether discontinuous absorption is any more unreasonable than discontinuous emission—or indeed whether "reasonable" and "unreasonable" have any meaning whatever in connection with much of the present day discussion, but there is no doubt of the clearness and frankness with which Planck has stated his hypotheses. Nor does there seem to be any doubt that he has introduced a constant "h," which is of far-reaching significance, whatever that significance may be, and the tremendous stimulating power of his ideas is evident to every one. C. E. M.

Electric Arcs. Experiments upon arcs between different electrodes in various environments and their explanation. By C. D. Child, Professor of Physics at Colgate University. The D. Van Nostrand Co. 5×7, cloth, 203 pp., 58 illustrations. $2.00.

This book deals with the arc primarily from the point of view of pure science rather than that of industry, though its conclusions have a practical as well as a scientific interest. It is a well-written, careful presentation of the often conflicting results of many investigators, made by one who has himself carried out important researches in this field. Though fifteen years have passed since Mrs. Ayrton's book on "The Electric Arc," our knowledge of it is still in many respects incomplete and unsatisfactory. Hence, instead of giving "a brief and definite statement of the laws governing the action of the arc," the author has been compelled, as he says, to present "an extended review of what different experimenters have thought about these laws," and, where the laws are unknown, to "discuss the results of those who have endeavored to find them." Such a method is likely to lead to a wearisome collection of details; but in this case the discussion is ably conducted, terse and constructive. Hundreds of references to original articles make the book especially valuable to future investigators. To most of us the final chapter on the theory of the arc is of chief interest. The author's conclusions as to the mechanism of the arc, though largely unproved by experiment, yet seem to follow convincingly from the preceding analysis of available data. They give the most complete and suggestive explanation of the phenomena so far advanced. G. S. F.

University of Wisconsin.

Rays of Positive Electricity and Their Application to Chemical Analyses. By Sir J. J. Thomson. New York: Longmans, Green and Co., 1913. Pp. vi + 129. Price $1.40.

This volume is one of a series of monographs on physics edited by Sir J. J. Thomson, O.M., F.R.S., and Frank Horton, D.Sc. It contains an account of the earlier experiments by Goldstein, Wien, and others, on anode or positive rays and also of the later ones in this same field by Sir J. J. Thomson himself. Full descriptions are given of the various types of apparatus used to investigate the properties of these rays. Accounts are given of the experiments which established the existence of multiply charged atoms, of allotropic forms of neon and of hydrogen, and of the molecular groupings represented by CH, CH_2, and CH_3. Reference is also made to the Doppler effect observed by Stark, Steubing, and Wendt in their investigations of the spectra of positive rays. The book concludes with a description of Sir J. J. Thomson's latest experiments on the evolution of helium and neon by substances bombarded by cathode rays. The work is full of suggestions and will prove a valuable aid to those following or wishing to become familiar with the manner in which positive rays have been used to add to our knowledge of the ultimate structure of matter.

J. C. M.

Recent Physical Research. By DAVID OWEN. London: The Electrician Printing and Publishing Co., 1913. Pp. i + 156. Price 3s. 6d.

This volume, which is a reprint of a series of articles published in the Electrician, aims at giving an account of a number of the more important of the recent advances in physics. The subjects dealt with are (1) rays of positive electricity and their applications to chemical analysis by Sir J. J. Thomson, (2) Rutherford's and Geiger's method of counting the alpha particles, (3) the work of Curie, Langevin, and Weiss, in developing a theory of magnetism, (4) Birkeland and Störmer's theory of the origin of auroræ, (5) the Brownian movement as investigated by Perrin, (6) the pressure of radiation, (7) Rubens, work on very long infra-red rays. (8) determinations of the wave-lengths of Röntgen rays, (9) the electron theory of conduction in metals, and (10) Heusler's aluminium-copper-manganese magnetic alloys. The various accounts are written with care and accuracy and the book should be useful to those for whom the original papers are not accessible. Subjects which might have been included in the book with advantage are: Paschen's work on series spectra, and the contributions of Wood, Lenard, and others, to our knowledge of phosphorescence and fluorescence phenomena. J. C. M.

Die Theorie der Strahlung und der Quantum. Verhandlungen auf einer von E. Solvay einberufenen Zusammenkunft (30. Oktober bis 3. November 1911). Edited by A. EUCKEN. Halle: Wilhelm Knapp, 1914. Pp. xii + 405. Price Mk. 15.60.

This book is the counterpart in German of the French report of the Solvay conference which was published in 1912 by Langevin and de Broglie. Eucken has added a valuable appendix giving an account of the development of the quantum theory down to the summer of 1913. The papers presented at this conference are of the highest order. H. A. Lorentz discusses the application of the equipartition of energy to radiation. Jeans presents the kinetic theory of specific heat by Maxwell's and Boltzmann's methods. Warburg describes experiments to verify Planck's formula for normal radiation and Rubens describes experiments with long waves. Planck discusses the laws of radiation and the quantum hypothesis; Knudsen, the kinetic theory and the properties of the ideal gas; Perrin, the proof of the existence of molecules. Nernst applies the quantum theory to physical and chemical problems. Kamerlingh Onnes deals with electrical resistance. Sommerfeld applies the quantum theory to radiant pulses. Langevin presents the kinetic theory of magnetism. Einstein discusses specific heats. Lord Rayleigh contributes a short letter. Other leaders of modern physics, Rutherford, Mme. Curie, H. Poincaré and others, took part in the discussions. The book is of the greatest interest and importance. W. F. M.

Physikalische Chemie der homogenen und heterogenen Gasreaktionen unter besonderer Berucksichtigung der Strahlungs- und Quantenlehre sowie des Nernstschen Theorems. By Dr. Karl Jellinek. Leipzig: S. Hirzel, 1913. Pp. xiv + 844. Price, Mk. 32.50.

This book contains much more than its name indicates. It opens with a study of Carnot's cycle and in the first two hundred pages there are given a development of the more modern thermodynamics, so far as it is needed in the applications to gases, and a sketch of the kinetic theory of gases. Nernst's principle of thermodynamics is considered in this portion of the book from its relations to entropy. Over one hundred and fifty pages are then given to an exposition of the theory of radiation and over one hundred pages more to the study of the relations of the theory of radiation to the theories of specific heats, of heats of reaction, and to Nernst's principle. The experimental work connected with the foregoing theories is described in two hundred pages more. The last hundred pages deal with the electrochemistry and the photochemistry of gases. The physical chemistry of gases takes up but a small part of the book. The author has studied his subject fully, he gives careful criticisms of the points of difficulty which occur so often in these modern theories, and has made a book which will be very useful for study and for reference.

W. F. M.

Die Entwickelung des Temperaturbegriffs im Laufe der Zeiten. By K. Meyer. Braunschweig: Vieweg and Sohn, 1913. Pp. 1 + 160. Price, Mk. 4.

The greater part of this small volume is devoted to the development of the concept of heat and temperature from the invention of a thermometer (about 1610) to the introduction of the Kelvin absolute thermodynamic scale. A very brief introductory chapter merely sketches the earlier peroid, before Galileo, and an equally brief final chapter states, in very good form, the hypotheses on which our present measurements rest, indicates the thermodynamic connection between the Kelvin scale and a gas scale, and gives a few results of measurements at extreme low temperatures. The main body of the book is well indexed and full of references, but is more largely concerned, of necessity, with quaint ideas than with important logical developments. No mention is made of the growth of the kinetic theory of matter in its relation to temperature, not of the relation of radiation theory to temperature, though both of these are of fundamental importance in the development of the concept. C. E. M.

Optique Geometrique. By J. BLEIN. Paris: Octave Doin et Fils, 1913. Pp. vi + 263. Price, 5 Fr.

The usual topics of geometric optics are clearly and briefly treated in this small volume. Of the eleven chapters three are devoted to the general subject of image formation, three to the general discussion of optical instruments, as to their magnifying power, field, resolving power, focal surface and brightness, and three to the more detailed discussion of aberrations and the characteristics of various types of objectives. There are enough diagrams to make the treatment clear, and the usefulness of the book is increased by an index and bibliography. C. E. M.

Researches in Color Vision and the Trichromatic Theory. By SIR WILLIAM DE W. ABNEY. New York: Longmans, Green and Company, 1913. Pp. x + 418.

As stated by the author in his preface, this book was written with a two-fold purpose, namely: to make his published contributions (extending over twenty-five years) more conveniently accessible; and, "to show that the trichromatic theory of color vision does not yet require a funeral oration over its remains."

It is divided into two parts, of which the first, comprising one third of the book, has been the subject of lectures to students. A general introductory chapter is followed by one on the eye and another on phenomena in vision. A description of measuring instruments is then followed by a chapter on the intensity of spectrum colors and one on their relative brightness or "luminosity." With a chapter on each of the subjects: "Complementary and Contrast Colors," "Numerical Registration of Colors," and "Color Disks," the first part is completed. Part II. opens with an interesting chapter on Extinction of Color and Light—a corresponding laboratory investigation of the commonly observed change in hue, as the day closes, of bright-colored flowers and foliage. This is followed by another on "Color Fields"—the areas on the retina capable of distinguishing the various colors—after which the author turns his attention to the theory of color vision and the various experimental observations incident to this study, of which that on color blindness occupies a large part. The style is lucid and the typographical effect pleasing. C. A. S.

Les Propriétés Optiques des Solutions. By C. CHÉNEVEAU. Paris: Gauthier-Villars, 1913. Pp. vii + 240.

This is essentially a second edition of the author's work entitled Recherches sur les propriétés optique des solutions et des corps dissous. It is of value to the physicist who is especially interested in the optical properties of solutions, and to the chemist who seeks a knowledge of chemical composition of bodies by means of their optical properties.

The first two chapters are devoted to a critical presentation of the theories of refraction and dispersion including those of Havelock and Langevin—the most recent. The next three present the results of the experimental study of refraction and dispersion of aqueous solutions (and mixtures) of inorganic substances, and also of those not entirely aqueous. Chapter VI., of special interest to chemists, deals with the determination of the chemical composition of organic substances by means of their optical constants. The book closes with a chapter, uniquely placed, on apparatus and methods of measurement. The large bibliography at the end shows the vast amount of work which has been done in this and closely allied fields. C. A. S.

Medizinische Physik. By PROF. DR. OTTO FISCHER. Leipzig: S. Hirzel, 1913. Pp. xx + 1120. Price Mk. 36.

This voluminous work attempts to develop those somewhat advanced principles of physics which lie beyond the scope of the ordinary elementary text, but which are indispensable from the point of view of the advanced student of medicine. The method of treatment while aiming to be non-mathematical involves quite freely the differential calculus, but not the integral. The various subjects are developed solely from examples of their application in the animal body. For instance the the center of motion ofgravity of the human body in walking serves as a basis about which to develop ideas of composition of velocities and accelerations, their tangent and normal components and methods of graphical representation including the hodographs. Rotations are taken up in connection with linkages and the detailed analysis of the motions of the arms and the legs is given. The humero-radial linkage serves to introduce the idea of instantaneous axis of rotation and generalized coördinates are used.

Under the excellent title of Muscle Mechanics are introduced the various ideas of mass and force. The composition of forces, and moments with the resulting effects of motion, rotatory and translatory, with reference to the center of mass are studied from examples of various muscle and bone combinations familiar to the anatomist.

In acoustics the development of the fundamental principles of wave motion and the applications to sound, together with Fourier's analysis, quite obviously, has nothing to do with the human body. An elaborate treatise of the physical phenomena occurring in the ear, its structure, theories of audition and the mechanism of the production of speech makes this section in keeping with the general plan.

Optics from the geometrical standpoint is handled at great length, occupying about five hundred pages. The applications of the laws of reflection and refraction not only to the human eye but to lenses of all sorts, the telescope, the microscope in great detail—together with general considerations of the lens errors, astigmatism, spherical and chromatic aberrations, go rather far afield from the general plan of confining attention to the human body. The Abbe theories of image formation, ultramicroscopy and the applications of polarized light to the polarization microscope and the polarimeter conclude this division and the entire work.

Of the tremendously important applications of electricity to modern applied physics and to medicine no mention is made whatever, it being suggested in the preface that medical men are better informed in these directions.

H. B. L.

UNIVERSITY OF CHICAGO.

Bausteine zur Flugbahn-und-Kreisel Theorie. By AUGUST DAHNE. Berlin, R. Eisenschmidt, 1914. Pp. i + 44.

This monograph is of primary interest to students of ballistics. Its author, an army officer, assuming familiarity with the gyroscopic properties of projectiles comments on the causes of the well-known deviations of the axis of rotation. Having previously demonstrated that in tops these deviations are caused primarily by friction effects on the axis, in this pamphlet he shows that for projectiles too the air friction (Magnus effect), is the true cause and that the experimental facts brought out during the last 36 years are in accord with this and do not bear out the Poinsot theory which is based on the principle of infinitely small rotations.

H. B. L.

UNIVERSITY OF CHICAGO.

Optique Physique. By R. W. WOOD. Translated from English by H. VIGNERON and H. LABROUSTE. In two volumes: Tome I. Optique Ondulatoire (translated by H. Vigneron), 1913; Tome II., Étude des Radiations (translated by H. Labrouste), 1914. Paris, Gauthier-Villars.

This is a careful translation of the second edition of Professor Wood's excellent treatise. Being spread over two volumes, the translation makes a much more pleasing typographical effect than that of the somewhat congested single volume of the original. Several paragraphs of new matter by the author, dealing principally with fluorescence and resonance spectra of vapors, appear in the second volume.

C. A. S.

Elements of the Precision of Measurements and Graphical Methods. By H. M. GOODWIN. New York: McGraw-Hill Book Company, 1913.

This is an elementary treatise of experimental errors and graphical methods for technical students. It will be found a very good presentation for those who believe in the applicability of the method of least squares to technical subjects.

C. A. S.

The Chemistry of the Radio-Elements. By FREDERICK SODDY, F.R.S. New York: Longmans, Green and Co., 1914. Pp. i + 46. Price, $ .60.

This brief monograph brings together a number of the more recent experimental contributions to our knowledge of the radio-elements. In attempting to fit all the known radio-elements into the periodic table the author brings forward the view that "isotopism" exists in nature and that as a consequence a number of the radio-elements being chemically non-separable are practically identical. The work includes chapters on the origin of actinium, the nature of the end products in radioactive disintegration, the structure of atoms, and the nature of the Argon gases. The work concludes with an extended list of references to recent papers on the subjects discussed. J. C. M.

A New Era in Chemistry. By HARRY C. JONES. New York: D. Van Nostrand Company, 1913. Pp. xii + 326. Price $2.00.

This work, attractively written, is a popular presentation of the advances which have been made in the different branches of chemistry in the last quarter of a century. The various subjects treated are given an historical setting and the book abounds with references of a personal nature which throw considerable light on the methods followed by the different investigators. The work contains chapters on the Law of Mass Action; the Development of Stereochemistry; the Phase Rule; Chemical Equilibrium; Osmotic Pressure; Electrolytic Dissociation; the Theories of Solution, Colloidal Chemistry; Ionization of Gases and Radiochemistry. In the treatment of these subjects the fundamental character and the scope of the contributions made by Van't Hoff, Arrhenius, Willard Gibbs, Ostwald, and J. J. Thomson are all clearly and well portrayed. J. C. M.

A First Course in Physics. (Revised Edition.) By R. A. MILLIKAN and H. G. GALE. New York: Ginn and Company, 1913. Pp. x + 442. Price, $1.25.

The changes which have been made in the revised edition of this little book include a number of interesting features. The list of portraits inserted has been extended to include a few of the more eminent of modern physicists. The portions dealing with mechanics and the principles underlying the dynamo and motor have been simplified, and in light more emphasis has been laid upon a combination of the wave and the ray methods in the treatment of image formation. In the interpretation of the phenomena of molecular physics frequent use is made of the kinetic theory of matter and of the atomic or electronic theory of electricity. Altogether the book constitutes an admirable introduction to the study of physics. J. C. M.

Geometrical Optics. By A. S. PERCIVAL. New York: Longmans, Green and Co., 1913. Pp. vi + 132. Price, $1.50.

Students and others desiring what may be called an intermediate treatment of geometrical optics will find this book exceedingly useful. In developing the subject the author has omitted all reference to matters of purely academic interest and has confined his treatment to those phases of the subject which are more especially capable of practical application. The proofs given are concise and the sequence is excellent. The numerical illustrations of the various formulae used in the book constitute a particularly attractive feature.

J. C. M.

First Course in Algebra. By W. B. FITE. Boston: D. C. Heath and Co., 1913. Pp. v + 285. (*Received.*)

Die Brownsche Bewegung und Einige Verwandte Erscheinungen. By G. L. DE HAAS-LORENTZ. Braunschweig: Vieweg and Sohn, 1913. Pp. i + 103. Price, Mk. 3.50.

Previous books on Brownian movements have been primarily reports from certain experimenters upon their own experimental results. The book before us, which grew out of a Doctor's thesis, is a very valuable summary of most of the important *theoretical* as well as experimental researches on Brownian Movements which have appeared up to the present time. It is invaluable for a student of this subject. Indeed I know of no other book in which the subject is treated in any such thoroughgoing way. The original features are mainly in the latter chapters and consist (1) in the discussion of the relation between the actual, complicated Brownian movement and the visible motion of the Brownian particle, and (2) in certain additions to the theoretical study of the spontaneous transport of Electricity by Heat.

R. A. M.

Photo-Electricity. The Liberation of Electrons by Light with Chapters on Phosphorescence, Photo-chemical Actions and Photography. By H. STANLEY ALLEN, M.A., D.Sc. London: Longmans Green & Co. Pp. i + 216. Price, $2.10.

The task of correctly portraying the present status of photo-electric research is a very difficult one, for the reason that, though the papers which have appeared in this field may be numbered literally by the thousand, there is as yet no unified and consistent theory in the light of which all the phenomena of Photo-electricity may be interpreted. How then is an author to choose between conflicting results and present only material which is of real significance? Or, if he strives to be quite impartial, how is he to avoid making his book a mere compendium of disconnected researches and of irreconcilable view-points?

Dr. Allen has met these inherent difficulties with notable success. He shows a thorough familiarity with the whole of the literature and at the same time he has shown much judgment in sifting it out and in getting each research into some sort of a theoretical setting. The chapters which present the relations to the Electron Theory of the phenomena of Fluorescence, Photochemistry and Photography are particularly valuable.

The fact that it takes a book of 216 pages to present at all adequately a subject as new and as undeveloped as is Photo-electricity shows with what amazing rapidity Physics is growing at the present time.

R. A. M.

Second Series. *August, 1914* *Vol. IV., No. 2.*

THE
PHYSICAL REVIEW.

THE CRYSTAL FORMS OF METALLIC SELENIUM AND SOME OF THEIR PHYSICAL PROPERTIES.

By F. C. Brown.

WHETHER light-sensitive selenium is made up of homogeneous crystal structure or of complex units is of importance in explaining the nature of light action in selenium. Thus the author has been led into a study of the crystal forms of metallic selenium, which study has been particularly fruitful in that new crystal forms of very large size have been isolated. These crystals are of such a size that the optical and photoelectric properties of individual crystals can be studied. Thus we are able to eliminate some of possible causes of complexity in selenium, such as polymorphic crystal mixtures, unknown condition of crystal contacts, irregular placing of the crystals, and impurities. The fact that these new crystal forms are light-sensitive opens up a large field of investigation of which this paper is only preliminary.

Muthman[1] described only one crystal form of metallic selenium of the hexagonal rhombohedral system. These crystals were produced by the sublimation of the vapor of selenium in air and were never larger than 0.2 × 0.5 mm.

Saunders[2] also describes only one form. His conclusions were based upon the changes in volume accompanying temperature changes, as determined by dilatometric measurements. The difficulty with Saunders's method is that different crystal forms having identical or only slightly varying density could not be detected.

Marc[3] following the experiments of Uljanin[4] who observed two distinct crystallographic structures under the microscope, and following his own

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[1] Zeit. f. Kryst., 17, 356, 1890.

[2] "The Allotropic Forms of Selenium," Journ. of Phys. Chem., Vol. 4, p. 423, 1900.

[3] Die Physikalische Chemischen Eigenschaften des Metallisches Selens, Verlag von Voss, 1909.

[4] Wied. Ann., 34, p. 241, 1888.

experiments on selenium cells, was led to believe in the existence of several forms of metallic selenium, which through the action of light underwent polymorphic transformations. By the investigations of the heat changes, the electrical conductivity and a microscopic investigation of the surface he was able to detect with certainty the existence of two polymorphic forms. The one form was crystallized by heating the selenium at lower temperatures in the neighborhood of 100°. It was essentially non conducting and was designated form *A*. At temperatures of 205° to 215° this selenium went over into selenium *B*, which was found to be quite conducting. A microscopic investigation showed the form *B* to be of uniformly round mounds, while that crystallized at 200° contained long crystals. The crystals observed however were only of the size of about 0.004 mm. Marc's results which led him to believe in the existence of two forms *A* and *B* and which led him to believe in the dynamic equilibrium of these forms, were not sufficient to convince him that there could not be other forms. One of the phases producing equilibrium he regarded as a vapor phase, thus giving only one solid form for two phases.

Recently White[1] did some very excellent work with selenium blocks crystallized at 200°. He found the interesting result that a greater change of conductivity was produced when the incident light was in the direction of flow of the current than when the illumination and electric current directions were at right angles to each other. He concluded that in the selenium block the highest resistance is at the electrode contacts and also that the greatest change of conductivity by illumination takes place at the electrodes. However in interpreting his results he did not consider the change of conductivity arising from increased potential gradient when the blocks were illuminated at the contact surfaces. He also observed a number of phenomena such as has been observed in the so-called selenium cells.[2] This work of White further suggests the advisability of obtaining single isolated crystals of selenium in which the physical conditions, may be known and controlled.

THE PRODUCTION OF ISOLATED CRYSTALS.

The new crystal forms that are the subject of discussion were in all cases produced by the sublimation of the vapor, either in a high vacuum or at atmospheric pressure. The vitreous selenium which it was desired to transform into crystals was placed in one end of a glass tube of 35 mm. inside diameter and of 30 to 60 cm. length. This glass tube fitted snugly

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[1] Phil. Mag., Ser. VI., Vol. 27, p. 370, 1914.

[2] Unquestionably the term "selenium bridge" as he suggests is a better term than "selenium cells."

into a cylindrical electrical oven of 30 cm. length, of which one end was closed. Near the open end of the oven there was a rather large temperature gradient whether the tube was evacuated or not. All the selenium that deposited in any form was usually within a distance of less than 7 cm. along the tube. A current of about six amperes in the oven was sufficient to heat the selenium to be transformed to about 270°. The selenium was kept at this temperature for a period varying between one day and one week. But there was no temperature regulator and consequently there was considerable variation in the temperature during the day, with either the dynamo or the storage battery as a source of current. The selenium was allowed to sublime on the walls of the glass tube, on a thermometer bulb, or on a form with wire electrodes for a light-sensitive selenium bridge. It was hoped that the thermometer would register the temperature at which the crystals formed on it, but the temperature gradient was so large that no reliance was placed in the readings.

The result may be stated in a general way as follows. The largest crystals in every case were formed at the highest temperature at which the selenium sublimed in the closed tube. This temperature is believed to be above 210°. It should have been ascertained more accurately, but the effort was rather discouraging because of the absence of automatic temperature control. These crystals became smaller as the cooler portion of the tube was approached, until about 3 cm. back of the largest crystals there was a continuous sheath of selenium of a silver luster surrounding the inner wall of the tube. On some occasions the hotter edge of this sheath was fringed with small well-developed rhombohedral-hexagonal crystals (see Fig. 1) such as described by Muthman[1]. Back of this silver-like sheath was sometimes discerned a rather black sheath which also had a metallic luster. Further along the tube at temperatures perhaps below 100° was usually a very thin deposit of red amorphous selenium. I was rather surprised to find so small an amount of selenium deposited in the red amorphous form, and so large a quantity in the metallic forms. I was also surprised in some cases to find so large a fraction of the selenium as one fifth of the total to be deposited in the form of large crystals. This is rather a sufficient argument against any view that might presuppose impurities to form the large crystals. The selenium that was placed in the tube to be sublimated was Merck's purified selenium.

Invariably the largest crystals were formed at the highest temperature, but it is not clear what determined the form they should take. The photograph in Fig. 2 shows distinctly varying formations that occurred

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[1] Loc. cit.

in two separate tubes. The acicular crystals were sometimes 11 mm. long and not more than 0.2 mm. in any other dimension. Usually they developed a number together in a cactus-like growth, with from 3 to 20 spines starting from a common center. These spines or needles were quite stiff and tough, rough handling and even dropping very rarely injuring them. The needles of any given cluster were usually very much alike in size and appearance. They always had a metallic luster by reflected light. If fully developed there were six surfaces. And yet some entire clusters apparently developed with any number of surfaces less than six. Under the microscope these crystals seemed the least transparent of any that were produced.

The crystals of the second type shown in Fig. 2 were large flat crystals with parallel surfaces usually less than 0.3 mm. apart. The length of one of these crystals indicated in the photograph was 9 mm., and for the greater part of the length the width was about 2 mm. These crystals frequently began growth by the addition of longer and longer crystals to the side of the last crystal laid down. This accounts for the triangular point frequently seen on these large crystals. Sometimes these flat crystals were nearly square and again diamond shaped. Frequently the flat would develop in places as a rolling surface and also be distorted at the edges. But in any given run with fixed resistance coils in series with the oven heater, there was an unmistakable resemblance between all the crystals formed in a given vicinity of the tube. The largest crystals of this form appeared in a tube in which the air pressure had been reduced to less than 0.01 mm. previous to the sealing off of the tube.

The crystal groups to the left in Fig. 3 are also typical of those that were formed in one tube. They were as long as 4 mm. and as wide as 0.8 mm. They were the result of uniform twinning in such a manner that an open or closed V-shaped gauge was formed by each spine.

Yet another interesting formation that appeared once was a large number of very flexible sheets, sometimes 4 mm. long and as wide as .6 mm. They developed largely one in a place. As they waved about freely by slight air currents, they much resembled a miniature aluminium leaf as used in an electroscope. The thickness of one of these was of the order of 0.01 mm. The other forms described above showed no such flexibility at all. The relative dimensions of two of these crystals may be estimated from the reproduced photograph in Fig. 5. The length of the longest is about 3 mm.

The crystals that formed at the edge of the metallic sheath were only occasionally like those described by Muthman. Fig. 3 shows a photograph of a much larger formation, some as large as 4 mm. which appeared

Fig. 1.

Magnified hexagonal crystals, natural size of single crystals not over 0.1 mm. in width.

Fig. 2.

(*a*) Groups of acicular crystals to the left. Natural size of longest ones about 11 mm. (*b*) Growth of lamellar crystals in lower right-hand corner, the largest of which as seen in the figure was about 9.0 × 2 × 0.4 mm.

Fig. 3.

The lower part of the photograph shows the growth of single and twinned crystals of maximum length about 4 mm. at the edge of a crystal mass of finer grain. The scattered

Fig. 4.

The above crystals are the same form as those shown in Fig. 3, but were photographed by transmitted light with a magnification of about 100 times.

Fig. 6.

Lamellar crystals of gradually increasing width. Magnification about 100. Note the striations making an angle of about 35° with the edge.

Fig. 5.

Two crystals of flexible lamellar type. The longest of the above was about 4 mm. long. The width and thickness may be estimated from the reproduction.

Fig. 9.

Crystals of metallic selenium by sublimation. Acicular hexagonal and acicular with side branches. Side branches perpendicular to stem are hexagonal. Those leaving at angle of 60° are lamellar plates showing parallel extinction.

F. C. BROWN.

under similar circumstances. Some of the same crystals were magnified much more and photographed by transmitted light as seen in Fig. 4.

The manner in which some of the semi-flexible lamellæ developed is shown by the photograph (transmitted light) in Fig. 6. The striations can be seen running across the surface at about an angle of 35°. With the aid of polarized light these striations become much more marked. They indicate the way in which the large sheet lamella are sometimes built up.

Yet other crystal forms of small dimensions that appeared under the microscope are shown in Fig. 7. The largest of these was about .05 mm. All of these were observed many times. However they were too small for a study of their electrical properties.

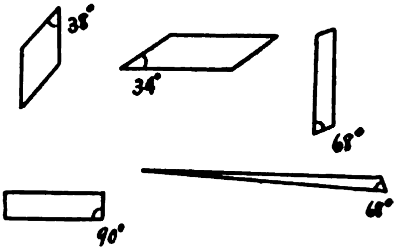

Fig. 7.

Unfortunately all the large crystal forms that I have described were formed at high temperatures. When we have obtained a satisfactory temperature regulator, we shall endeavor to obtain large-sized crystals at about 150°. No doubt more constant and better defined temperatures will produce more regular and larger crystals of the types that I have described, and perhaps new ones.

It is probable that all the crystals belong to the third crystal system, but before this can be decided definitely I believe that it will be at least advisable to obtain more large crystal forms, in order to determine accurately the angles between the faces.

Naturally the question arises as to why selenium crystallizes in so many forms at so nearly the same temperatures. I wish to proclaim a most profound ignorance as to this question. However there is strong indication that the partial pressure of the vapor of selenium and the temperature gradient in the tube at the place of formation are the prime factors. For example I have noted repeatedly that the thin lamellar form appeared when the oven temperature was low, where the amorphous selenium was placed for sublimation. Without doubt the conditions for formation of the crystals should be studied more carefully.

A photograph of some interesting crystals formed at atmospheric pressure is shown in Fig. 9.

THE PROPERTY OF DOUBLE REFRACTION.

All the forms of metallic selenium that have thus far been observed display a distinctly metallic appearance by reflected light. The surfaces were nearly always quite plane and consequently, as the photographs show, the crystals appeared very light or very dark depending on the position of the surface angle with regard to the direction of maximum illumination. It is to be observed in Figs. 2 and 3 that the diffuse light is much less than that from the white paper which served as a background. Even the fine-grained crystal masses formed at lower temperatures and shown in Fig. 3 show a distinct black with contrasting white specks here and there. Likewise all the crystal forms except the acicular ones transmit light even through great thicknesses, and it is interesting to note that wherever the crystal transmits light it displays the property of double refraction. When examined under crossed nicols the transmitted light is most frequently some tinge of deep red, but various formations and thicknesses of crystal show the transmitted light to be distinctly a blue, yellow, orange, green or even white. The fully developed rhombohedral hexagonal crystals in Fig. 1 show a deep ox-blood red under crossed nicols. The transmitted light seems to appear as a body color in the crystal. This may be due to a deviation or scattering of the light inside the crystal. In a lamellar crystal such as shown in Fig. 6, several variations of color are observable at different places. The striations become particularly distinct under crossed nicols by the variation of color and color density.

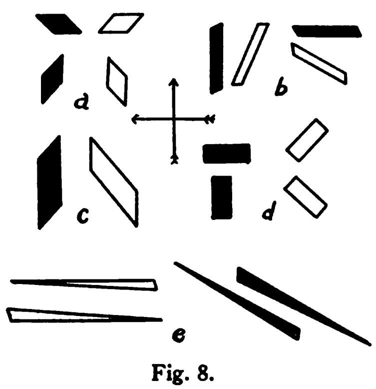

Fig. 8.

The elementary crystal forms and the position in which they show extinction of light under crossed nicols are shown approximately correct in Fig. 8. The nicol prisms are placed as shown by the arrows. The dark positions are those of light extinction and the light positions those in which the light was readily transmitted, in some color or other. The crystal *a* is for one having acute angles of about 35°. Crystal *c* is obviously the multiplication of *a*. Crystal *b* is one having 68° angles. Its axis obviously runs lengthwise of the crystals. In *d* is a distinctly right-angled lamellar form, also with optic axis running lengthwise of the crystal. The crystal *e* which is also representative of a large number that were observed, showed the optic axis to run parallel to the very short edge. The acute angle here was about 68°. Thus this crystal may be regarded as built up of a large number of *b* crystals of gradually increasing length. It happens that all the crystals represented in Fig. 8 were drawn under the same magnification, but they might have been chosen in widely varying sizes. It appears contrary to first expectation that the crystal in Fig. 6 for example is not built up of units of *e*, but rather of units of the crystal *b*. Under crossed nicols it shows extinction parallel to the striations. The crystal in Fig. 5 when viewed flatwise shows extinction parallel to the length of the crystal, indicating the optic axis to be lengthwise of the crystal. This long and flexible crystal may therefore be regarded as built up of the crystals of type *d*, in Fig. 8. The crystals in Fig. 1 show extinction parallel to the long crystallographic axis.

It may be of passing interest to deviate from the topic of this paper to note that the glittering red plates of the red crystalline variety of selenium show double refraction and also that they are not at all metallic in appearance, nor do they show electrical conductance even under very high pressures.

Some time ago the depth of penetration of light into selenium[1] was deduced to be greater than 0.014 mm. This result is very much larger than that of other observers. Pfund,[2] for example, found the depth to be about 10^{-6} cm. White[3] on the other hand believed his selenium blocks indicated a greater penetration than any result indicated above. It seems that the depth of penetration should and does vary with the size, character and positions of the crystals. I have observed a large amount of light transmitted through a lamellar crystal such as shown in Fig. 2 of thickness as great as 0.3 mm. This transmitted light was a deep red and was deviated from its path on emergence through a large angle, perhaps 40°. Thus in a mixture of small crystals adjacent to each other the path might be devious indeed. There is no reason to believe that the light or the light action might not travel several millimeters in a well-formed crystal. Evidence will be mentioned later which indicates that the total number of electrons liberated does not vary greatly whatever may be the deviations of the light.

THE EFFECT OF PRESSURE.

The pressure effect on selenium was discovered in 1905[4] showing a very large increase of conductivity for increased pressure up to 1,000 atmospheres. Montèn[5] extended the study independently up to 3,000 atmospheres and observed conductivity changes more than a hundred-

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[1] Phys. Rev., 34, p. 201, 1912.

[2] Phys. Rev., 28, p. 324, 1909.

[3] Loc. cit.

[4] Phys. Rev., Vol. 20, p. 185, 1905.

[5] Arkiv för Matematik, Astronomi och Fysik, Bd. 4, No. 31, p. 1, 1908.

fold. About the same time a study of the light sensitiveness under varying hydraulic pressures[1] very decidedly indicated that the pressure produced a genuine change in the selenium and did not alter the contact resistances. This argument was based on the presumption that no part of the light action was of a nature of alteration of contact resistances. A recent consideration of the similarity of certain physical properties in light-sensitive selenium and crystal contacts[2] has led the author to intimate that the early presumption referred to above might be wrong. White's recent work[3] with selenium blocks showed the largest change of resistance by light to take place at the electrodes.

A study of a single crystal, in contact with plane electrodes, under varying pressure should reveal important information as to the seat of the pressure effect in ordinary light sensitive selenium as well as in the individual crystal. This pressure effect was easily studied by placing the crystal between two surfaces as shown in Fig. 10. The pressure was increased by merely adding weights. This pressure was applied in the direction of the flow of current. The curves in Fig. 11 show the remarkable variation of the conductivity of a single lamellar crystal such as shown in Fig. 2 with change in pressure. The lower curve is the conductivity in the dark and the upper one is for the conductivity in the light. In the dark the conductivity increases about 120 times for an increase of pressure of 180 atmospheres. The crosses on the curves show the conductivity values both in the light and in the dark as the pressure was released. Thus it appears that the selenium crystal attains a given equilibrium under a given pressure. Under higher pressures the con-

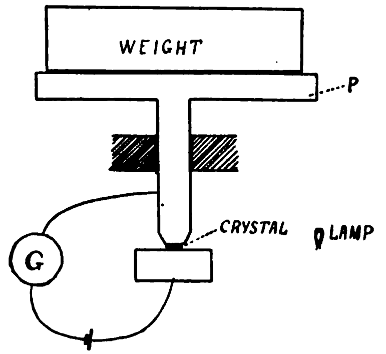

Fig. 10.

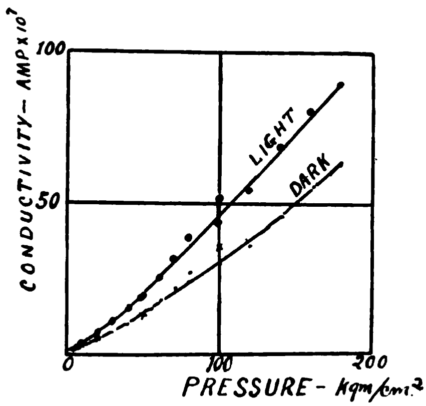

Fig. 11.

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[1] See paper by Brown and Stebbins, PHYS. REV., 26, p. 273, 1908.

[2] PHYS. REV., N. S., Vol. 1, p. 245, 1912, and Proc. Iowa Acad., 1913.

[3] Loc. cit.

ductivity was unsteady, so that thus far the action of higher pressures is uncertain. However I succeeded in increasing the conductivity of one crystal about a million times by pressure. Under these conditions it was not light sensitive. The fact that there was only a slight hysteresis as illustrated by the fact that the points on the curves shown were not taken in exactly regular order, is strong evidence that the pressure effect is in the selenium crystal and not at the electrode contacts.

The Variation of the Light-Sensitiveness with Pressure.

Experimental Results.—With the apparatus shown in Fig. 10 it was easy to determine the light-sensitiveness with different pressures. The interesting result followed that the light action increased with increase of pressure. In the following table is given the conductivity (c) in the dark, which prevailed under the varying pressures recorded in the upper curve of Fig. 11, and also the proportional change of conductivity, $\Delta C/C$, produced by constant illumination. The conductivity is indicated in terms of divisions deflection of the galvanometer, G. It is observed that the conductivity in the dark varied by a factor of 124. Within this large range, the light-action as measured by the change of conductivity is directly proportional to the conductivity in the dark. It may be concluded for pressures up to 180 atmospheres, that the percentage increase of conductivity by a given illumination is constant. This result leads to a remarkable interpretation as to the nature of light action.

C.	$\Delta C/C$.	C	$\Delta C/C$.
.5	.6	21.6	.43
2.0	.5	28.5	.46
4.5	.45	35	.50
7.4	.42	46.5	.46
10	.44	55	.45
13	.42	62	.43
17.8	.40		

Interpretation of Result $\Delta C/C$ = Constant.—First it will be seen that the resistance exists in the selenium itself and not at electrode contacts. For suppose the light to produce a change of resistance ΔR in the selenium. Then the proportional decrease of resistance by the light, $\Delta R/(Rs + Rc)$, would get constantly greater inversely proportional to the change of resistance, where Rs and Rc are the resistances of the selenium and the contacts respectively. This result would follow whether the light action were at the contacts or in the selenium itself. For pressures of only a few grams the above ratio was sometimes observed to increase with increase of pressure, but with pressures of 50 gm. or more the ratio of $\Delta C/C$ (or $\Delta R/R$) was constant as shown in the preceding table. It is concluded therefore that contact resistances are of very secondary or negligible consequence in connection with the ordinary light-electric phenomena in selenium crystals. This result is quite consistent with other experiments to be published later.

The constancy of the ratio of $\Delta C/C$ could be explained if it were possible that the increased amount of the conducting component produced by pressure were the only component that could still further be acted upon by light. But this seems so unreasonable, and it is certainly inconsistent, with the results obtained by the author in his various works on selenium cells.

The only reasonable interpretation, it seems, must be worked out somewhat along the following line. Suppose a number of isolated conducting centers throughout the crystal and that this number is a function of the pressure. Generally such an insulated center would not conduct at all, but as the pressure becomes greater a large number of these centers would come within each other's confines, thus increasing the conductivity of the crystal. If the number of centers is small, the conductivity should increase as the number of centers. If a given illumination produces a definite number of new centers, then these new centers should bridge over gaps between old centers approximately proportional to the number of gaps filled, which should vary as the number of old centers or circuits. Of course if the selenium should have a very large number of gaps already completed without the light action, then the light would not have the opportunity of completing a circuit so frequently for every center acted upon. In agreement with this notion I found that when pressures, sufficient to increase the conductivity a million fold were applied that the light sensitiveness diminished to practically zero. It may be convenient to modify this explanation later to satisfy the electron theory of conduction. However it may be mentioned that as a result of further experiments with these crystals Dr. Sieg and myself are compelled to revise the electron theory of electrical conduction. These experiments will be described later.

THE ACTION OF LIGHT.

Essentially in the Body of the Crystal.—In view of White's[1] experiments with selenium blocks in which he observed the greatest change of conductivity at the electrode contacts, it seemed advisable to try the same experiment with a single crystal. This time one of the lamellar crystals of about 4 mm. length was chosen. The ends were clamped to silver electrodes as shown in Fig. 12, leaving about 2 mm. between. A movable

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[1] Loc. cit.

slit of 0.1 mm. width was placed about 1 cm. in front of the crystal in the path of a parallel beam of light from an arc lamp. The slit was moved along in front of the selenium by a screw attachment. The conductivity was read when the illumination was on the contacts and at various positions between. The result showed the change of conductivity by constant illumination to be almost unvarying throughout the distance between the electrodes and to be slightly less when illuminated at the contacts. This result seems to lead unmistakably to the conclusion that in the crystal studied, light action is essentially a genuine action throughout the selenium crystal. The action was not a surface action, as evidenced by the fact that if the light impinged on the back side of the crystal the effect was unaltered.

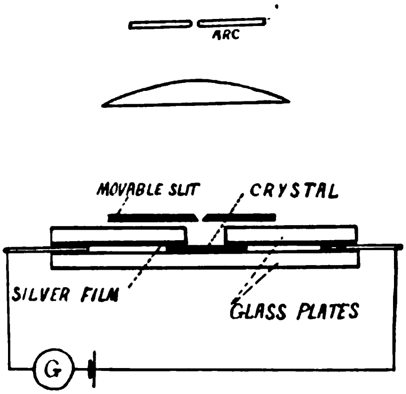

Fig. 12.

The Action of Light Spreads Inside of the Crystal.—With the arrangement shown in Fig. 12 the slit was adjusted midway between the two electrodes. Then the slit width was widened .05 mm. on each side at a step, the idea being that if each fraction of the light altered the part of the selenium in its path, that the change of conductivity would be in direct proportion to the width of the beam. This should follow from the fact that the parts of the selenium crystal were resistances in series. However the following is typical of the results obtained.

Slit Width Mm.	Convectivity.	Slit Width Mm.	Convectivity.
0	40	.4	101
.1	72	.5	109
.2	81	.6	118
.3	92	.7	126

This indicates that the first unit of light that falls on the selenium by diffused light in the selenium crystal or some sympathetic mechanism acts to either side of the path or throughout the crystal. Ever after that as the path is widened there is only the action of the selenium in the path or its equivalent. The first unit of light produces an excess of conductivity of 22 units, which can only be ascribed to a spreading action inside of the crystal or its equivalent. The above argument presupposes that not all the light absorbed in the selenium can go toward altering the conductivity, or further that the absolute alteration of the value of conductivity decreases with increased illumination.

The Variation of the Conductivity Change with the Light Intensity.—In the previous part of the paper, it was noted that the change in conductivity by a given illumination was proportional to the conductivity in the dark. This may be written in the form $dC/dI = k \cdot C$, providing that for very small illumination intensities the constant k should remain unchanged. By the integration of the above equation is obtained the value of the conductivity for any intensity, as $C = C_0e^{kI}$, where C_0 is the conductivity in the dark.

I have not yet had the time to check this equation with more direct experiment.

ACTION OF TEMPERATURE.

Dieterich[1] has recently shown that the character of the wave-length sensibility curves can to a large degree be controlled by the heat treatment of the selenium during annealing, particularly by the temperature at which the selenium was heated. For example, he found that the annealing at about 200° produced a pronounced maximum in the red end of the spectrum, while if the annealing were about 150°, there was a relatively high maximum about .55 μ. Unfortunately I have not thus far been fortunate in producing large crystals by sublimation at both the above temperatures. Therefore we were not able to investigate individual crystals as to the character of the sensibility curves. However the next best step was taken. A selenium cell form (soapstone frame with parallel platinum wires for electrodes) was placed in a highly evacuated tube in which the selenium was to be sublimated. It was so situated lengthwise of the tube that at the end of higher temperature large crystals of the lamellar type formed in considerable abundance with smaller ones nearby the electrodes. About a centimeter further along the frame where the temperature was perhaps 200° was deposited an even finer crystalline structure of unknown texture. After remaining in the tube several days, almost the entire framework revealed in the presence of intense illumination a large number of glittering crystal faces, resembling in a striking manner the appearance of freshly fallen snow in bright sunlight. Except for the crystal faces in the selenium that were at just the right angle to the observer the background was relatively dark.

By the apparatus recently described by Brown and Sieg[2] we obtained the sensibility curve for the crystal masses at two places in this selenium bridge. For the crystal mass supposedly sublimated at the higher temperature, the upper curve in Fig. 13 was obtained, while for the selenium deposited at the lower temperature the lower curve in the figure was

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[1] PHYS. REV., N.S., June, 1914.

[2] PHYS. REV., N.S., 2, p. 487.

obtained. Thus, contrary to Mr. Dieterich's result referred to, the red maximum appears in the crystals formed from the higher temperature. However it must be noted that this maximum is not in the same position as the maximum which Mr. Dieterich obtained at lower temperature. The cause of the shifting of the maximum will be searched for by the investigation of separate crystals.

General Considerations.

The present status of the theory of light sensitive selenium points towards a separtion of the complex elements along three lines, the light sensitiveness inherent in a given crystal structure, the varying action because of varying refraction, scattering and penetration, and the minor alterations due to the indirect action of the light on the contacts. In view of White's work it seems that contact resistances play a greater part in selenium blocks than in selenium crystals. Whether varying pressure will alter the character of the curves, will depend perhaps on the likeness or unlikeness of the light action for different parts of the spectrum. This point is under investigation at the present time. Also an attempt is being made to locate what characteristics of sensibility curves are seated essentially in individual crystal forms.

This much at least is settled: that the entire action of light in certain crystals at least is in the body of the selenium and not located either at the electrodes or at the surface layer.

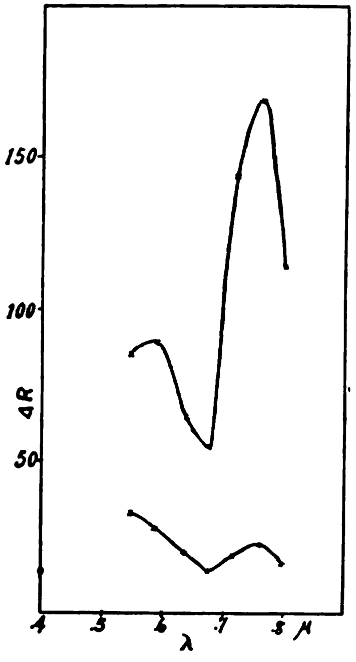

Fig. 13.

Perhaps in the so-called selenium cells the varying temperature action as well as the varying pressure and light action are involved in connection with contact resistances. By placing high pressures, a few atmospheres, on a single crystal of selenium we should be able to eliminate this contact resistance and thereby arrive at the true temperature coefficient of a pure form of selenium and also the true light sensitiveness at various temperatures.

The action of light seems to be in centers inside the crystals and does not necessarily involve the existence of more than one complete crystalline form. These centers inside the crystal resemble the so-called active centers in zinc sulphide as described by Rutherford.[1]

The specific conductivity of the crystals tested varied between 200 ohms and 10^7 ohms depending on the pressure applied. Judging from the conductivity, it would seem that the selenium in White's selenium blocks existed under an internal strain as a result of crystallization in a rigid form. If so we should expect different stress inside the blocks from what it was near the surface. Possibly some such basis might account for the increased photoelectric properties at the electrode contacts.

SUMMARY.

1. A large number of new crystals of metallic selenium have been formed, some of which are of very large size.
2. All of these forms except one are very transparent selectively to light, a large amount of light penetrating to a greater depth than 0.2 mm.
3. All the forms tested are conducting, showing a specific conductivity varying between 200 and 10^7.
4. All the crystal forms but one have been observed to be doubly refracting.
5. All the crystal forms increase in conductivity when illuminated.
6. The action of light is in the selenium itself and not at the contacts.
7. Mechanical pressure produces a genuine change in the selenium which may alter the conductivity more than a thousand times.
8. The absolute change of conductivity in one crystal by constant illumination was proportional to the conductivity in the dark, when that conductivity was altered by pressures between 1 and 180 atmospheres.
9. The temperature at which the crystals sublimate in mass has been shown to influence the character of the wave-length sensibility curves.
10. It has been shown that the production of individual crystals of metallic selenium of large size opens up a large field of investigation, which promises to be free from some of the possible complexities in selenium cells.

In conclusion I desire to thank Mr. M. H. Teeuwen for his assistance in designing and constructing apparatus, Mr. Scott Walker for assistance in taking observations, and Professor G. F. Kay, of the Geology Department, for the use of apparatus and also for his kind and generous help in the crystal studies.

THE PHYSICAL LABORATORY,
THE UNIVERSITY OF IOWA,
April 11, 1914.

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[1] Proc. Roy. Soc., *A*, 83, p. 561, 1910.

ON PONDEROMOTIVE FORCE UPON A DIELECTRIC WHICH CARRIES A DISPLACEMENT CURRENT IN A MAGNETIC FIELD.

By Robert H. Goddard.

Introductory.

According to Lorentz's theory of a material dielectric as being composed of elementary charges, such a body should, when carrying an electric displacment current in a magnetic field, experience a ponderomotive force similar to that which acts upon a conductor when carrying a conduction current in a magnetic field. Also, a ponderomotive force should be present in the analogous case of a body, of permeability greater than unity, when carrying a magnetic displacement current in an electrostatic field. The present paper deals with these two researches, neither of which has hitherto been successfully performed.

An attempt made by Whitehead[1] to demonstrate the presence of the first of these forces has given a negative result. Lorentz,[2] R. Gans,[3] and Koláček,[4] have shown this fact to be in agreement with electrical theory, without however suggesting a way by which a positive result could be attained.

Theory of the Ponderomotive Action for Electric Displacement in a Magnetic Field.

(*a*) *General Theory.*—The following theory shows that the Whitehead experiment is but a particular case of a more general research, a statement of which will now be made. Let there be given a vertical magnetic field, H, produced by two co-axial, circular, coils, in which an alternating current is flowing. Suppose a strip of homogeneous dielectric, D, Fig. 1, to be suspended from an arm, a, which is pivoted at an axis O; the strip being in the said magnetic field, between the upper and lower coils.

Suppose, further, that condenser plates, K_1 and K_2, are placed upon each side of the strip, at right angles to the arm, a; there being impressed upon these plates an alternating E.M.F., of the same period as the current

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[1] J. B. Whitehead, Jr., American Journal of Science, 4 ser., Vol. 14, pp. 109–128, 1902.

[2] H. A. Lorentz, Konink. Akad. van Wetten. te Amsterdam, Vol. 5, pp. 622–628, 1903.

[3] R. Gans, Phys. Zeit., Vol. 5, pp. 162–4, 1904.

[4] F. Koláček, Phys. Zeit., Vol. 5, pp. 45–47, 1904.

in the coils, but differing in phase by a quarter period. Both the displacement current in the dielectric, and the magnetic field, will thus be in the same phase. For experimental reasons the condenser plates K_1 and K_2, and the surfaces of the strip are taken as cylindrical surfaces concentric about O; the radii being large compared with the thickness of the dielectric strip. Let distances, measured outward along the radius, be r; and counter-clockwise along the arc, be s. Consider an element, $drds$, of the dielectric strip.

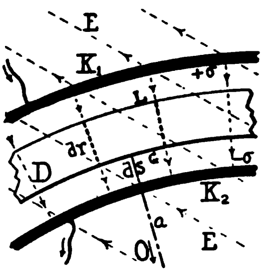

Fig. 1.

It is required to find the conditions, if any exist, for the presence of a moment of force, about O, due to the electric displacement in this element, $drds$, in the presence of the magnetic field—analogous to the force upon a conductor which carries a conduction current in a magnetic field. Before proceeding to a discussion of the moments which must act upon the element,[1] a few preliminary remarks are necessary. Let the surface density be σ at K_1, and $-\sigma$ at K_2. The charges upon K_1 and K_2, at the ends of the imaginary stream-tube which passes through the element, will then be σds and $-\sigma ds$, respectively; if we consider, for convenience, that the vertical width of the strip is one centimeter. We shall then have a current in the lead-wires of the condenser, due to the continuation of this elementary current tube,[2] of magnitude

$$i_w = \frac{d\sigma}{dt} ds. \tag{1}$$

Let the dielectric in question be of constant, k, and, for simplicity, be surrounded by a medium of constant unity; and let G be the element of surface on the inner, and L the element on the outer, boundary. Then, inasmuch as the dielectric is homogeneous,[3] the circumstances will remain identically the same if the dielectric is removed, and charges are given the boundaries G and L of respective densities

$$\sigma_G' = \frac{k-1}{k} \sigma, \tag{2}$$

and

$$\sigma_L' = -\frac{k-1}{k} \sigma. \tag{2'}$$

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[1] Lorentz, *loc. cit.*, derives expressions for the *reaction* of the moments upon the condenser plates. The expressions, however, hold only for the case in which the dielectric completely fills the space beteen these plates.

[2] After Maxwell's fundamental conception of a *closed circuit* as being a conduction, plus a displacement, current.

[3] Were the dielectric heterogeneous, it would be necessary also to consider apparent bodily charges.

These charges are the so-called apparent charges, to which the presence of the dielectric is equivalent.

Let us now define a *virtual* or *apparent current*,[1] in the space occupied by the dielectric, as the *time-rate of change of apparent density of charge at the boundary*. We shall then have for the apparent current across the element

$$i_v = \frac{d\sigma'}{dt}ds = \frac{k-1}{k}\frac{d\sigma}{dt}ds, \tag{3}$$

this current being in the same direction as that in the lead-wires of the condenser.

Moment I.—The moment of force upon the dielectric element, due to this apparent current, together with the vertical magnetic field H will be

$$dM_1 = \frac{a(k-1)}{ck}\frac{d\sigma}{dt}Hdrds, \tag{4}$$

where c is the ratio of the electrostatic to the electro-magnetic units.

Moment II.—Another couple is to be considered, owing to the combined presence of the apparent charges produced by the condenser K_1K_2, and the electrostatic field which must accompany the varying vertical magnetic field, H. This electrostatic field is represented in direction and intensity by the vector E, Fig. 1. It is directed in circles about the axis of symmetry of the coils which produce the magnetic field. We may take the point (rs) as coinciding with the lower right-hand corner of the dielectric element.

The moment due to the apparent charge at G is the product of the apparent charge at G by the moment of the s-component of E (*i. e.*, aE_s). By equation (2) this is

$$\frac{k-1}{k}a\sigma E_s ds.$$

The corresponding moment, due to the apparent charge at L is, by (2′),

$$-\frac{k-1}{k}a\sigma\left(E_s + \frac{dE_s}{dr}\right)ds,$$

making the total moment

$$dM_{II} = -\frac{k-1}{k}a\sigma\frac{dE_s}{dr}drds. \tag{5}$$

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[1] This apparent current corresponds to the "polarization current," and should not be confused with the conception of "apparent current" of which use is made in the theory of media, heterogeneous with respect to permeability. It will be noticed that the present treatment makes use of the fact that the presence of the dielectric is equivalent electrostatically to apparent charges on the boundaries; and magnetically, to the polarization current.

Moment III.—It is, of course, necessary to consider apparent charges as being produced by the field E. The apparent densities, let us say σ'', at G and L will be, respectively,

$$\sigma_G'' = -\frac{k-1}{4\pi k}E_r \tag{6}$$

and

$$\sigma_L'' = \frac{k-1}{4\pi k}\left(E_r + \frac{dE_r}{dr}dr\right). \tag{6'}$$

An apparent current, as above defined, will then take place in the space occupied by the dielectric, in the r-direction; and this, together with H, will give a moment, to the first order, of

$$dM_{III} = \frac{(k-1)a}{4\pi ck}\frac{dE_r}{dt}Hdrds. \tag{7}$$

It is important to notice that the mean value of dM_{III} is other than zero, for the reason that H and dE/dt differ in phase by a half period.

Moment IV.—The apparent charges at G and L produced by the field E must be considered as being acted upon by the field E itself.

The moment of the force E upon the apparent charge at G is

$$-\frac{(k-1)a}{4\pi k}E_rE_sds,$$

and that upon the apparent charge at L is

$$\frac{(k-1)a}{4\pi k}\left(E_r + \frac{dE_r}{dr}dr\right)\left(E_s + \frac{dE_s}{dr}dr\right)ds,$$

so that the total moment is

$$dM_{IV} = \frac{(k-1)a}{4\pi k}\left(E_s\frac{dE_r}{dr} + E_r\frac{dE_s}{dr}\right)drds. \tag{8}$$

It remains to apply this theory to particular cases, in which the piece of dielectric is of a size sufficiently large to permit of experimentation.

(*b*) *Application to Particular Cases. Case* 1.—In the Whitehead experiment,[1] a small block of dielectric was suspended between two condenser plates, the arrangement being analogous to the element $drds$, with the field E wholly in the s-direction.

Although the case is different from that in which the element is a part of a strip, it is nevertheless possible to determine, in a general way, what moments will be effective. The moment dM_{II} is equal to the line-in-

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[1] Whitehead, loc. cit. Three other forms of apparatus were tried by Whitehead, in which the arrangements were such that the field, E, was directed in circles concentric with the center of the dielectric block. In each of these forms, however, there was present a comparatively strong electrostatic controlling force.

tegral, taken in the positive direction of rotation, of the force E_s, about the contour of the element (*i. e.*, *curl* E) multiplied by the apparent density, $-[(k-1)/k]\sigma$. This is true, owing to the fact that at the ends, dr, of the element, both the apparent charges, and E resolved tangentially, are zero. But this contour integral of the tangental component of E is, by Maxwell's equation,

$$\text{curl } E = -\frac{1}{c}\frac{\partial H}{\partial t}\,drds, \tag{9}$$

for it will be noticed that $Hdrds$ is the flux of the magnetic field within the contour. Moment dM_{II} thus becomes

$$dM_{II} = \frac{(k-1)a}{ck}\sigma\frac{dH}{dt}\,drds. \tag{10}$$

The sum of moments I and II is, then, by (4) and (10)

$$dM_{I} + dM_{II} = \frac{(k-1)a}{ck}\frac{d}{dt}(\sigma H)drds. \tag{11}$$

Since this is the time-rate of change of a periodic quantity, the mean value will be zero. Furthermore, since $E_r = 0$, moments dM_{III} and dM_{IV} will be zero,[1] whence the total moment will vanish—which accounts for the negative result of the Whitehead experiment.

Case 2.—If a strip of dielectric is employed, instead of a small block, the theory here presented holds strictly true for any given element; and it is at once evident that, if the element is so placed in the field, E, that E_s is zero, then moment dM_{II} will vanish. If, at the same time, moments dM_{III} and dM_{IV} can be eliminated, there will remain only moment dM_{I}—which is the moment desired.

In considering the resultant moment for the entire strip, it is evidently desirable that the integral of dM_{I} with respect to s should be a maximum, and the sum of the integrals of dM_{II}, dM_{III} and dM_{IV}, should be a minimum. At the same time, it should be borne in mind that the strip must have as boundaries, surfaces cylindrical about the axis O, in order to permit of a rotation about this axis, during the experiment.

These conditions are satisfied, as nearly as possible, by the strip of dielectric shown in Fig. 2. The vertical width of the strip is moderate, which permits two coils being placed close to it, one above and the other beneath. The diameters of these coils do not exceed the length of the condenser K_1K_2, so that the magnetic force is inconsiderable beyond

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[1] In the Whitehead experiment, the ends, dr, of the dielectric, must be considered as having had apparent charges due to the force E_s; but owing to symmetry, E was everywhere the same within the immediate neighborhood of the dielectric, for any particular value of r. The resultant moment due to these apparent charges, was, therefore, zero.

the dotted circles of electric force, E. The strip is sufficiently thin for the thickness, l, to correspond to dr; it thus being unnecessary to express integration with respect to r. With this arrangement, the various moments are as follows.

Moment I.—Owing to the magnetic field being everywhere vertical, and the displacement current between the condenser plates being everywhere radial with respect to O, each element experiences the total moment dM_1, so that the moment for the entire strip (or, what is equivalent, the part of the strip within the condenser) is

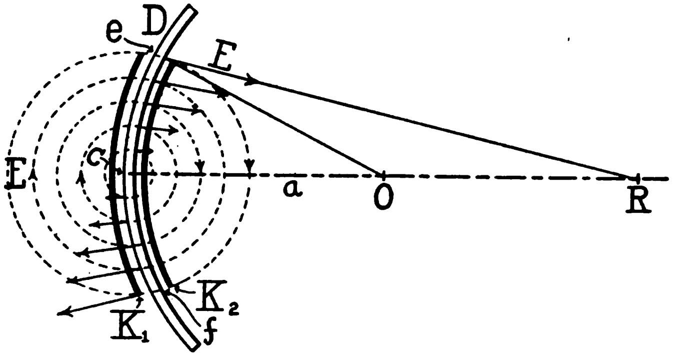

Fig. 2.

$$\int_e^f \frac{dM_I}{ds} ds = \frac{a(k-1)}{ck}\frac{d\sigma}{dt} Hsdr,$$

or

$$= \frac{a(k-1)}{ck} iHl, \qquad (12)$$

where i is the total current through the strip.

Moment II.—The second moment would vanish completely if E were everywhere normal to the boundaries of the strip, and also if there were no apprent charges at the ends. The force, E, is not normal, but (for an infinitely thin strip) passes through the point R, distant $2a$ from c, instead of through the point O.

There are, however, ***no apprent charges at the ends*** of the strip, owing to the fact that the ends project into a region in which both the fields E, and that due to K_1K_2, are negligible. These projecting parts of the strip are of importance also in another respect; namely, in permitting the strip to swing through a small angle, about the axis O, without experiencing an electrostatic controlling force tending to keep it symmetrical with respect to the condenser K_1K_2.

An expression for the integral of dM_{II} may be derived in the following way. The strip may be considered as ending at the points e and f, inasmuch as the fields of force do not extend beyond these points. Let us denote by ϵ the ratio of the line integral of E resolved tangentially along the *sides* of the strip, to the line integral of E resolved tangentially along the *sides and ends* of the strip. Then, making use of (9), as before, and integrating, we have

$$\int_e^f \frac{dM_{II}}{ds}ds = \frac{\epsilon(k-1)a}{ck}\sigma\frac{dH}{dt}ls, \tag{13}$$

which, when combined with (12) gives for the sum of the first and second moments,

$$(1-\epsilon)\frac{(k-1)a}{ck}\frac{d\sigma}{dt}Hls+\epsilon\frac{a(k-1)}{ck}\frac{d(\sigma H)}{dt}ls=(1-\epsilon)\frac{(k-1)a}{ck}\frac{d\sigma}{dt}Hls. \tag{14}$$

A method of measuring $(1 - \epsilon)$ will be given below under "Details of Method and Apparatus."

Moment III.—It is evident that the radial components of E are equal in magnitude but opposite in direction, at points on the boundary, equal distances on each side of the center, c, The integral of the third moment thus vanishes, by symmetry.

Moment IV.—From what has just been said concerning Moment III., it is evident that the apparent charges produced by the radial components of E are equal in magnitude, but opposite in sign, at points equally distant on each side of c. Further, it may be seen from the figure that the tangential component is in the same direction at all points of the strip, and is of equal magnitude at equal distances from c. Hence the integral of the fourth moment also vanishes.

The sum of all four moments for the entire strip thus reduces to the expression (14).

THEORY OF THE PONDEROMOTIVE ACTION FOR MAGNETIC DISPLACEMENT IN AN ELECTROSTATIC FIELD.

The theory of ponderomotive action herein developed will apply equally well to a body, of permeability greater than unity, which carries a magnetic displacement current in an electrostatic field. In this case, in place of the two coils, previously described, there must be substituted a condenser consisting of two plane, circular, plates, placed co-axially, one above the other; and in place of the condenser K_1K_2, there must be substituted two coils of such forms as will give a field of force similar in shape to that of this condenser. The strip corresponding to D might be composed of fine iron filings imbedded in paraffine.

The case of electric displacement in a magnetic field, is, however, more amenable to accurate experimentation; and a research will now be described in which the theory here presented is applied to a system such as that shown in Fig. 2.

General Method.

Two difficulties which were encountered in the Whitehead experiment, namely, electrostatic disturbing forces in an apparatus not perfectly symmetrical, and the impossibility of using a comparatively large mass of dielectric, were both overcome in the present experiment by the use of currents of very high frequency.[1] It was thus possible to secure a current of several tenths of an ampere through a condenser of surprisingly small capacity. A strip of dielectric, supported on the arm of a torsion balance, was permitted to swing freely between the plates of this condenser.

A measured value of the force was obtained from the constant of torsion of the fiber, together with the deflection of the mean position during torsional vibrations, produced by a reversal of the force.

A calculated value of the force was obtained from the following measurements: the thickness of the dielectric strip; the mean magnetic field, per unit current, within the space occupied by the dielectric; the dielectric constant at the particular frequency employed; the factor $(1 - \epsilon)$; and the proportion of total current that passed through the strip.

Details of Method and Apparatus.

(*a*) *Coils and Condenser.*—The magnetic field which acted upon the dielectric was produced by two coils shown as M_1, M_2, in the plan and elevation of the coil-condenser apparatus, Fig. 3. These coils were

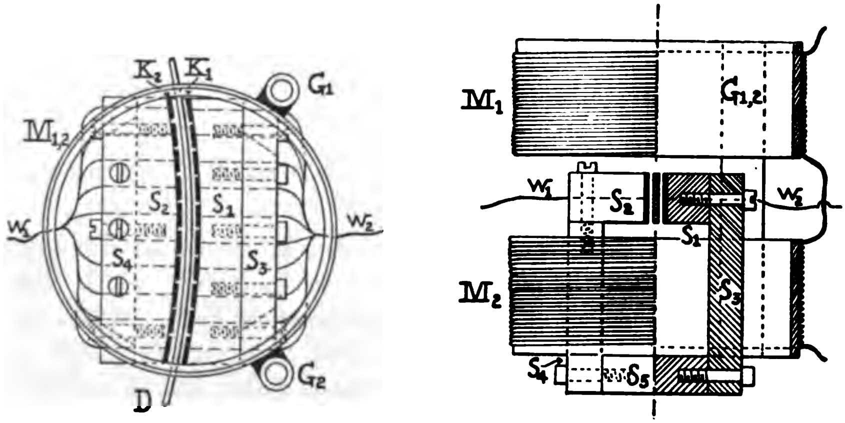

Fig. 3.

Coil and Condenser. ⅓ actual size.

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[1] The frequency in Whitehead's experiment was 133 cycles per second.

2.2 cm. apart, and each consisted of a single layer of 28 turns of cotton-covered copper wire, 1 mm. in diameter, wound on a section of glass tube 7.45 cm. in diamater. They were soaked in melted paraffine, after being wound.

The condenser K_1K_2, in which the dielectric swung, was composed of two rows of separate brass segments, in order to eliminate eddy-currents otherwise present from the alternating magnetic fields of M_1 and M_2; each segment being connected by a wire with the leads, w_1 and w_2. The rows K_1 and K_2, were of 1.3 cm. vertical width and were separated by an air-gap of 0.4 cm. The surface of each row was cut accurately to the arc of a circle, with center distant 20 cm. from the middle of the air-gap.

The coils and condenser were held in position by a support consisting of five pieces of hard rubber, $S_1 \cdot \cdot \cdot S_5$, fastened with hard rubber screws. The coil M_1 was fastened with sealing-wax to two glass rods G_1 and G_2, which were, in turn, fastened to M_2.

(*b*) *The Suspended System.*—The torsion arm, *B*, Fig. 4, consisted of a glass tube 0.25 cm. in diameter and 40 cm. long weighing 0.09 grains per linear cm., with a lead weight, *W*, at the one end, and two branches which supported the dielectric, *D*, at the other end.

The dielectric, used in the experiment to be described below was a strip of hard rubber 15 cm. long, 0.13 cm. thick, and the same vertical width as the condenser K_1K_2; namely, 1.3 cm. It was given the proper curvature by being heated, and placed while hot upon a cylindrical surface of brass, of 20.065 cm. radius. The strip projected 4 cm. beyond the ends of the condenser K_1K_2.

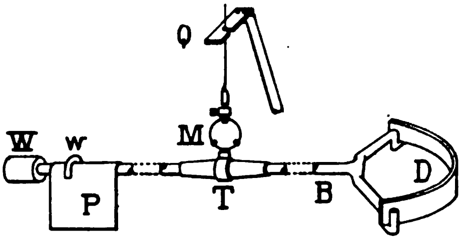

Fig. 4.
The Suspended System.

A mirror, *M*, reflected the image of a vertical wire, illuminated from behind by a Nernst filament, to a scale 160 cm. distant. It was found later, however, that greater accuracy could be attained by observing an illuminated transparent scale, distant 3 meters from *M*, through a telescope magnifying 70 diameters.

The suspended system was adjusted for level while free from the coil-condenser apparatus. The arm *B* was made level by means of the 2 gram rider, *w*; while, at the same time, the distance between the axis of suspension and the middle of the strip was made 20 cm., by calipers—this latter adjustment being made by moving *B* lengthwise in the brass holder *T*.

The upper edge of the strip was made level by turning the glass tube in the holder T until this edge remained, during torsional vibrations, at a constant small distance from a fixed needle-point, held just above the strip.

The proper height of the strip was easily secured by means of the wire, supporting the fiber, which passed through a torsion-head of the ordinary type. The height of this wire was varied until the top edge of the strip D was level with the top of the condenser K_1K_2.

The strip was made to swing exactly in the middle of the air-gap, in the following way; a template was made, with one edge cut to the arc of a circle of 20.2 cm. radius, and with a small hole through the point corresponding to the center of this circle. Next, the piece S_2, Fig. 3, was removed, and the curved edge of the template was brought in contact with the row of segments K_1. A fine wire with a small weight attached, supported by the torsion head, was passed through the hole in the template; and the torsion head was then moved until the wire, handing freely, passed through the center of this hole.

In order subsequently to place the suspension fiber in exactly the position occupied by this wire, a guide Q was made, as follows: a small hole was drilled in a flat plate and a slot was cut from the edge of the plate to the hole, as shown in the figure. A rod was fastened to this plate, and was held in such a position that the wire, above mentioned, passed through the center of the hole. While in this position, the lower end of the rod was fastened, with sealing-wax, to a glass plate, 60 cm. square, which supported the coil-condenser apparatus.

(*c*) *The Suspension.*—Quartz fibers were found more satisfactory than phosphor-bronze strip. The fibers used were 0.05 mm. in diameter, 30 cm. in length, and were cut from fibers 4 meters long, produced by the bow-and-arrow method.

The torsion-head was held in an iron clamp, supported by an apparatus-stand of the ordinary pattern. This stand rested on the same wooden table as the other apparatus. The means for producing the oscillations was on a separate table, as far from iron, and other conductors, as possible.

(*d*) *Elimination of Air-currents.*—The suspended system and coil-condenser apparatus were enclosed in a box of cardboard, 52 cm. square and 21.5 cm. high. This box was double-walled, 3 cm. thick, the interspace being packed with cotton; and the top was made in sections, so as to permit removal without disturbing the suspension. Within this box was a small single-walled box, which surrounded closely, but without touching, the coil and condenser, and the suspended system. Both boxes, as well as the coil-condenser apparatus, were fastened to the above-mentioned glass plate, being so placed that the fiber was over the middle of this plate. Small windows of plate glass permitted observation of the mirror, *M*; and the fiber suspension was enclosed in a glass tube, fastened to the torsion-head.

After the apparatus had been adjusted, it was covered completely with several layers of cotton; but even with this protection, it was found necessary to keep the temperature of the room constant by means of a thermostat. When this was done the zero was very steady.

Owing to the fact that the coils produced heat close to the dielectric strip, it was necessary to enclose the coils and lead-wires completely with heavy cardboard—the condenser alone remaining exposed. Two openings, into the space thus enclosed, served to remove the heated air, by convection.

In order to set the suspended system into vibration, when completely enclosed, a strip of paper, *P*, Fig. 4, was fastened to the arm *B*. A minute jet of air could be directed upon this strip in the following way. A glass tube, with the end drawn to a fine opening, was fastened in the walls of the boxes, with the small end directed toward *P*. A rubber tube, closed at one end, was attached to the end of the glass tube that projected from the boxes. By gently pressing this tube, a torsional vibration of any desired small magnitude could be produced.

It was found that when the apparatus was set up in cold weather, an electrostatic charge was acquired which manifested itself in a strong controlling force. This difficulty was overcome by placing, for a few hours, a half milligram of radium bromide in a glass tube, within the double-walled box.

(*e*) *Source of High-frequency Current.*—The alternating current was produced by a Chaffee arc, or gap. This consists essentially of an aluminum cathode and a copper anode, surrounded by an atmosphere of moist hydrogen. The air-cooled gap described by Chaffee[1] was used with modifications of certain mechanical features.

Details of the gap are shown in Fig. 5; the right half of the figure being in section. At *e* is shown the aluminium electrode which fitted tightly in a brass rod, or terminal, *t*. This terminal was free to slide in a brass tube *b*; which latter was provided with radiating vanes, as shown. Opposite the aluminium electrode was a copper electrode, held in a similar manner. The terminals were provided with hard-rubber handles *r*, for adjusting; and were fastened, when adjusted, by long screws with hard-rubber heads, *s*.

The brass tubes, *b*, were threaded, and engaged in the side-plates, *c*;

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[1] E. L. Chaffee, Proceedings, American Acad. of Arts and Sci., Vol. 47, pp. 286–312, 1911.

which latter were fastened to a hard-rubber base. Holes were drilled in one side-plate to provide an inlet, *i*, and an outlet, *a*, for the hydrogen produced electrolytically by a current of 3 amperes. The side-plates were separated by a hard-rubber ring, *h*.

This form of the gap was of advantage in that the ring of hard-rubber was free from inlet and outlet holes, and could therefore easily be replaced; and, further, the terminals were of a shape convenient to be held in a chuck whenever it became necessary to expose fresh, plane, surfaces at the ends of the electrodes.

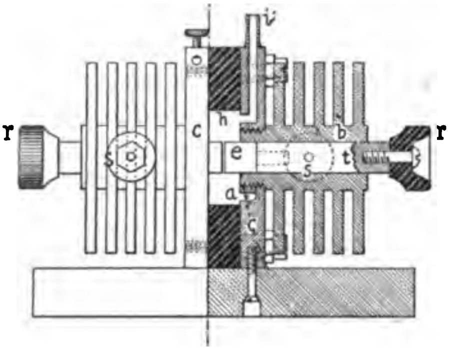

Fig. 5.

Air-cooled Chaffee Discharge Gap. ½ actual size.

(*f*) *Arrangment of Connections.* The connections are shown in the diagram, Fig. 6. The source of direct current for the gap, *G*, consisted of two 250-volt generators, in series, one generator being provided with a field-rheostat to regulate the voltage. These generators delivered 530 volts at the main switch S_1, as read by the volt meter V_0. Owing to electrostatic charges from the generators, produced by belt-friction, it was found necessary to connect one point of S_1 to earth through a long glass tube filled with dilute copper sulphate solution.

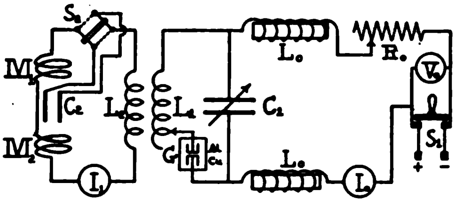

Fig. 6.

Diagram of Connections.

A direct current ammeter I_0 measured the current in the primary circuit. This circuit also contained two inductances or choke-coils, L_0

(ordinary alternating current arc-light inductances), together with a variable resistance and a bank of lamps in series, R_0. The current was reduced to about 0.2 ampere, by means of these resistances.

The primary condenser, C_1, consisted of two circular brass plates, in air, 30 cm. in diameter, and 0.2 to 0.5 cm. apart. Adjustment of this latter distance was made by a micrometer screw. The primary inductance, L_1, was a single layer of 1 mm. diameter copper wire, cotton covered, wound on a glass tube, 1.4 cm. diameter and 3.5 cm. long. The secondary, L_2, was wound directly over these primary turns, after the latter had been dipped in melted paraffine. Close coupling, as just described, was found necessary, as otherwise the secondary current was very irregular. Adjustments of R_0, C_1 and the potential V_0, were made until the secondary current was a maximum.

A commutator, S_2, served to reverse the secondary current, usually 0.3 or 0.4 ampere, through the condenser C_2 (corresponding to K_1K_2 of Fig. 3). This commutator was of small capacity, and consisted of four small mercury cups at the top of four glass rods, 10 cm. high. Even with this arrangement there was a noticeable change in frequency, on reversing, due to a change in the area enclosed by the secondary circuit, and a consequent change of inductance.

The hot-wire ammeter, I_1, was an R. W. Paul high-frequency galvanometer, having sufficient resistance in series with the moving coil to give a complete scale deflection with 0.5 ampere. This instrument was purposely placed between L_2 and the coils M_1M_2, in order to be as near the node of potential as possible. This was done to reduce to a minimum the capacity of I_1 to earth. It should be remarked, in passing, that the secondary current produced no sensible heating of the dielectric strip.

(*g*) *Measurement of the Magnetic Field.*—It was found possible to measure directly, with a sufficient degree of accuracy, the mean magnetic field, in the space occupied by the dielectric, per unit current in the coils, M_1, M_2. A glass strip, the same width and thickness of the dielectric, and of a length *ec*, Fig. 2, was given the same curvature as the dielectric. Upon this strip were wound eighty turns of fine, enamelled, copper wire. The resulting search-coil was fastened with wax, in the gap of the condenser K_1K_2, with one edge at *e* and the other at *c*; thus filling the space occupied by half the dielectric. The flux of the magnetic field through this coil was, by symmetry, half the flux through the entire dielectric strip.

This flux was measured with a Grassot fluxmeter, by using as much current as the coils M_1M_2 could safely stand; namely, 7 amperes. The coils M_1M_2 were, of course, first disconnected from the secondary circuit,

Fig. 5, and the dielectric was removed from the condenser K_1K_2. The arrangement of connections was such that the deflection of the fluxmeter resulted from a *reversal* of the current in the coils M_1M_2, rather than from applying and removing this current; thus affording a deflection of twice the magnitude obtainable in the ordinary way.[1]

The fluxmeter was calibrated by using a circular search-coil, of 15 turns, 3.80 cm. in diameter, placed in the middle of a solenoid. This solenoid carried a current of 4 amperes, and consisted of a single layer of cotton-covered copper wire wound upon a glass tube. The constants were: length, 49.2 cm.; mean diameter of windings, 5.6; and number of turns per cm., 1.063.

It was necessary to determine the area-turns, or effective area, of the curved search-coil already described. This was done by comparing the two search-coils in the field of an electromagnet. The electromagnet employed for this purpose was one used by Professor O. W. Richardson in determining the constants of thermions, which he kindly placed at the writer's disposal; the field of which had been found to be very uniform over a considerable area. The effective area of the search-coil found in this way was 52.25 cm. It should be remarked that all the measurements recorded were means of a number of determinations, in close agreement, taken at various parts of the fluxmeter scale.

The value of H thus obtained, per unit current in M_1M_2 was 5.23, to within 1 per cent. A test of the flux, in the space occupied by the part of the dielectric strip *outside* the condenser K_1K_2, was made with the curved search-coil. It was found that this flux was but 2 per cent. of that within the gap of the condenser.

(*h*) *Measurement of* $(1 - \epsilon)$.—A measurement of $(1 - \epsilon)$ was made possible, for the apparatus under discussion, by the fact that the dimensions of the coils M_1M_2 were small compared with the wave-length (100 meters). For this reason, all the parts of the magnetic field with which we are concerned were, at any instant, in the same phase. Thus from the theorem

$$\text{curl}\, E = -\frac{1}{c}\frac{\partial H}{\partial t}\, drds, \tag{9}$$

we have

$$\text{curl}\, E \propto H drds. \tag{15}$$

Now, since the apparent charges exist only between the condenser

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[1] A cylindrical commutator was devised which automatically performed this operation in the proper order. The deflection could thus be magnified n times, by giving the commutator n turns. It was found, however, that there was too rapid a drift toward the zero to make this method practicable, owing to the short free period of the fluxmeter (one minute). With a free period of 15 or 20 minutes, satisfactory results could have been obtained.

plates K_1K_2, on the inner and outer boundaries of the dielectric, we may assume, as already stated, that the dielectric strip does not extend beyond the points e and f, Fig. 2; and also that the ends are portions of the circumference of a circle passing through e and f, with center at c.

The ratio of the part of *curl E* eliminated to the total *curl E* for the strip,—in other words, $(1 - \epsilon)$—will then be by (15)

$$(1 - \epsilon) = \left(\frac{a}{b} f_c\right) \frac{1}{f_d}, \tag{16}$$

where a = the sum of the arcs of the circle, cut by the dielectric strip at e and f (0.26 cm.),
b = the circumference of this circle (21.67 cm.),
f_c = the flux of H within this circumference, for a given current in the coils M_1M_2, and
f_d = the flux of H within the boundaries of the dielectric strip, for the same current in M_1M_2.

An experiment was thus made necessary to determine H as a function of the distance from the axis of symmetry, c, of the coils M_1M_2. This was done by measuring the flux, as described under (*g*), through five circular search-coils, for the same current in M_1M_2; each coil being placed with the center coinciding with c. The measurements made with these coils, reference to one of which has already been made, are given in the following table.

Coil No.	Diameter.	No. of Turns.	Fluxmeter Deflection.	Nos. $\propto \frac{\text{Flux of H}}{\text{Turns} \times \text{Circum.}}$
1	1.275	147	33.2	0.566
2	2.540	33	37.6	1.425
3	3.80	15	37.4	2.08
4	5.08	8	36.5	2.845
5	6.32	5	35.5	3.56

From the numbers in the last column it will be noticed that for a single turn, the flux of H, for each search-coil, divided by the corresponding circumference, is a linear function of the distance from the axis of symmetry, c; as shown clearly by Fig. 7. Hence E, by (15), is also a linfunction of this distance; and H is therefore practically the same at all points in the region considered.

It thus becomes possible to substitute for the flux f_c through the circle in question, and the flux f_d within the boundaries of the dielectric, the respective areas enclosed by these figures; namely, 37.37 cm.^{2} and 0.963 cm.^{2} The ratio (16) is then found to be 0.4659. This is the ratio of the part of *curl* E at the *ends* of the strip, where there are no charges, to the *total curl*; it is thus the fraction of total *curl* which does not give a perfect differential, with respect to the time, when (13) is added to (12); and is, therefore, the quantity $(1-\epsilon)$.

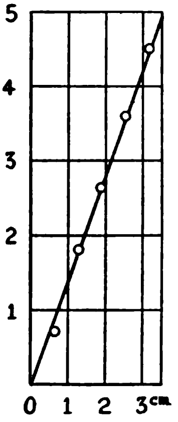

Fig. 7.

Ordinates; numbers proportional to E. Abscissæ; radii of search coils

(*i*) *Measurement of Dielectric Constant.*—The apparatus used in the measurement of dielectric constant was the condenser shown in Fig. 8, together with a wave-meter of the Dönitz type. The supports of the condenser plates, P_1P_2, were hard rubber throughout. The plates were 4 cm. in diameter and 1 mm. thick. P_1 was fixed whereas P_2 could be moved by a micrometer, N. The hard rubber rod, r, which supported the movable plate, traveled in brass bushings, b, and was provided at the end with a steel ball, kept pressed against the micrometer under the influence of the spring, s. This rod was prevented from turning by a pin which moved in a groove in one of the hard rubber supports. The dielectric to be examined, in the form of a plate or disk, D, was fastened to a hard rubber slider, J, free to slide lengthwise on the base of the instrument.

The wave-meter consisted of a variable air condenser of semi-circular plates, by Cossor, of 0.003 microfarad maximum capacity, together with an inductance. The latter was a single loop of copper rod, 81 cm. by 23.5 cm., attached to the terminals of this condenser. Across these terminals were connected, in series, a 1,000 ohm telephone receiver and an aluminium-tellurium rectifier, after Austin.[1]

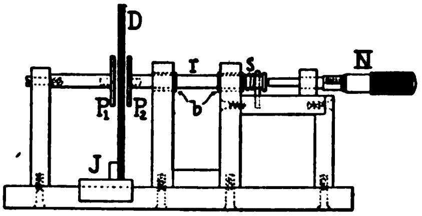

Fig. 8.

Condenser for Measurement of Dielectric Constant. (⅓ actual size).

This type of rectifier was found much superior to a silicon-steel contact, at the frequencies used; and a loud hissing sound could be heard in the telephone while the pointer was passing over about 40′ of arc. Such sharp tuning as this was only possible when the wave-meter was at least 4 meters from the oscillating system.

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[1] L. W. Austin, Bulletin, Bureau of Standards, Vol. 5, 1, pp. 133–147.

The procedure consisted in substituting, in the secondary circuit shown in Fig. 6, the condenser just described and a suitable small inductance, in place of C_2 and M_1M_2. The wave-meter was kept set for resonance at the frequency of the secondary, with C_2, M_1M_2, and the dielectric strip in position. The micrometer was adjusted until this frequency was attained by the new secondary circuit. The dielectric was then removed from between P_1 and P_2, and the micrometer adjusted until the frequency was exactly the same as before. Since no inductance was changed in the process, and further, since the frequency was the same in both cases, the capacities must also have been the same. If we denote by k, the dielectric constant; d_1, the thickness of the dielectric; and d_2, the difference in the two micrometer readings, we have

$$k = \frac{d_1}{d_1 - d_2}. \tag{17}$$

This method assumes that the field within the dielectric is everywhere parallel to the axis of the condenser plates; so that the removal of the dielectric and adjustment of the plates until the frequency becomes again the same will amount merely to the substitution of an equivalent thickness of air. It will be noticed that, with the condenser P_1P_2, the current density was of the same order as that in the experiment with the movable dielectric strip.[1]

(*j*) *Proportion of Total Current in the Dielectric Strip.*—In order to obtain an idea of the magnitude of the distributed capacity together with certain other leakages, experiments were first performed to find the proportion of the total current, as measured by I_1, that passed through the dielectric strip, when the coils M_1M_2 were removed from the immediate neighborhood of K_1K_2—the electrical connections remaining the same. A curved glass strip, of the same shape and size as the hard rubber strip used in the actual measurements, was placed in the gap and then withdrawn—the change in capacity of the wave-meter necessary to produce resonance being noted.

The corresponding readings, in order, were 45° and 37° 30′, showing that the increase in capacity due to the glass strip was 20 per cent. of the original capacity. The change in capacity, calculated from the thickness of the strip and the dielectric constant ($K = 2.55$ by the method previously described) was 26.4 per cent. In this calculation the dielectric was considered as being part of an infinite plate, between

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[1] In measurements of dielectric constants, in other researches, in which small current-densities are employed, it would be advantageous to use several condensers in series, each similar to that shown in Fig. 8, with plates of large area, close together. In this way the end corrections would be small, while at the same time the frequency could be very high.

condenser plates of infinite extent. From this it follows that 75.8 per cent. of the total current, as measured by I_1, passed through the dielectric. In other words, the loss of current due to the part of the field of K_1K_2, lying outside the strip, together with the distributed capacity of M_1M_2, and the capacity to earth of the hot-wire ammeter, I_1, was but 24.2 per cent.

The proportion of the total current through I_1 which passed through the strip while the measurements of ponderomotive force were being made was found by repeating the preceding experiment after M_1 and M_2 had been replaced in their normal positions, as shown in Fig. 3. In this case, the introduction and removal of the glass strip caused a change in wave-meter capacity of from 84° to 78°. Thus the total capacity was now greater than before, whereas the proportion of total current passing through the dielectric was less. The proportionate increase in capacity due to introduction of the glass strip was 7.6 per cent. so that but 28.8 per cent. of the total current passed through this strip.

It was hoped that each of the factors which enter into the expression for the ponderomotive force could be measured to within 1 per cent. The corrective factor under consideration is important, but unfortunately it involves the most difficult of all the measurements—depending as it does upon a *difference* of reading of but a few degrees—each reading not being made closer than 20′. Thus the error of this measurement exceeded that of all the others combined, except under the very best conditions of tuning of the wave-meter.

It was assumed that the fractional change in capacity, on introducing and removing the strip, was due solely to an influence upon those electrostatic lines which passed through the strip. This, however, was not the case, as there was necessarily present a bending, toward the upper and lower edges of the dielectric strip, of the electrostatic lines from the condenser K_1K_2 which passed immediately above and below. It is thus seen that a part of the increase of 7.6 per cent. in capacity was due to displacement currents lying outside the dielectric strip, which could not have taken part in the ponderomotive action. The correction is, however, neglected as being small in comparison with the combined error of the various measurements. An idea of the magnitude of this neglected correction, as compared with the effect of distributed capacity *et cetera*, could have been obtained by comparing the change in capacity first described with that resulting from the introduction and removal of a strip of much greater vertical width.

(*k*) *Measurement of Frequency.*—Although it was not necessary to measure the frequency, in order to calculate the ponderomotive force, it was nevertheless of interest to do so, as the values obtained for the dielectric constant would otherwise be meaningless. The wave-meter capacity, let us say C, could be obtained by calibration, and the inductance L, of the loop of copper rod, calculated; but the usual expression for the frequency, n, namely,

$$n = \frac{1}{2\pi\sqrt{LC}}, \tag{17}$$

could not be used for an accurate determination, owing to the fact that the capacity to earth, let us say, x, and, especially, the inductance of the condenser, y, could not be neglected.

If we make a second measurement of the frequency, n, by using a smaller loop, the inductance of which, L_0, may be calculated; at the same time, adjusting the condenser to the proper capacity, C_0, for resonance, we shall have by equating the two expressions corresponding to (17),

$$(L - L_0)x + (C_0 - C)y + C_0L - L_0C = 0. \tag{18}$$

This procedure may be repeaed while the oscillator is emitting waves of some other frequency, n'. The only difference, in this case, will be that the condenser must be set at two new values of capacity. Let us say C' and C_0', instead of C and C_0. We shall then have

$$(L - L_0)x + (C_0' - C')y + C_0'L - L_0C' = 0. \tag{19}$$

Solving (18) and (19) for x and y, we find

$$x = \frac{L_0C - C_0L}{L_0 - L} - \frac{C_0 - C}{L_0 - L}\left(\frac{L(C_0' - C_0) + L_0(C - C')}{(C_0 + C') - (C_0' + C)}\right),$$

$$y = \frac{L(C_0' - C_0) + L_0(C - C')}{(C_0 + C') - (C_0' + C)}.$$

RESULT.

The writer regrets that, owing to ill-health, it was impossible to make a series of measurements, using various dielectrics, with the degree of accuracy of which the method is capable. It was hoped that at least hard rubber, "Galalith," and black "'Bakelite' Molding Mixture No. 150," might be tried, which substances have dielectric constants, at 3×10^6 cycles per second, of, repectively, 2.15, 2.74, and 3.52.

Fortunately, however, a preliminary measurement was made, with hard rubber as dielectric, which showed the order of magnitude of the force in question. Instead of M_1 and M_2, two other coils were used in this experiment, each 7 cm. in diameter and having 12 turns.

(*a*) *Observed Force.*—The mean deflection was obtained from the mean of several swings of the torsion balance, first to the right of the zero position and then to the left, by reversing the switch S_2. The shift of the mean position occurred in the expected direction.

The deflections, in cm., were

	Right.	Left.
	35.6	
	33.8	30.6
	35.1	31.5
	34.6	30.7
Mean	34.77	30.93
	Difference,	3.84

During the time these observations were being made, the readings of I_1 were 30 and 72 divisions, respectively. The readings of this instrument were found to be proportional to the square of the current; further, the deflection of the torsion balance was, by theory, also proportional to the square of the current; whence both these deflections were proportional to each other. Thus the deflection corresponding to 72 divisions was $[72/(72 + 30)] \times 3.84$, or 2.71 cm.

The constant of torsion, μ, of the fiber used was obtained from the expression for a torsional semi-vibration

$$t = \pi\sqrt{\frac{IL}{\mu}}, \tag{20}$$

where t = the time of a semi-vibration, 450 seconds,
I = moment of inertia of the suspended system, 9,504, and
L = the length of the fiber, 30 cm.;
whence

$$\mu = 1.386.$$

The moment of inertia was obtained from a comparison of the periods of torsional vibration, when two small lead-riders, of equal masses, were placed on the torsion arm, B, Fig. 4, first at one distance, and then at another distance from the axis of suspension. In each case the stress on the fiber, and hence the constant of torsion, was the same. The other quantities involved in the calculation were; arm of the force, 20 cm.; scale distance, 160 cm.

Inasmuch as μ is the couple required to twist unit length of the fiber through unit angle, the actual ponderomotive force, F_m, upon the dielectric was $2.70/(20 \times 30 \times 160) \times 1.386$, or

$$F_m = 0.000391 \text{ dyne}.$$

(*b*) *Calculated Force.*—The expression for the force, F_c, is, from (14)

$$F_c = \frac{p(k - 1)(1 - \epsilon)}{k} \frac{Hil}{10}, \tag{21}$$

where p = the proportion of total current flowing through the dielectric strip,
k = the dielectric constant, 2.15,
$(1 - \epsilon)$ = the corrective factor, 0.4659,
H = the magnetic field per unit current in the particular coils used in this experiment. The value of H may be taken as 0.3, that for the coils M_1M_2, thus being $0.3 \times 5.23 = 1.569$.
i = the total secondary current as measured by the deflection, 72 divisions of the hot-wire ammeter. By the calibration curve of the instrument, this deflection corresponded to 0.408 ampere.
l = the thickness of the dielectric strip, 0.13 cm.

The frequency was 3.1×10^6 cycles per second.

The value of the calculated ponderomotive force, F_c, was

$$F_c = 0.000732 \text{ dyne.}$$

While it should be remembered that this determination of the force is only approximate, it is nevertheless suggestive that the corrective factor which was neglected, as explained under (*j*), would tend to decrease the calculated value of the force, F_c. Further, the current in the coils M_1M_2 was not so great as that indicated by the hot-wire ammeter, owing to distributed capacity. This, if considered, would also reduce the value of F_c.

In conclusion, I wish to express my gratitude to Deans West and Magie for my opportunity, as research instructor in physics at Princeton University, 1912–1913, of using the facilities of the Palmer Physical Laboratory, where this work was performed. I also wish to thank Professor Webster, of Clark University, for my use, as honorary fellow in physics, of the laboratory of that university, where positive results, concerning the force in question, were first obtained.[1]

SUMMARY.

1. According to Lorentz's theory of a dielectric, a ponderomotive force must act upon such a body when carrying a displacement current in a magnetic field, analogous to the force which is known to act upon a conductor when carrying a conduction current in a magnetic field. An experiment performed by Whitehead, to demonstrate the existence of

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[1] Presented in a paper before the American Physical Society, at the Cambridge Meeting, April 27, 1912.

this force, has given a negative result; which result has been shown by others to be in agreement with electrical theory.

2. In the present paper, the theory of a more general research is developed, which includes the Whitehead experiment as a special case, and suggests, further, an experiment by which a positive result may be attained.

3. An apparatus has been designed, on the lines suggested by this theory, whereby the quantities involved in a determination of the force in question may be obtained with a considerable degree of accuracy.

4. An experiment, of a preliminary nature, has shown that the ponderomotive force, upon a piece of hard rubber as dielectric, is of the expected order of magnitude.

WORCESTER, MASS.,
April, 1914.

DETERMINATION OF e/m FROM MEASUREMENTS OF THERMIONIC CURRENTS.[1]

By Saul Dushman.

IN a recent paper on "The Effect of Space Charge and Residual Gases on Thermionic Currents in High Vacuum,"[2] Dr. I. Langmuir has shown that the thermionic current from a heated metallic surface in high vacuum is limited by (*a*) temperature, (*b*) space charge.

With a given potential difference between the electrodes, the current from the heated cathode increases with the temperature in accordance with the equation of Richardson,

$$i = a\sqrt{T}\epsilon^{-\frac{b}{T}}. \tag{1}$$

However, above a certain temperature, this current becomes constant; further increase in temperature does not cause any corresponding increase in thermionic current. The temperature at which this limitation occurs increases with increase in anode potential. It was shown in the above paper that this effect is due to the existence of a space charge produced by the emitted electrons, and it was furthermore deduced that at a fixed cathode temperature, the electron current ought to increase with the three-halves power of the voltage (until the saturation current as defined by the Richardson equation is attained), that is, for electrodes of any shape, the space charge current

$$i = k \cdot V^{3/2}, \tag{2}$$

where V denotes the potential difference, and k is a constant depending on the shape of the electrodes, their area and the distance between them. For the case of the heated filament in a concentric cylindrical anode (infinite length),

$$i = \frac{2\sqrt{2}}{9}\sqrt{\frac{e}{m}} \cdot \frac{V^{3/2}}{r}, \tag{3}$$

where i is the thermionic current emitted per unit length of filament, and r the radius of the anode.

While in the above investigation, a number of results had been ob-

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[1] An abstract of this paper was read at the Chicago Meeting, November 29, 1913. See Phys. Rev., 3, 65, 1914.

[2] Phys. Rev., 2, 450, 1913.

tained which were in accord with equation (2), it was considered, in view of the skepticism exhibited by many investigators with regard to the actual existence of a pure electron emission that a separate experimental test of the validity of equation (3) would be of interest, especially as it leads to a new and probably very accurate method for the determination of the ratio e/m.

DESCRIPTION OF APPARATUS.

In order to be able to apply equation (3) to the case of a finite length of anode, the latter was inserted between two other anodes on the guard-ring principle. A tungsten filament 15 cm. long and 0.25 mm. diameter was supported by means of a glass tube about 7 cm. in diameter and 36 cm. long (see Fig. 1). The current required to heat the filament was carried by a separate wire at the top, thus avoiding any weakening in the tension exerted by the spring, owing to heating. The anode consisted of three molybdenum cylinders 2.54 cm. in diameter, made from sheet metal about .37 mm. thick. The two edges of the sheet were brought together to form a cylinder and heavy molybdenum strap riveted to the outside of the cylinders, held these edges close together. At the same time, these straps were drilled and tapped to fit 2.5 mm. molybdenum rods. The latter were held rigidly in the side arms by means of molybdenum springs. The outside anodes were each 2.54 cm. long, while the center one was 7.62 cm. long. The connections through the glass were made by means of platinum leads.

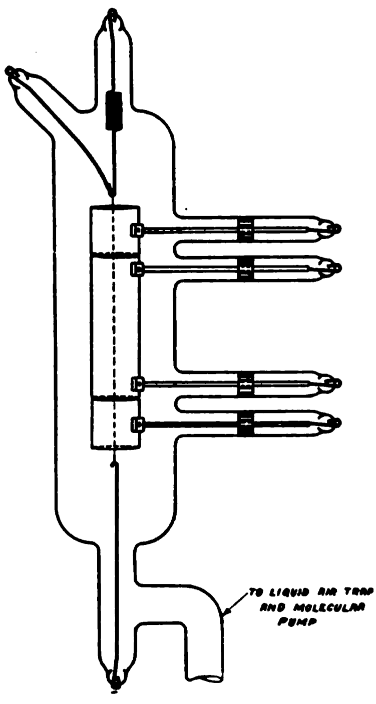

Fig. 1.

The tube shown in Fig. 1 was connected through a large-bore liquid air trap to a Gaede molecular pump. The latter was connected in series with a Gaede "box" oil pump and the rough vacuum line (pressure of about 1 cm.).

A McLeod-gage was inserted between the molecular pump and the oil pump, as well as a tap connected to the atmosphere through a phosphorus pentoxide tube for letting in dry air into the vacuum tube before stopping the molecular pump.

The tube could be exhausted at a temperature of 350° C. by means of an electrically heated oven which was lowered over it.

VACUUM OBTAINABLE.

As stated by Dr. Langmuir in his paper, the most essential conditions for obtaining pure thermionic currents are an extremely high vacuum and absolutely gas-free electrodes.

A special investigation, the results of which will be published shortly in a separate paper, has shown that in order to obtain a very high vacuum even with the molecular pump, it is necessary to insert a liquid air trap between the tube to be exhausted and the pump, and also to heat the tube to over 300° C. for a period of at least one hour. The water-vapor absorbed in the glass walls is thus removed much more quickly and the liquid air trap prevents the diffusion backwards of stopcock grease vapor and other condensible gases. After this preliminary heating, the vacuum is found to be improved considerably. With a pressure of about 0.01 mm. on the rough side of the pump, it is possible in this manner to obtain a vacuum in the tube, which is certainly less than 2×10^{-7} mm.

PREPARATION OF ANODES.

However, under the action of the electrons emitted from the heated cathode, there is a continuous liberation of gas from the anodes, which not only leads to an actual decrease in the thermionic currents themselves, but also prevents the space charge limitation from occurring, owing to the production of positive ionization. It is, therefore, very important to free the anodes thoroughly from occluded gases and volatile oxides which may dissociate gradually when the anodes are heated. This may be accomplished either by heating the anodes to a white heat in a tungsten vacuum furnace,[1] or by powerful electron bombardment in the exhausted tube itself.

The latter method was adopted in this case. A potential varying from 1,000 to 5,000 volts was applied to the electrodes by means of a transformer, the filament being maintained at such a temperature that anywhere from 50 to 200 milliamperes thermionic current was obtained. Under the influence of the electron bombardment at high voltages, the temperature of the anodes was raised to 1000° C. or more and the gases contained in the metal were gradually eliminated. This progressive clean-up of the gases was accompanied by a disappearance of the blue glow, which always indicates the presence of positive ions and by a corresponding increase in the magnitude of the thermionic currents actually

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[1] Coolidge,

emitted at any given temperature of the filament. Whenever the blue glow ceased, the voltage was raised, thus heating the anodes to a still higher temperature and liberating more gas from them. By this procedure, the anodes were finally raised nearly to a white heat, and made gas-free.

METHOD OF DETERMINING THE TEMPERATURE OF THE FILAMENT.

The temperatures of the filament were determined from the relation,

$$T = \frac{11{,}230}{7.029 - \log_{10} H},^{1}$$

where H is the intrinsic brilliancy of the filament in international candles per square cm. of projected area, and T denotes the temperatures in degrees Kelvin.

The variation in H with the current carried by the filament was determined by photometric measurements carried out on a special lamp, made up from the same coil of tungsten wire as that used in making up the filament of the discharge tube described above. From these data, a curve was obtained giving the relation between the current carried by the filament and the resulting temperature.

EXPERIMENTS.

In Table I. is given a typical record of observations made on the magnitude of the thermionic currents during the process of evacuation. It will be observed that at the very beginning, when the vacuum was comparatively bad, the electron emission was very low. Replacing the ice and salt mixture around the trap by liquid air caused only a small increase in the thermionic currents obtained. But after the anodes were heated to a high temperature and made fairly gas-free, the increase in the electron emission was considerable. Thus, at 2130° K. the currents observed increased from 0.08 to 2.30 milliamperes, while at 2210° the increase was from 0.1 to 18 milliamperes, and similarly for other temperatures.

These observations are most easily interpreted in terms of Dr. Langmuir's surface film theory. According to this theory, "the effect of gases in changing the electron emission, is due to the formation of unstable compounds on the surface of the wire. In the cases observed (so far), the presence of the compound decreases the electron emission. The extent to which the surface is covered by the compound depends on the *rate of formation* of the compound and on its *rate of removal* from the

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[1] I. Langmuir, PHYS. REV., 2, 452, 1913.

TABLE I.

Effect of Gradual Clean-up of Gas in Electrodes.

T = temperature of filament; V = voltage between electrodes; i_1 = thermionic current to center anode only; i_3 = current to all three anodes. ¶ Currents measured in milliamperes.

Time.	T	V	i_3	Remarks.
8:00 A.M.				Tube heated in oven to 330° C. for 1 hour.
9:00	2000°	20	0.065	
	2030	20	.14	
	2080	20	.04	*Ice and salt mixture on trap.*
	2130	20	.08	Pressure on rough pump side at about 50 μ. Estimated pressure in tube about 0.001 μ.
	2175	20	.08	Current gradually decreased to 0.04
9:15	2175	50	.04	
	2210	20	.10	
	2210	50	.10	
	2250	50	.22	
	2300	50	.44	
		20	.40	
	2250	50	.26	
	2300	20	.48	
9:30	2315	20	.72	
		50	1.00	Had to stop pump. Pressure on rough pump side still high (50 microns). Cleaned out pump. Put on *liquid air* on trap. Baked out tube in oven for 1 hour.
		95	1.00	
12:00	2180	21	.07	Pressure on rough pump side, 25 μ. Estimated pressure in tube about 0.0005 μ.
	2230	21	.25	
	2330	21	1.1	
12:30 P.M.	2230	21	.38	
	2190	21	.30	
	2230	21	.75	
	2330	21	2.2	Note the gradual increase in thermionic current.
12:45	2250	1,600	1.6	Applied high voltage to bombard anodes.
	2250	2,000	1.2	
	2290	2,000	2.6	Current decreased rapidly to 1.6.
	2330	1,500	8	Current decreased rapidly to 3.
1:00	2330	3,000	40	Current decreased rapidly to 15.
	2365	3,000	90	Current decreased rapidly to 30.
2:00	2400	3,500	130	Current decreased rapidly to 30.
	2360	3,000	70	Current decreased rapidly to 40.
3:00	2400	4,000	140	Current does not decrease so much. No blue glow apparent. Anodes red hot. Changed to low voltage battery.
4:00	2230°	21	12	
	2360	21	14	
	2170	21	9	
	2130	21	3.5	
	2230	21	12	
	2250	28	19	

Time.	T	V	i_3	Remarks.
	2290	28.5	21	
	2160	28	7	
	2160	50	7	Temperature saturation.
	2210	50	18	
	2240	50	27	
	2290	50	48	Space charge effect.
	2330	50.6	49	
			i_1	Measured current to centre anode only.
4:45	2240	50	15.5	See Curve 1, Fig. 2.
	2260	50.2	20	
	2290	50.2	27	
	2310	50.4	28.5	
	2320	50.6	29.0	
	2260	50.2	21	
	2310	50.4	28.5	
		43.4	24.5	
		73.4	32.5	Temperature saturation.
		94.9	32.0	
	2330	95.0	40	
	2345	95.2	55	
	2360	95.3	65	
	2440	96.5	75	
	2460	97.0	77	See Curve 1, Fig. 2.
6:00	2480	97.0	77	Let in dry air.
6:30				Re-exhausted. Fresh liquid air on trap.
7:00	2250	50	10	
	2300	50.4	20	
	2340	50.6	31	
	2390	50.6	34.5	
	2270	50.6	8	
	2270	73	8	
	2310	96	15	
	2340	96	32	
	2390	96	53	
7:30	2440	96	110	See Curve 3, Fig. 2.
			i_3	Applied high voltage to bombard anodes again.
	2340	4,800	66	Current decreased to 40.
	2390	5,000	110	Current decreased to 75.
	2410	5,000	125	Current decreased to 115.
8:00	2450°	5,000	250	Anodes at bright red heat.
	2450	5,000	300	
	2320	5,000	110	
	2380	5,000	225	
	2400	5,000	250	Current does not decrease so much.
			i_1	Measured current to centre anode only at low voltages.
9:00	2440	88	71	

Time.	T	V	i_s	Remarks.
	2400	87	70	See Curve 4, Fig. 2.
	2360	87	68	
	2344	87	60	
	2320	87	50	
	2300	87	38	
	2275	87	28	
	2238	87	17.5	
	2216	87	12.0	
9:30	2350	87	65	
	2380	87	69	
	2450	88	70	Space charge current calculated = 73.
	2200	106	9.5	
	2240	106	20.5	
	2320	106	49	
	2360	107	72	
	2370	107	79	
	2420	108	94	
	2440	108	94	
	2460	108	95	Space charge current calculated = 99.
	2540	108	95	See Curve 4, Fig. 2.

surface." Now even low velocity electrons striking the anode liberate gas from it. If the pump is unable to remove the gas as fast as liberated, the rate of formation of the film on the surface of the cathode exceeds its rate of destruction and consequently the electron emission decreases. This was exhibited strikingly in the actual experiments during the treatment with high voltages. The volume of gas liberated during the interval of only a few minutes was so great that the electron emission would decrease very rapidly.[1] It was only towards the end of the high voltage treatment that the thermionic currents remained fairly steady, thus indicating that little or no gas was being liberated.

The subsequent observations on the thermionic currents at lower voltages show that the vacuum was now considerably better than before the high voltage treatment. Otherwise no space charge effects could have been observed; for as long as gases are given off by the electrodes or the glass walls, there is no indication of space charge effects. In Curve 1, Fig. 2, are plotted the observations obtained between 4:45 and 6:00 P.M. The space charge currents calculated for 50.6 and 97 volts are 32 and 84.5 milliamperes respectively; the values actually obtained were 29 and 77.

The effect of bad vacuum conditions on the thermionic currents is

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[1] The high voltage was applied for only a few minutes at a time, until blue glow occurred; the filament was then allowed to cool for a couple of minutes to permit the pump to remove the liberated gases.

shown by the observations made between 7:00 and 7:30 P.M., and plotted in Curves 2 and 3, Fig. 2. After letting in dry air and re-exhausting, it was evidently not possible to obtain a good enough vacuum until the anodes were heated again by high voltage bombardment.

It will be seen from these curves that the effect of the presence of gas is to remove the limitation set by space charge effect, while decreasing the actual electron emission. Curve 3 is interesting because it shows the complete absence of any space charge effects. Evidently the amount of gas present in the tube had kept on increasing during these measurements, so that the space charge effect disappeared completely.

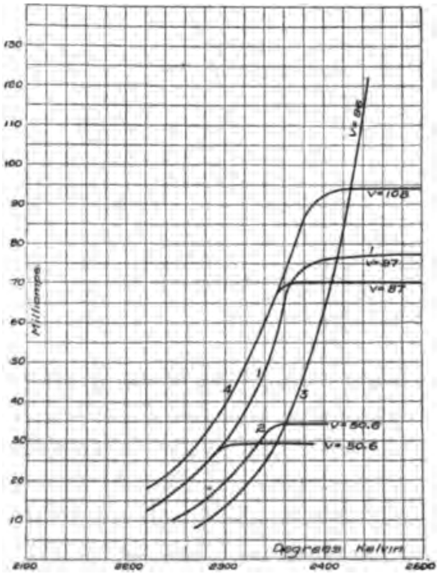

Fig. 2.

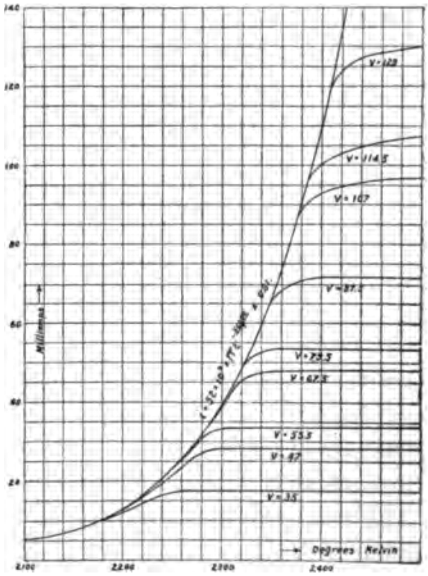

Fig. 3.[1]

After bombarding the anodes again, the results plotted in Curve 4, Fig. 2, were obtained. Owing to the improved vacuum conditions, the electron emission at any temperature was increased, while the space charge currents observed were not very different from those calculated in accordance with equation (3), as shown below.

Table II gives the actually observed space charge currents at different voltages, together with a number of the observations (made under the same conditions) on the total electron emission at different temperatures. The results are plotted in Fig. 3. It is seen that heating the anodes with high voltage again led to no further changes in the values of the thermionic currents observed.

The values under i_1 (calc.) were calculated by means of equation (3)

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[1] The ordinates give the current in milliamperes *not* microamperes.

Denoting the constant, $2\sqrt{2/9}\sqrt{e/m}$ by k_0, the values of this constant for different values of e/m are as follows:

$10^{-7} \times e/m$.	$10^3 \times k$.
1.775	14.71
1.766	14.68
1.755	14.62

The last seems to be the most accurate value of e/m, according to the

TABLE II.

Space Charge Currents Obtained at Different Voltages under Best Vacuum Conditions.

Tube exhausted as before. Anodes bombarded with electrons of high velocity by using high voltage. Measured current to center anode only. Pressure on rough pump side, 18 microns. Estimated pressures in tube, less than 0.0004 micron.

T	V	i_1	i_1 (calc).
2125°	47.0	3	
2185	47.0	9	
2236	47.0	18	
2280	47.0	27	
2320	47.0	28	28.5
2340	35	17	17
2300	35	17.3	
2240	35	16.2	
2208	35	12	
2180	35	8.5	
2240	68	19.5	
2262	68	27	
2318	68	45	
2340	68	47	
2400	68	48	49
2340	55.5	34	36
2340	76.0	54	
2400	75.2	54	57.5
2400	88.5	72	
2450	87.5	71	72.5
2480	89	72	74
2480	107.0	96.5	98
2430	108.5	97.7	100
2440	107.5	95.0	
2396	107.5	92.0	
2300	111	49	
2390	114.2	99	
2440	114.5	104.8	
2476	114.8	106.4	
2500	115	107.3	
2522	115	108.0	109
2440	128.5	126	
2480	128.8	128	
2500	129	130	130
2520	129	130	

Treated anodes with high voltage again, and measured thermionic currents to centre, anode only. Pressure on rough pump side down to 10 microns; estimated pressure in tube less than 0.0002 micron.

T	V	i_1	i_1 calc.
2350°	50.8	31.8	32
2180	49	15.5	
2180	70.5	15.0	
2240	71.0	37.5	
2315	71.5	52.2	54
2400	93.5	81.0	80
2490	114.0	111.0	109
2500	136.0	144.0	140

observations of Bucherer and others and is in good accord with the value calculated from the Zeeman effect. It was, therefore, used to calculate the space charge currents for different voltages for the length of filament, corresponding to that of the center anode (7.62 cm.). The results are given in Fig. 4. From this curve, the values given in Table II., under i_1

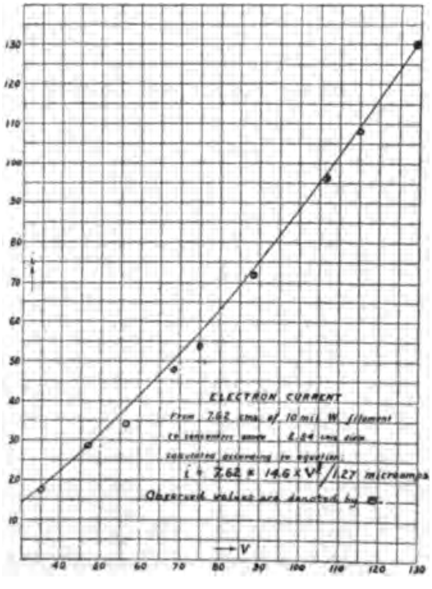

Fig. 4.

(calc.), were taken. The actually observed thermionic currents are also indicated in Fig. 4.

Sources of Error.

No attempt was made in this investigation to attain great accuracy; the object being merely to show the validity of the space charge equations.

It may, however, be well to indicate briefly the sources of error involved in the above measurements.

The largest errors undoubtedly occurred in the measurement of V, the potential difference between the electrodes. The voltmeter used was of the ordinary Weston type. As direct current was used to heat the filament, a correction had to be made for the voltage drop through the filament. This drop differed, of course, with the temperature, and was determined by means of a separate set of measurements. To correct for the possibility that the center anode might not be situated symmetrically with respect to the voltage drop through the filament, the thermionic currents were observed for both directions of current through the filament, and the means of pairs of observations are recorded in Table II.

Other sources of error besides that involved in measuring V occurred also in the determination of i_1, the thermionic current to the center anode, and of r, the radius of the cylinder. In neither case was an accuracy greater than 1 to 2 per cent. obtained.

Besides these sources of error involved in the actual experimental manipulation there is another factor which must be taken into account when one attempts to use the method for an accurate determination of e/m.

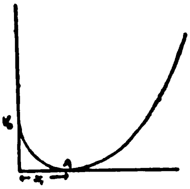

Fig. 5.

It will be observed that in deriving his space charge formula, Dr. Langmuir intentionally assumed that the electrons emitted from the hot cathode have no initial velocity. This is, however, manifestly not true. Thus the average velocity of electrons emitted from a hot cathode at 3000° K. is about 0.4 volt. In consequence of this initial velocity, the curve giving the potential distribution between anode and cathode will have a form such as that indicated in Fig. 5, where V_0 denotes the potential due to the initial velocity of the electrons, and V, the potential of the anode, both potentials being measured with respect to the minimum point A.

It is therefore evident that instead of using for V in equation (2) the potential difference as measured, we ought to add to the observed value of V another term V_0.[1] Since for small wires, the diameter of the wire does not affect the value of i calculated according to equation (3) we may consider the small distance x_1 which the electrons have to travel before reaching the minimum point as merely an increase in the diameter of the wire and we may therefore neglect it.

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[1] It must also be observed that in all the calculations we have assumed an average velocity for the electrons, whereas in the more rigorous calculation Maxwell's distribution law ought to be taken into account.

A more accurate form of equation (3) is therefore:

$$i = \frac{2\sqrt{2}}{6}\sqrt{\frac{e}{m}}\frac{(V_{\text{obs}} + V_0)^{3/2}}{r}.^{2}$$

As in all the experiments recorded above, V_0 was less than 1 per cent. of the observed potential difference it was neglected.

There is no doubt, however, that by carrying out these measurements under more exact conditions the method described in this paper could be used to determine e/m with a very high degree of accuracy.

Effect of Gases.

As has been pointed out by Dr. Langmuir the effect of gases is two-fold. Firstly, it tends to eliminate space charge effects owing to the presence of positive ionization; secondly, the actual thermionic emission is decreased below the value obtained in a good vacuum.

It has already been pointed out that such effects were observed during the process of evacuation. In Table III. are given some data obtained at the end of an experiment with gas-free electrodes in presence of nitrogen. Nitrogen of the same degree of purity used in the manufacture of the nitrogen-filled tungsten lamps was allowed to enter the discharge tube

Table III.

Effect of Nitrogen on Thermionic Currents.

P Rough Side of Pump.	T	V	i_R	i_S	i_0	Remarks.
200 microns	2300°	15	39	9	28	P in tube estimated at 0.01 micron.
		80	39	63	28	
1,000 microns	2325	100	51	86	30	
		80		63	25	
		60		41	18	
		40		22	18	
	2400	79	109	61	85	
		58		39	70	
		38		21	57	
		15		9	20	
	2450	22.5	180	9	75	
		30		14	68	
		52.5		32	75	Blue glow.
		72.5		54	87	Blue glow.
		93.5		79	105	Blue glow.

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[2] The effect of taking into account the initial velocity of the electrons and Maxwell's distribution law has been discussed very fully by W. Schottky in a recent paper (Physik. Zeitsch. *15*, 624) who has also pointed out that in making use of the space charge equation for an accurate determination of e/m, failure to consider the above factors might involve errors of over 2 per cent. (Physik. Zeitsch. *15*, 528, footnote.)

through a narrow-bore stopcock connection sealed between the liquid air trap and the pump. The rate of flow of the gas was measured approximately by noting the pressure on the rough pump side. From this reading it was possible to obtain an estimate of the pressure in the tube.

Under i_r is given the value of the electron emission at the corresponding temperature as obtained from the curve $i = a\sqrt{T}\epsilon^{-\frac{b}{T}}$ given in Fig. 3. The values under i_s are the space charge currents calculated for the corresponding voltages by means of equation (3) and plotted in Fig. 4. The actually observed thermionic currents are given under i_0. It will be observed that *even with blue glow* the thermionic currents obtained were much lower than those calculated from the Richardson equation.

This decrease in thermionic currents was certainly not due to space charge effects, as is seen from the data at $T = 2325$. In some other experiments in which air was allowed to enter the tube, the decrease in thermionic current was so pronounced that even with 3,000 volts on the anode the sudden appearance of gas in the tube (whether originating in the anode or allowed to flow in through a side connection) immediately caused the electron emission to fall to almost zero.

General Conclusions.

The above experiments are thus in uniform agreement with the observations and explanation given by Dr. Langmuir in his paper on space charge effects, and in the light of all our subsequent experiments his conclusions on the nature of electron emission from heated metals appear to be well justified. The *perfect definiteness* of the results obtained which are independent of vacuum conditions after a sufficiently high vacuum has once been attained, the *reproducibility* of the observations even after allowing gas to enter the tube and then re-exhausting, and the quantitative agreement obtained not only in the above experiments but in all the experiments so far carried out in this laboratory point to the existence of a *pure electron emission per ipse*, which is not a secondary effect due to chemical reactions, as assumed by a number of other investigators, and which is a *function of the temperature only*.

In conclusion the writer wishes to express his indebtedness to Dr. I. Langmuir for kindly suggestions and interest during the progress of the above investigations.

Summary.

The space charge formula developed by Dr. I. Langmuir for the thermionic current from a heated filament to a coaxial cylindrical anode has been tested experimentally over a range of voltages from 35 to 140, for

the case of a tungsten filament in a concentric molybdenum cylinder.

The results obtained are in good agreement with this formula and indicate that the method ought to prove a very accurate one for the determination of e/m. The observations obtained on the effect of gases on the thermionic currents are also found to be in accord with Dr. Langmuir's surface film theory and justify the conclusion that there exists a pure electron emission from heated metals which is a function of the temperature only and is not a secondary effect due to presence of gases as assumed by a large number of investigators.

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THE EFFECT OF A LONGITUDINAL MAGNETIC FIELD ON SPARK POTENTIALS.

By Robert F. Earhart.

THE present paper may be regarded as a continuation of an article published in the February number of this journal on the "Discharge in a Magnetic Field."[1] The former paper sought to extend the work of Paalzow and Neison[2] by studying the variation in the magnitude of a discharge current between parallel plate electrodes. The electric force and magnetic force were parallel. The quantitative measurements showed that for gas pressures exceeding the critical pressure, the effect of a magnetic field was to diminish the current. As the pressure was reduced the effect of the field diminished and as the critical value was approached the effect vanished. For pressures less than the critical pressure small fields increased the current strength but strong fields were not so effective as the lesser ones in some cases.

The effect of the weaker fields in increasing the current was explained by Townsend[3] in commenting on some experiments of Strutt.[4] He suggests that the helical motion imparted to an electron by the longitudinal field will so increase its path that ionization by collision will take place with a smaller potential gradient between the electrodes.

For higher pressures it would seem that this would be offset by another effect. In a luminous discharge at the higher pressures the motion of an ion or electron will probably be complex because collision is frequent and the forces on any electrified unit will vary continually in magnitude and direction. If this be true the motion of an ion will at times have a component velocity normal to the lines of force and thus be subject to a force tending to divert it to the walls of the tube. If a longer tube of the same section should be substituted the opportunity for the charged body to be diverted would be increased. Again if the tube should be maintained of constant length but its section increased together with the area of the electrodes the probability of the ion reaching the electrodes would be increased.

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[1] Phys. Rev., N. S., Vol. III., p. 103, 1914.

[2] Wied. Ann., I.-XIII., p. 207, 1897.

[3] Phil. Mag., VI., 26, p. 730, Oct., 1913.

[4] Proc. Roy. Soc., *A*, 89, p. 68, 1913.

Having this in mind it seemed possible to attempt a test of the hypothesis. A discharge chamber *D* in Fig. 1 was placed between two brass electrodes *A* and *B*. These were mounted on the poles of a large electromagnet by means of fiber collars. The pole pieces *M*, *M* in the figure were 5.08 cm. across the face so that the midportion of the field was quite uniform. This was tested and with satisfactory results. Fields of 10,000 C.G.S. could be obtained with the gap made necessary by the insertion of the discharge chamber. The spark chamber was made from a disk of hard rubber sealed to the electrodes. The disk itself served to measure the distance between the electrodes and at the same time confine the discharge to the uniform portion of the electric and magnetic fields.

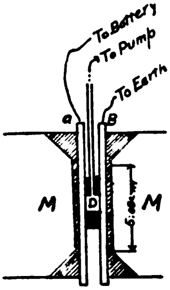

Fig. 1.

The length of the discharge path was made 2.5 mm., 5.0 mm. and 10.0 mm. and in each case apertures of three sizes were used. The apertures were 3, 6 and 9 mm. approximate radius. They were bored out with a ¼, ½ and ¾ inch drill. The ratio of their areas is therefore 1, 4, 9. The electrical arrangement was similar to the one described in the previous paper and is not reproduced. In each of the nine cases the gas pressure was varied between .2 mm. and 5 to 6 mm. The fields were of strength 0, 500, 1,000, 2,000, 5,000, 10,000 C.G.S. units. Air was the only gas used.

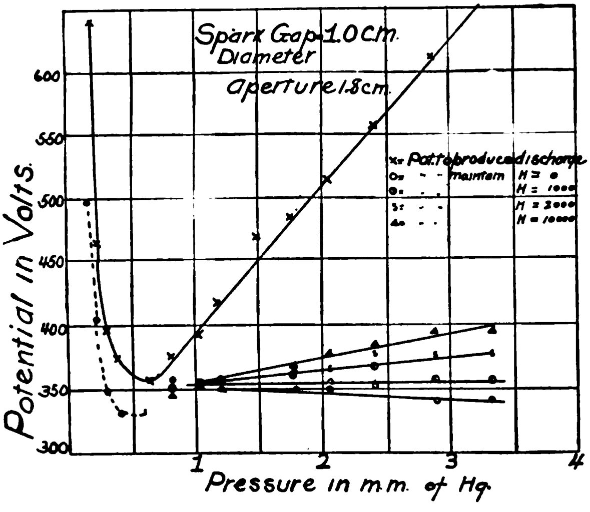

Fig. 2.

It is well known that for a particular pressure a definite potential is required to start a discharge but that once started a lower potential will maintain it.

All these experiments show that for pressures above the critical value the potential required to start the discharge is unaffected by the magnetic field but that the potential required to maintain the current flow must have a higher value when a field is on than for a zero field. This is shown in figure 2, where the distance separating the electrodes is 1 cm. and the aperture is 18 mm. in diameter. The variations in the potential required to maintain the discharge are quite appreciable. This is in agreement with the current-potential curves obtained in the previous experiment because the current value depends on the excess of potential over the value required to maintain the current rather than its relation to the potential required to start the discharge. The graphs show only the results for fields of strength 0, 1,000, 2,000, and 10,000 units. It will be noted that these curves converge near the critical pressure. For pressures below the critical value the discharge potential is reduced by the magnetic field. These are shown by full black circles. Here the variations due to different fields are small. The ones represented are for a field of 10,000 units. The values for the fields from 1,000 to 10,000 differ very slightly from the ones shown. Figs. 3, 4 and 5 exhibit the variation due to size of aperture. It will be noted that a higher potential is required to maintain the discharge for the smaller aperture. Fig. 5 should be compared with Fig. 2 to show the effect of length of path. All of the nine families of curves (four only are reproduced) are consistent with one another save a vagary exhibited by the tube 1 cm. long having the smallest aperture. In that case all the effects due to variations of the field were duplicated but the entire family of curves was displaced upward along the *Y*-axis, meaning that higher potentials were required.

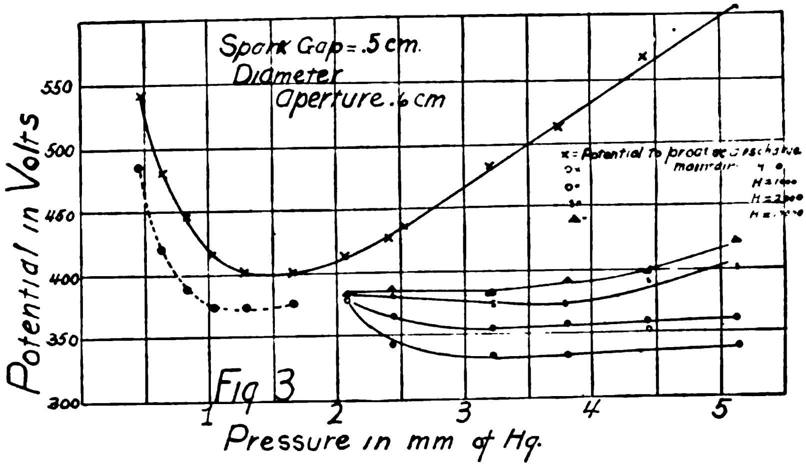

Fig. 3.

In my judgment this is due to another effect superposed on the one sought. In a paper entitled "Some Characteristic Curves for Gases at Low Pressures"[1] the author reported on some experiments made when

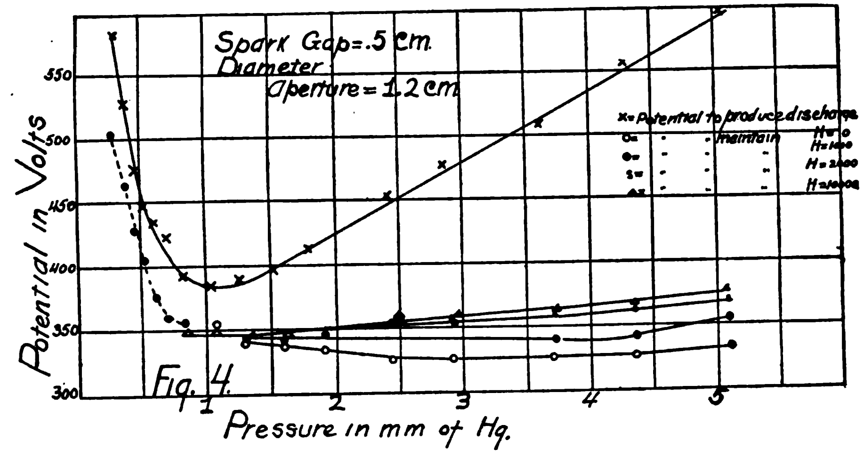

Fig. 4.

thin diaphragms having small apertures were interposed between the electrodes, also when the discharge was made to pass through a tube serving as a constriction in the path. It was found that the effect of

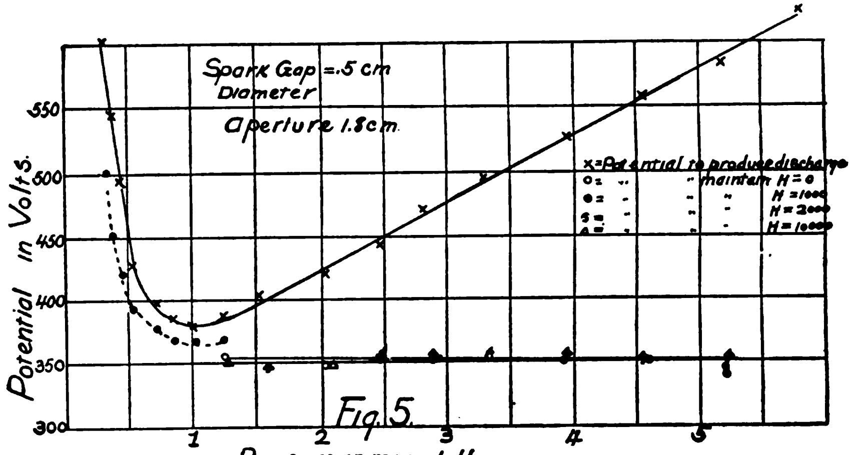

Fig. 5.

introducing such a constriction was determined by the length of the constricted portion and that the potential required to maintain a discharge of given strength had to be raised. The only effect on the characteristic

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[1] Phys. Rev., N. S., Vol. I., p. 85, February, 1913.

curve thus obtained was to displace it upward along the potential axis. It seems that in the case of the long tube with the smaller aperture that the superposition of constrictive effect on the other is to require a higher potential to produce the same variations of change due to the magnetic field.

The conclusions are:

1. Spark potentials for pressures above the critical pressure are unaffected by a longitudinal field.

2. Spark potentials below the critical pressure are decreased by a longitudinal field.

3. When the pressure exceeds the critical one, the existence of a longitudinal field requires a higher potential to maintain the discharges than if no field existed.

4. The variations noted in (3) are increased with increasing fields and become larger as the gas pressure is increased.

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April 20, 1914.

ON THE PASSAGE OF CATHODE PARTICLES THROUGH GASES AT LOW PRESSURE.

BY L. W. MCKEEHAN.

THE work to be described was undertaken primarily to determine whether a very narrow pencil of cathode rays is scattered by traversing a high vacuum. Such an effect may be expected if the electric field of each electron in the atoms composing the walls of the vessel or outside it, is confined to a few narrow tubes or filaments extending to infinity. This structure of the electric field was suggested by Sir J. J. Thomson in his Electricity and Matter (p. 63, 1903).

The cathode rays were obtained from a plane aluminium cathode in a spherical bulb about 15 cm. in diameter. They passed into the test chamber through a cylindrical brass tube 5 cm. long and of .02 cm. internal diameter set in a plane anode directly opposite the cathode. The test chamber was about 60 cm. in length and 3 cm. in diameter, and terminated in a transverse tube of larger diameter that served as a photographic camera. All glass to glass, or glass to brass joints were sealed with sealing wax. Diffusion through the hole in the anode was so slow that a good vacuum could be kept in the test chamber without lowering the pressure in the discharge tube to such a value that the photographic effect of the rays would be much reduced. The attainment of exceedingly high vacua was not attempted, since at pressures easily reached by liquid air and cocoanut charcoal (5×10^{-5} cm. of mercury by a MacLeod gauge of about one liter capacity) no scattering was detected.

Proper precautions to prevent any electrostatic fields, or any except axial magnetic fields, were taken by using a brass lining within the test chamber, by cleaning all metal surfaces exposed to rays, and by placing the axis of the apparatus parallel to the magnetic field due to the earth and to the induction coil.

After leaving the hole in the anode the narrow cone of rays traversed about 30 cm. before striking a small aperture or solid obstacle, and the rays not stopped traversed about 30 cm. more before reaching the photographic plate. A series of exposures on the same plate could be taken by the method used by Sir J. J. Thomson for positive rays.[1]

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[1] Phil. Mag., VI., p. 228, 1911.

The absence of scattering in a high vacuum was shown by the fact that the shadows cast on the photographic plate were geometrically obtainable by drawing straight lines enveloping the obstructions from a point on the axis of the apparatus, which, in case the pressure in the discharge tube was fairly high, was within the anode, but which was near the cathode, in case the pressure in the discharge tube was low enough to make the scattering due to the gas unimportant. The accuracy with which measurements could be made on the photographs was limited by the width of the transition from fully exposed to unaffected areas, about .002 cm. The sharpness of this transition proved that the rays were not uniformly distributed across the aperture in the anode, but were principally axial, as was expected from the necessary obliquity of the electric field close to the anode except along the axis of the aperture.

From the dimensions of the apparatus it was clear that an average angle of scattering exceeding 30 seconds of arc would have been detected with certainty if it had occurred. One series of photographs with rays of varying velocity passing through a slit .029 cm. wide is shown in Fig. 1, exposures 1, 3, 5, 7, and 9, on the center line of the figure. Fig. 2 shows the shadow cast by a cross of copper wire placed in the path of the rays. In exposures 1, 2, and 3 the cross was connected to the anode, which was always to earth, and the geometrical shadow is obtained. In the other exposures the cross was disconnected from the anode, and received a negative charge from the rays which it stopped. These latter photographs (one of which has been intensified in the print) show the shape of the field so clearly that they suggest a method for measuring the electric intensity on a convex surface in a high vacuum.

The rays were not homogeneous in velocity but most of them were of approximately the same speed, as shown in Fig. 1, exposures 1, 3, 5, 7, and 9, and in Fig. 2, where the more diffuse spots at one side of the center line are produced by magnetic deflection between the slit and the photographic plate. The coil for this purpose consisted of a few turns of wire wound on a rectangular frame with semicircular ends. This was placed outside the test chamber with the axes of the two coinciding. If such a coil of length $2l$ and of width a is placed with its center at a distance d from a source of axial cathode rays of velocity v and corresponding e/m the deflection x on a photographic plate at a distance from the source equal to $2d$ corresponding to ni ampere-turns is given to terms of the second order by the equation

$$x = \frac{4nie}{mv} \cdot \frac{d}{a} \left[\frac{(l+d)^2}{\{a^2 + (l+d)^2\}^{\frac{1}{2}}} - \frac{(l-d)^2}{\{a^2 + (l-d)^2\}^{\frac{1}{2}}} \right].$$

The average velocity of the fastest group of rays used, measured in this way, varied from 2.7×10^9 to 8.0×10^9 cm./sec.

Upon raising the pressure in the test chamber the scattering due to the gas became apparent in a reduction of intensity in the photographs at pressures of air as low as 3×10^{-4} cm. of mercury, but even at 1.2×10^{-3} cm., the highest pressure at which photographs were obtainable with an exposure of one hour or less in this apparatus, the geometrical pattern obtained was of the same size, and as clear cut, as at the lower pressures, although very faint, and overlaid by scattered radiation extending with decrease of intensity to more than a centimeter from the center of the photograph. The conclusion to be drawn is that at this pressure a considerable number of these electrons (velocity about 5×10^9 cm./sec.). have free paths in excess of 60 cm., or more than 100 times the mean free path of a molecule of the gas.

Recent experimental work on slow cathode rays by J. Franck and G. Hertz[1] indicates that for such rays the mean free path is $4\sqrt{2}$ times that of a gas molecule, that is, for the case discussed, 3.4 cm. Extrapolation from the values given by Townsend[2] for negative ions of sub-atomic dimensions gives 6 cm. as the mean path between ionizing collisions under the same set of conditions. The still higher value indicated by these experiments suggests that the high-speed cathode rays must, in many cases, pass completely through the atom without measurable change of direction.

Positive rays could be obtained without alteration of pressure by reversing the discharge current. In Fig. 1, exposures 2, 4, 6 and 8, taken in this way show that the positive rays are much scattered at a pressure that does not affect the cathode rays. The atomic size of the positive rays explains the greater chance of a collision and consequent deflection.

An attempt was made to deduce the law of scattering of cathode rays from the variation of intensity with distance from the center of the photograph, using approximately circular apertures to limit the beam of rays after sorting them magnetically with a solenoid. The hole in the anode was made several millimeters in diameter to permit a greatly increased number of rays to be dealt with, and since discharge tube and test chamber could not be kept at different pressures the average velocity of the rays was altered by Whiddington's method.[3] Conditions were held constant for longer periods of time by supplying gas through a very fine capillary tube and removing it continuously with a Gaede mercury pump.

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[1] Verh. d. D. Phys. Ges., 15, p. 34, 373, 929 (1913); 16, p. 12 (1914).

[2] Ionization by Collision, p. 30 (1910).

[3] Camb. Phil. Soc. Proc., 17, p. 251, 1913.

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August, 1914.

PLATE I.
To face page 142.

FIG. 1.

FIG 2.

FIG. 3.

L. W. MCKEEHAN.

In each experiment a series of exposures for widely differing times was obtained under conditions kept as nearly constant as possible, but in different experiments the nature of the gas, its pressure, and the velocity of the rays were varied independently. Fig. 3 shows the photographs obtained in oxygen at 4.56×10^{-4} cm. pressure with rays of velocity 6.5×10^{9} cm./sec. The time of exposure varies from five seconds to one hour.

If the photographic intensity were directly proportional to the number of rays received, a single exposure on each plate would suffice to give the relation between distance from the center and number of rays. This was not the case as experiment showed, for the ratio of intensities at two given distances from the center was not the same on the different exposures. However, if conditions are really constant during an experiment, and if the intensity of the photograph on a small area of the plate depends only on the number of rays of approximately the same velocity received there, although not proportional to it, the law of scattering for a given gas atom and for a given velocity of rays can be found by measuring the distances from the centers of the several exposures in the appropriate experiment to circular zones in which the photographic intensity is identical. Each pair of unequally timed exposures gives the relation between distance from the center and relative number of cathode rays, and all pairs on the same plate should give the same relation. As long as the greater part of the pencil of rays is not diffused, the scattering is single and not compound[1] since the probability of more than one deflection is small.

In the experiments actually performed in this way consistent results were not obtained. The chief reason for this failure lies in the wandering of the beam of rays within the discharge tube, due to surface charges and fluctuations of discharge potential. These alterations of direction were directly observed on fluorescent screens placed within the test chamber, and were so considerable as to vary the intensity of the photograph by as much as one half in consecutive exposures of several minutes duration.

Another difficulty inherent in the method is the small amount of scattered radiation that can be obtained within the limits of single scattering, and the consequent faintness of the photographs. This faintness enhances errors due to fogging by light or by X-rays, to original non-uniformity of the sensibility at different points on the plate, to non-uniform development, and to reversion of the latent image. The layer of the emulsion affected by homogeneous cathode rays is so thin

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[1] Sir Ernest Rutherford, Phil. Mag., VI., 21, p. 669, 1911.

that inversion began at low intensity, a second inversion took place at the center in all long exposures, and a third inversion was just detectable in one photograph. The use of sensitized paper in place of dry plates was found to overcome the last mentioned defect, but no better results were obtained.

One of the assumptions made above is open to attack on theoretical grounds. Scattered rays should have smaller velocities than unscattered, and the photographic effect should not therefore be quite proportional to number of rays, but should depend slightly on the angle of deflection. It can be seen that this effect should be small for small angles, but it is inseparable from the photographic method.

A consideration of the number of rays available, and of their ionizing power at the necessary low pressure made it seem inadvisable to attempt an electrical method of counting, especially since the chief defect of the experiments would not be overcome by such a change. The general problem of the deflection of an electron by matter is being attacked in this laboratory by a more direct and powerful method,[1] from which definite results are expected.

In conclusion thanks are due to Sir J. J. Thomson for the suggestion of the original problem, and for the facilities afforded during that part of the work performed in the Cavendish Laboratory.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
April 27, 1914.

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[1] A. F. Kovarik and L. W. McKeehan, PHYS. REV., II, 3, p. 149, 1914.

THE SPECIFIC HEAT OF SOLIDS AND THE PRINCIPLE OF SIMILITUDE.

By Richard Chace Tolman.

THE writer has already presented[1] a number of derivations in widely different fields of physical science, all of which are based on a single postulate which may be called the *principle of similitude.* In the present article we shall make use of this same principle in deriving a formula for the specific heat of homogeneous, isotropic, elastic substances.

We shall find that the formula thus obtained for specific heat agrees with that of Debye[2] for the specific heat of solids and liquids at low temperatures. In our proof, however, it will *not* be necessary to make use of any of the various forms of quantum theory. In particular, the derivation of Debye rests on the special assumption borrowed from the quantum theory of Planck that

$$E_\nu = \frac{h\nu}{\epsilon^{\frac{h\nu}{kT}} - 1} d\nu$$

is the energy in an elastic sphere at temperature T associated with modes of vibration whose frequencies lie between ν and $\nu + d\nu$. In our derivation, however, *we shall make no assumptions as to the form of the relation connecting energy with temperature and frequency of vibration.*

This possibility of deriving a satisfactory formula for the specific heat of solids and liquids without the special assumptions of the quantum theory is of particular interest at the present time, since in spite of brilliant justification from the experimental standpoint, the theoretical foundation of the quantum theory must still be regarded as a field where further research is necessary.

It is also interesting to point out that the derivation of the Debye formula which we shall present will necessitate no new investigations in the field of elasticity theory, but will merely make use of the well-known expressions for the velocity of wave motions in elastic solids.

Part I. The Principle of Similitude.

The principle of similitude may be stated as follows:—*The fundamental entities of which the physical universe is constructed are of such a nature*

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[1] Tolman, Phys. Rev., *3*, 244 (1914).

[2] Debye, Ann. d. Physik, *39*, 789 (1912).

that from them a miniature universe could be constructed exactly similar in every respect to the present universe. For a discussion of this new postulate we must refer the reader to the article already mentioned, and shall here confine ourselves merely to a recapitulation of those deductions from the principle which are necessary for our immediate purpose.

Let us consider two observers, O and O', provided with instruments for making physical measurements. O is provided with ordinary meter sticks, clocks and other measuring apparatus of the kind and size which we now possess, so that he can duplicate all our measurements in the present physical universe. O', on the other hand, is provided with a shorter meter stick, and correspondingly altered clocks and other apparatus so that he can make measurements in the miniature universe of which we have spoken and in accordance with our postulate obtain exactly the same numerical results in all his experiments as does O in the analogous measurements made in the real universe.

If now O and O', instead of each remaining in his own universe, should both make measurements of the same physical objects and occurrences, it is evident that they would not obtain the same results, since their units of measurement are different. Let us suppose that O and O' both measure the *same* length, then if O's meter stick is x times as long as that of O', it is evident that their measurements of the given length would be connected by the equation $l' = xl$.

In the article already referred to it was found possible to obtain a whole series of transformation equations of the type $l' = xl$, connecting the measurements made by O and O' of given lengths, time, intervals, masses, forces, charges, etc. These relations were derived by combining with the postulate of similitude, certain principles taken from the Einstein theory of the relativity of motion and from the electron theory. We shall now present for future reference a table of such transformation equations.

TABLE OF TRANSFORMATION EQUATIONS.

For length	$l' = xl$	(1)
For area	$S' = x^2S$	(2)
For volume	$V' = x^3V$	(3)
For an interval of time	$t' = xt$	(4)
For frequency	$\nu' = \nu/x$	(5)
For velocity	$v' = v$	(6)
For acceleration	$a' = a/x$	(7)
For quantity of electricity	$e' = e$	(8)
For mass	$m' = m/x$	(9)
For density	$\rho' = \rho/x^4$	(10)
For force	$f' = f/x^2$	(11)
For pressure	$P' = P/x^4$	(12)
For energy	$E' = E/x$	(13)

For energy density.................................. $u' = u/x^4$ (14)
For absolute temperature........................... $T' = T/x$ (15)
For the coefficient of compressibility................. $\chi' = x^4\chi$ (16)[1]
For Poisson's Ratio................................ $\sigma' = \sigma$ (17)[1]

PART II. APPLICATION OF THE PRINCIPLE OF SIMILITUDE TO DETERMINE THE FORM OF THE FUNCTION.

The general method of applying the principle of similitude to determine the form of functional relations connecting physical magnitudes is to consider some construct which could exist either in the actual universe or in the miniature universe which to observer O' appears the same as the actual universe. It is evident from the principle of similitude that the properties of a construct which has the same general character in both universes will have to obey the same general laws, whether measured by observer O or by observer O', while a further condition will be imposed upon the magnitude of these measurable properties by the transformation equations which we have just developed. These two sets of conditions will permit the attainment of definite information as to the necessary form of the functional relation connecting the measurements of different properties of the construct.

In the article already mentioned this method was used for deriving the law of Charles for an ideal gas, for showing that the specific heat of such an ideal gas would be independent of the temperature, for deriving Stefan's law for the energy density of a hohlraum, for obtaining a relation between energy density and frequency in a hohlraum, and for deriving various relations in the fields of electromagnetic and gravitational theory. We may now apply the method to obtain information as to the form of the functional relation connecting the energy density of an elastic substance with its temperature and elastic constants.

In the first place it is evident that a given homogeneous, isotropic, elastic substance is a construct which would appear to be homogeneous, isotropic and elastic both to observer O and to observer O', and hence by the principle of similitude experiments made on it by either observer would have to lead to the same general laws. It is also evident that the energy density u, of such a substance will be completely determined by

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[1] Of these transformation equations the only ones not already derived in the article referred to are those for the coefficient of compressibility and for Poisson's ratio. The coefficient of compressibility may be defined by the equation

$$\chi = \frac{1}{V}\frac{dV}{dP},$$

while Poisson's ratio, σ, is that between the longitudinal and transverse strain of an elastic substance. The derivation of the transformation equations for these two quantities is obviously merely a matter of substituting equations already given above.

its temperature T and the three constants—density ρ, compressibility χ, and Poisson's ratio σ—which completely determine its elastic properties.[1] Suppose now the measurements of O have led to the following relation, between the energy density and the elastic properties and temperature of the substance,

$$u = F(\rho, \chi, \sigma, T) \tag{18}$$

then by the principle of similitude the measurements of O' will have to lead to a similar relation,

$$u' = F(\rho', \chi', \sigma', T') \tag{19}$$

where F must have the same form as in the previous equation.

We have, however, already found transformation equations, Nos. (14), (15), (10), (16), and (17), connecting the measurements of energy density, temperature and elastic constants made by O' with those made by O. Substituting in equation (19) we have

$$\frac{u}{x^4} = F\left(\frac{\rho}{x^4}, x^4\chi, \sigma, \frac{T}{x}\right)$$

or solving for u and combining with (18) we obtain,

$$u = x^4 F\left(\frac{\rho}{x^4}, x^4\chi, \sigma, \frac{T}{x}\right) = F(\rho, \chi, \sigma, T) \tag{20}$$

Since the total energy of an elastic body will be proportional to its volume we may write,

$$E = Vx^4F\left(\frac{\rho}{x^4}, x^4\chi, \sigma, \frac{T}{x}\right) = VF(\rho, \chi, \sigma, T) \tag{21}$$

Since x may be any number we have thus obtained from the principle of similitude an equation which imposes very definite conditions on the form of the functional relation connecting the energy content of an elastic body with its temperature and elastic properties.

By considering some familiar results of the theory of elasticity we may complete the solution of our problem.

PART III. APPLICATION OF ELASTICITY THEORY.

In the previous section we have derived from the principle of similitude a functional equation, No. 21, which presents certain necessary relations between the energy content of an elastic body and its volume, elastic constants, and temperature. Let us now see what conclusions we can draw from the *theory of elasticity* as to the energy content of an elastic body.

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[1] We shall assume a substance in which the elastic properties are independent of the temperature.

Let us consider an elastic body of some definite shape and size, fixed at its outer surface. At a given temperature such a body will contain a certain amount of energy and we may think of this energy as due to the presence of systems of standing waves which undergo continual reflection at the fixed boundary. These waves may either transverse or longitudinal. In the former case the theory of elasticity has shown that their velocity will be $\sqrt{\mu/\rho}$ and in the latter case $\sqrt{(\lambda + 2\mu)/\rho}$, where ρ is the density of the material and μ and λ its elastic constants. These constants are connected with the measurable quantities compressibility, χ, and Poisson's ratio, σ, by the equations

$$\mu = \frac{1}{\chi}\frac{3(1 - 2\sigma)}{2(1 + \sigma)} \tag{22}$$

and

$$\lambda = \frac{1}{\chi}\frac{3\sigma}{1 + \sigma}. \tag{23}$$

If now we should change the dimensions of the body or its elastic properties, it is evident that we should also change the frequency of the standing waves which it contains at thermal equilibrium.

If we consider changes in the *volume* of the body without alteration of shape, it is evident that the linear dimensions of the body will always be proportional to the cube root of the volume, $V^{\frac{1}{3}}$. Hence the wave lengths of the different standing waves which the body contains will be proportional to the cube root of the volume or their frequencies to its reciprocal.

On the other hand, if we change the *elastic properties* of the body we should also change the frequencies of the different standing waves since we change the velocity of propagation. For transverse waves the frequencies will be proportional to the velocity of transverse waves, $\sqrt{\mu/\rho}$; and similarly for longitudinal waves the frequencies will be proportional to $\sqrt{(\lambda + 2\mu)/\rho}$.

Hence we may write for the frequencies of the different standing waves which a body of given shape will contain

$$\nu_1 = \frac{a_1}{V^{1/3}}\sqrt{\frac{\mu}{\rho}},\quad \nu_2 = \frac{a_2}{V^{1/3}}\sqrt{\frac{\mu}{\rho}},\quad \nu_3 = \frac{a_3}{V^{1/3}}\sqrt{\frac{\mu}{\rho}},\ \text{etc.,}$$

$$\nu_1' = \frac{a_1'}{V^{1/3}}\sqrt{\frac{\lambda + 2\mu}{\rho}},\quad \nu_2' = \frac{a_2'}{V^{1/3}}\sqrt{\frac{\lambda + 2\mu}{\rho}},\quad \nu_3' = \frac{a_3'}{V^{1/3}}\sqrt{\frac{\lambda + 2\mu}{\rho}}\ \text{etc.,} \tag{24}$$

where ν_1, ν_2, ν_3, etc., are the frequencies of the various transverse, and ν_1', ν_2', ν_3', etc., are the frequencies of the various longitudinal standing waves. a_1, a_2, a_3, etc., and a_1', a_2', a_3', etc., are constants which depend on the shape of the body but are independent of its volume or elastic properties.

Now let us assume that at thermal equilibrium the average energy associated with each loop of a standing wave is determined merely by its frequency and the temperature. Then in the case of our elastic body we may write for the energy associated with a standing wave of frequency ν,

$$E_\nu = nF'(\nu, T)$$

where n is the number of loops in the standing wave under consideration, and F' is the unknown functional relation connecting energy with frequency and temperature.

For the total energy of the body we may write

$$E = \Sigma E_\nu = \Sigma nF'(\nu, T)$$

where the summation Σ is to include all the different waves in the body.

We have already seen, however (equation 24), that the frequencies of the different waves are determined by equations either of the form

$$\nu = \frac{a}{V^{1/3}}\sqrt{\frac{\mu}{\rho}}$$

or

$$\nu = \frac{a'}{V^{1/3}}\sqrt{\frac{\lambda + 2\mu}{\rho}},$$

where a and a' are constants depending on the shape of the body. Hence we may write for the total energy of the body

$$E=\Sigma E_\nu = \Sigma nF'(\nu,T) = \Sigma nF'\left(\left[\frac{a}{V^{1/3}}\sqrt{\frac{\mu}{\rho}}\right], T\right)$$
$$+ \Sigma' n' F'\Big(\Big[\frac{a'}{V^{1/3}}\sqrt{\frac{\lambda+2\mu}{\rho}}, T\Big), \quad (25)$$

where the summation Σ includes all the transverse waves and Σ' includes all the longitudinal waves. In this equation n, n', a and a' are constants independent of the volume or elastic properties of the body.

Let us now transform this equation by substituting for μ and λ their values in terms of compressibility and Poisson's ratio as given in equations (22) and (23). We shall obtain

$$E=\Sigma E_\nu=\Sigma nF'(\nu, T)=\Sigma nF'\left(\left[\frac{a}{V^{1/3}}\left(\frac{1}{\rho\chi}\frac{3(1-2\sigma)}{2(1+\sigma)}\right)^{1/2}\right], T\right)$$
$$+ \Sigma' n' F'\left(\left[\frac{a}{V^{1/3}}\left(\frac{1}{\rho\chi}\frac{3(1-\sigma)}{1+\sigma}\right)^{1/2}\right], T\right). \quad (26)$$

We thus have obtained from the theory of elasticity a functional relation between the energy of an elastic body, and its volume, density, elastic constants χ and σ, and its temperature.

We have already obtained, however, from the principle of similitude another functional relation, equation 21, between these same quantities. This equation was

$$E = Vx^4F\left(\frac{\rho}{x^4}, x^4\chi, \sigma, \frac{T}{x}\right) = VF(\rho, \chi, \sigma, T).$$

By comparing these two expressions we see since the x's must cancel and since volume must enter to the first power, that $F'([\,], T)$, which occurs in equation (26) must be of the form,[1]

$$E_\nu = k\frac{T^4}{[\,]^3} = k\frac{T^4}{\nu^3}, \tag{27}$$

where k is some constant.

The total energy of the body will then be given by the equation

$$E = A V\rho^{3/2}\chi^{3/2}\left\{ B\left(\frac{2(1+\sigma)}{3(1-2\sigma)}\right)^{3/2} + \left(\frac{(1+\sigma)}{3(1-\sigma)}\right)^{3/2}\right\} T^4, \tag{28}$$

or differentiating with respect to T we obtain for the heat capacity of any elastic isotropic body,

$$C = 4A V\rho^{3/2}\chi^{3/2}\left\{ B\left(\frac{2(1+\sigma)}{3(1-2\sigma)}\right)^{3/2} + \left(\frac{(1+\sigma)}{3(1-\sigma)}\right)^{3/2}\right\} T^3. \tag{29}$$

A and B are numerical constants which are the same for all materials and their values may be determined once for all by measurements on two substances.

We have thus derived from the principle of similitude and the theory of elasticity not only the important fact that the *heat capacity of an elastic substance is proportional to the third power of the temperature* but also the complete form of the equation connecting heat capacity with temperature and elastic properties.

PART IV. COMPARISON WITH THE EQUATION OF DEBYE.

It can be shown that the above equation agrees with the form taken at low temperatures by Debye's equation for the specific heat of monatomic elastic substances. Debye's equation No. 13[2] reads

$$C/C_\infty = 77.938(T^3/\theta^3).$$

Substituting for θ the value given by Debye's equations (7), (3^1) and (2), and for C_∞ the value $3Nk$, we may rewrite his formula in the form

$$C = 326\frac{k^4}{h^3}V\rho^{3/2}\chi^{3/2}\left\{ 2\left(\frac{2(1+\sigma)}{3(1-2\sigma)}\right)^{3/2} + \left(\frac{(1+\sigma)}{3(1-\sigma)}\right)^{3/2}\right\} T^3,$$

which agrees in form completely with our equation No. 29.

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[1] Since ρ and χ occur to the same power, T has to enter to the fourth power in order that the x's cancel.

[2] Loc. cit.

The undetermined factor B which occurs in our equation is seen to have the numerical value 2 in Debye's formula. From our derivation it is evident that B is the ratio of the total energy associated with transverse waves to that associated with longitudinal waves. Our considerations however, were not sufficient for a theoretical determination of the magnitude of this ratio.

The factor $326(k^4/h^3)$ in Debye's formula corresponds with our undetermined numerical factor $4A$. k and h in this factor have the usual significance; k is the so-called gas constant for a single molecule, and h is Planck's Wirkungsquantum.

Debye's calculations have led to a definite numerical value of A since he has made the definite assumption that the energy associated with a mode of vibration of frequency ν is

$$\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}.$$

We, on the other hand, have merely assumed that the energy associated with each loop of a stationary wave is *some* function of frequency and temperature, and then determined, except for a constant numerical factor, what this function must be (see equation 27).

With regard to the temperature range over which we can expect our formula for specific heat to hold, we can merely say that at low temperatures where the amplitudes of wave motions are small we may expect *actual substances* to behave like homogeneous media, and hence to obey this formula.[1] For comparison with experimental data we may refer to the article of Debye[2] or to an article by Nernst and Lindemann.[3] We may call attention to the fact that from our derivation we shall expect the same formula to apply to all substances and not merely to monatomic substances as Debye's derivation suggested. As a matter of fact this appears to be true in the case of the measurements of Nernst and Lindemann[4] on potassium and sodium chlorides.

At moderately high temperatures, where there will be a freer motion of the individual atoms, we shall no longer expect that solid bodies will behave like homogeneous media, and hence shall expect increasing deviations from the formula which we have dervied. According to Debye,

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[1] The possibility that actual substances behave like continua at very low temperatures was first suggested to the writer by Professor G. N. Lewis. This idea also forms the basis of the considerations of Debye and of Born and Karman, Phys. Zeitschrift, 13, p. 297 (1912). In the case of anisotropic substances we might provisionally take an average value of σ.

[2] Loc. cit.

[3] Sitzungsber. Preuss. Akad., p. 1160 (1912).

[4] Loc. cit.

however, we may expect the specific heat to be proportional to the third power of the temperature within one per cent, up to temperatures as high as $T = \theta/12$ where θ varies for different substances from 72 to 467.[1]

At high temperatures we know that the specific heat of solid substances will obey the law of Dulong and Petit (or of Kopp in the case of polyatomic substances). Hence we might devise as Debye has done a formula which would give a smooth connection between the two known ends of the specific heat curve. For the present, however, we cannot see any satisfactory theoretical derivation for this intermediate part of the curve.

University of California,
Berkeley, Cal., U. S. A.
May 12, 1914.

[1] Loc. cit., pp. 802 and 817.

NEW BOOKS.

X Rays. By G. W. C. Kaye. New York: Longmans, Green & Co., 1914. Pp. xix + 252. Price $1.25 net.

This little volume is a welcome addition to the literature of this interesting subject. It covers much the same field as Pohl's book but the treatment is less theoretical. The first seven chapters deal with cathode ray phenomena and some features of high tension generators and x-ray tubes, this is followed by about twenty pages devoted to measurement. The treatment of secondary radiation and the brief discussion of interference phenomena in crystals will be very useful to those wishing a general idea of these recent developments but who lack the time required to read the original papers. Appendices contain a brief description of the Coolidge tube and the production of high vacua. The tables and diagrams are well selected and many references are included. All mathematical theory has been omitted but the resumé of experimental data is as complete as the size of the volume permits.

J. S. S.

Molekularkinetik und Molarassociation als Physikochemische Grundvorstellungen. By Dr. Carl Drucker. Leipzig: Akademische Verlagsgesellschaft, 1913. Pp. 1 + 33.

Dr. Drucker in this "Antrittsvorlesung" emphasizes the importance of the molecular association theory as a working hypothesis for the explanation of a large number of physico-chemical problems. He shows that it should be regarded as a fundamental hypothesis, parallel to and in some cases superior to that of molecular kinetics. This theory is characterized by the assumption of the existence of various molecular complexes, each having definite properties, and of the existence of an equilibrium between them, depending upon the imposed physical conditions, such as temperature and pressure.

The pamphlet gives merely an outline of the theory and a prophesy of its future importance without specific quantitative applications. However, the author has added a large number of references to investigations in which the theory has evidently been used with success.

K. E. G.

Recueil de Constantes Physiques. By Henri Abraham and Paul Sacerdote. Paris: Gauthier-Villars, 1913. Pp. xvi + 751.

This volume supplements in an admirable manner the standard collections of physical constants. It brings together a great variety of data otherwise not easily found. The tables of the ordinary constants such as density, boiling point, fusion, specific heat, etc., are augmented by the inclusion of much new material, and a great variety of useful information not found in previous compilations has been added. By the free use of diagrams many relations, which are not conveniently expressed in tabular form, are rendered available. This method is particularly useful in describing the properties of alloys, mixtures, solutions, etc. All the essential formulae and relations are given with an explicit statement of the notation employed. The volume is well indexed and lists of the authors from whom data have been taken accompany the various tables.

E. L. N.

Die Hydrierung durch Katalyse. By Paul Sabatier. Leipzig: Akademische Verlagsgesellschaft, 1913. Pp. 1 + 20.

This is an address given by Professor Sabatier when he received the Nobel prize in December, 1912. It is a nontechnical description of his general method of producing direct hydration by catalysis and his chemical theory of catalysis. It also contains the history of its conception and development, and the principal applications of the method.

K. E. G.

Dialogues Concerning Two New Sciences. By Galileo Galilei. Translated from the Italian and Latin into English by Henry Crew and Alfonso de Salvio. New York: The Macmillan Company, 1914. Pp. xiii + 300. Price, $2.00 net.

To anyone who is interested in the history of physics or who wishes to send his students to the sources of that science this book will be of great value. When the principal interest of a book is in the ideas which it contains, one can always tolerate a translation, particularly when, as in this instance, the translator has caught the style and spirit of the original. Everyone knows the principal contributions which Galileo made to the science of motion, but most of us do not know how many other notions, which are now the common property of mathematicians and phyaicists, of less fundamental importance, perhaps, than these, but still of great interest, were first presented or acutely discussed by that great genius. An examination of these dialogues on the strength of materials and on motion will reveal, on almost every page, something to attract attention.

The style of the translators is easy, fluent, and clear. The book is beautifully printed in an antique form and type. The diagrams are reproductions of the originals.

W. F. M.

Die Photographie. By Otto Prelinger. Leipzig: B. G. Teubner, 1914. Pp. iv + 110.

Die Luftfahrt. By Dr. Raimund Nimführ. Leipzig: B. G. Teubner, 1913. Pp. viii + 132.

Unsere Kohlen. By PAUL KUKUK. Leipzig: B. G. Teubner, 1913. Pp. ix + 120.

These booklets are three volumes of Teubner's series, entitled "Aus Natur und Geisteswelt," which is published for the distinct purpose of counteracting the modern tendency towards overspecialization. Each volume is written by an authority on the subject, but in accordance with the aim of the whole series the treatment is elementary and nontechnical. As means for the diffusion of knowledge the booklets certainly accomplish their purpose.

Dr. Prelinger's book presents in a popular form—in addition to a description of the various processes employed in photography and the physical and chemical principles underlying them—a short historical introduction and a condensed chapter on photography in natural colors.

One half of the volume, entitled "Die Luftfahrt" contains a general presentation of the physics of the atmosphere and the fundamental laws of aerostatics and aerodynamics. The second half gives an account of the technical development of balloons and aeroplanes.

"Unsere Kohlen" is written mainly from a geological and geographic point of view. It contains mere references to the physics and chemistry of coal and to the technical side of the subject. Only a few pages are devoted to the occurrence of coal in countries outside of Germany.

K. E. G.

Natural Sources of Energy. By A. H. GIBSON. New York: G. P. Putnam's Sons, 1913. Pp. vi + 131.

This is an interesting little book on the world's energy problem discussing the present and future available sources of energy. It concludes with the consolingly optimistic view that the gradual depletion of the supply of fossil fuel will be more than compensated for by bringing into use the perennial supply of solar energy in its various available forms.

C. A. S.

Electrical and Magnetic Calculations. By A. A. ATKINSON. New York: D. Van Nostrand Company, 1913. Pp. vii + 310. Price, $1.50.

The book, a fourth edition, contains a large number of problems in elementary magnetism and electricity and in the applications of these subjects to electrical engineering. Each chapter contains a short introduction explaining the terms and principles to be used, some illustrative problems with the solutions given in detail, and finally a large number of original problems. These are well selected and contain many valuable practical hints. A number of tables of magnetic and electrical constants are scattered through the book.

As a collection of useful problems the book is of value to teachers in technical high schools or small colleges where only the rudiments of electrical engineering are taught. As a text, however, the book is obsolete. Many of the units described are out of date and loose definitions and statements are frequently found. The "metre des Archives" and the corresponding kilogram still figure as standards of length and of mass though long ago replaced by the international standards. Similarly the electrical units given in the book are still those proposed by the Chicago conference twenty years ago. "*g*" is called the force of gravity and a statement like this: "One volt delivering one ampere of current per second represents a power of one watt" is meaningless. Even an elementary text should not lag so many years behind modern knowledge and practice. This is especially to be regretted when it occurs in an otherwise very useful book.

K. E. G.

Modern Seismology. By G. W. WALKER. New York: Longmans, Green and Company, 1913. Pp. xii + 88. Price, $1.40.

The author of this book is a practical seismologist. He gives in it an outline of the theory of seismographs including a study of the conditions of sensitiveness, and of the methods employing in registration; a description of the types of seismographs in present use and directions for their installation. In a second part, he considers the theory of a solid isotropic earth and the application of the seismographic records to the study of particular earthquakes as well as to the testing of theories of the earth's structure. The book contains a number of plates giving specimens of records which illustrate special phenomena and the peculiarities of particular instruments. W. F. M.

Investigacion Experimental Sobre Las Corrientes de Turbulencia en Tubos Capilares. By TRIFON UGARTE. Buenos Ayres, 1913. Pp. 1 + 89.

This extensive thesis contains a full discussion of the formulas which have been proposed to represent the efflux of liquids from capillary tubes as a function of the pressure, the time and the temperature, and of the observations upon which they are based. The author presents an elaborate series of experiments upon the same subject, and verifies the types of these formulas, finding in certain cases that the constants of the formulas are of unexpected simplicity and of general interest.

W. F. M.

Magnetooptische Untersuchungen mitbesonderer Berucksichtigung der magnetischen Zerlegung der Spektrallinien. By P. ZEEMAN. Pp. xi + 242. Price, Mk 9.

This is an exact translation of the original English edition of the author's work, Researches in Magneto-Optics, MacMillan and Co., London, 1913, by Max Iklé. C. A. S.

Die realistische Weltansicht und die Lehre vom Raume. By E. STUDY. Braunschweig: Vieweg and Sohn, 1914. Pp. ix + 145. Price, Mk 4.50.

This book, by a distinguished mathematician, will be found interesting by anyone who is philosophically inclined. The author assumes the realistic position, and criticizes acutely and with humor the opposing positions of idealism, positivism, and pragmatism. Proceeding to consider the problem of space, he accepts as possible hypotheses for the geometry of space, the Euclidean and the various forms of the non-Euclidean geometry. He accepts as a definition of a point that it is a system of n related numbers, and proceeds from this and similar definitions to describe these geometries by equations. The book ends with a discussion of the part played by the axioms in geometry.

W. F. M.

The Norwegian Aurora Polaris Expedition, 1902–1903. Vol. I. By Kr. Birkeland. New York: Longmans, Green and Co., 1913. Price, $10.00.

This great volume contains the results of more than ten years' hard and continuous study of magnetic storms and of the origin of terrestrial magnetism. The general thesis which is supported by the author is that magnetic storms are caused by cathode rays projected from the sun. The mathematical theory of the action of such rays, when they are deflected by the magnetic field of the earth, is worked out, and illustrated by the records of magnetic storms. The consequences of theory are also illustrated by a model earth, or terella, past which, when it is magnetized, cathode rays are sent; and the terella is also made a source of cathode rays, and other phenomena, analogous to the zodiacal light, Saturn's rings, and spiral nebulæ are observed. The observations with the terella will be of special interest to the physicist.

W. F. M.

Second Series. *September, 1914* *Vol. IV., No. 3.*

THE

PHYSICAL REVIEW.

THE TEMPERATURE COEFFICIENT OF RESISTANCE OF METALS AT CONSTANT VOLUME AND ITS BEARING ON THE THEORY OF METALLIC CONDUCTION.[1]

BY CHARLES A. KRAUS.

IT is a well established fact that, excepting at very low temperatures, the resistance of pure metals increases very nearly as a linear function of the temperature.[2] If we grant the postulate that the electric current consists in a transfer of electricity in the form of discrete charges, it obviously follows that a change in resistance must be due either to a change in the number of carriers or to a change in their speed. In the theoretical treatment of the problem of metallic conduction it is usually assumed or implied that the number of carriers in a metal is independent of the temperature. This assumption greatly simplifies the treatment of the problem, but, as will be shown below, such an assumption is not always justified.

If the current in metals is carried by the negative electrons, then some manner of equilibrium must exist between the negative electrons, the positively charged residuals, and the neutral molecules. To what extent the metal atoms are dissociated is still an open question. Unless the dissociation is very high, the influence of temperature on the dissociation process must make itself felt; but whether the dissociation increases or decreases with increasing temperature is a question that can not be answered *a priori* either in the affirmative or negative. On the one hand, as in the case of many homogeneous equilibria with which we are familiar, the dissociation may increase with increasing temperature,

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[1] Contributions from the Research Laboratory of Physical Chemistry of the Massachusetts Institute of Technology, No. 99.

[2] There are numerous cases where the resistance-temperature curve passes through a minimum instead of approaching a zero value as the temperature approaches the absolute zero. This abnormal course of the curve is doubtless due to the presence of impurities. The nature of the phenomenon underlying the action of impurities on the resistance is as yet unknown.

while, on the other hand, metallic systems may behave analogous to electrolytic solutions, in which case the dissociation as a rule decreases with increasing temperature. It is to be borne in mind that the metals constitute highly condensed systems and that therefore a very small change in physical condition may have a large influence on the equilibrium in question. Thus, for example, temperature change is accompanied by change in density and it will be shown below that the change in the resistance of metals with temperature is in a large measure a consequence of the accompanying density change.

If the conduction of the current is due to electrons having a considerable freedom of motion in the body of the metal, then, other factors remaining equal, the resistance should increase with increasing temperature, since the frequency of collision increases with temperature according to the kinetic theory. The general form of the resistance-temperature curve of metals is therefore in harmony with the usual theory of metallic conduction. The fact should not be overlooked, however, that the observed resistance change is a resultant effect of all factors concerned among which are included effects due to volume change.

It is evident that with decreasing volume, the number of collisions per unit of time will increase and that the resistance will therefore increase. On the other hand, in most cases, it is found that the resistance actually decreases. The number of carriers under these conditions must therefore increase by an amount more than enough to compensate the decrease in resistance due to the increase in the number of collisions.

In determining the influence of volume change on the number of carriers present in a metal the obvious course is to determine the resistance of the metal at different temperatures keeping the volume constant. Without going into a detailed discussion, it is evident that with increasing density of a metal the number of carriers may be expected to increase. For example, in the case of mercury we know that the vapor is a non-conductor. If we were able to pass mercury through the critical state, we would doubtless find a continuous gradation of conducting power all the way from that of metallic liquid mercury to non-conducting mercury vapor. Similarly, when a metal is dissolved in a non-metallic solvent, we have a continuous gradation of conductance from that of a highly conducting metal to that of a poorly conducting solution.[1] So also, metals which melt with decrease of density increase in resistance on melting, while metals which melt with increase of density decrease in resistance on melting.

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[1] In this case, however, the condition approached as the metal becomes highly attenuated is materially modified by the presence of the solvent medium in which the metal is imbedded. Kraus, Trans. Am. Electroch. Soc. *21*, 119 (1912).

Data are available for calculating the resistance-temperature coefficient at constant volume for a number of metals. With the exception of mercury, all are solids. Taking into account the crystalline structure of solid metals, the influence of pressure on their resistance can not be interpreted with certainty. In the case of mercury, however, we have a metallic system of very simple molecular structure the results with which may be interpreted unambiguously if our kinetic conceptions regarding metals have any foundation in fact.

The temperature coefficient of resistance of a metal at constant volume is given by the equation:

$$\frac{\partial R/R}{\partial T} = \frac{dR/R}{dT} - \frac{\partial R/R}{\partial p} \cdot \frac{\partial v/v}{\partial T} \bigg/ \frac{\partial v/v}{\partial p}.$$

The values of the last four coefficients have previously been measured and found to be as follows for liquid mercury at 20° and at one atmosphere pressure:

$$\frac{dR/R}{dT} = 8.9 \times 10^{-4}; \quad \frac{\partial R/R}{\partial p} = -3.3 \times 10^{-5}; \quad \frac{\partial v/v}{\partial T} = 1.82 \times 10^{-4};$$

$$\frac{\partial v/v}{\partial p} = -3.8 \times 10^{-6}.$$

From these we find by calculation

$$\frac{\partial R/R}{\partial T} = -6.9 \times 10^{-4}.$$

This result shows that when liquid mercury at 20° is heated at constant volume its resistance *decreases* by 0.069 per cent. per degree, whereas when heated at constant pressure its resistance increases 0.089 per cent. per degree.

The result obtained above for the resistance-temperature coefficient of liquid mercury at constant volume is very significant and admits of only one interpretation, namely: *the increase in resistance of liquid mercury with increasing temperature is due to its decrease in density.* The total resistance change due to density change is 0.156 per cent. per degree.

Since the influence of increasing temperature, according to kinetic conceptions, is to increase the resistance of a metal, owing to reduction of the mean free path, and since at constant volume the resistance of liquid mercury decreases with increasing temperature, it follows that *at constant volume the number of carriers in liquid mercury increases with the temperature.*

If carriers are present in metals at all, they must be in some manner of equilibrium with each other and with the neutral molecules from which they are derived. This equilibrium is controlled by two main factors, namely: density and temperature, and these two factors influence the equilibrium in opposite directions. The dependence of the equilibrium on temperature and density is in entire conformity with our knowledge of equilibria in general and in particular with electrolytic equilibria. For example, the enormous decrease in the dissociation of electrolyte with increasing temperatures is in very considerable measure due to the decrease in the density of the solvent with increasing temperature. This is well illustrated by solutions of potassium iodide in methyl alcohol beyond the critical point where, for constant concentration of the electrolytes, the conductance increases enormously with increasing density of the solvent.[1]

The magnitude of the effect of density on the conductance of mercury has a very important bearing on the question as to the number of carriers present in liquid mercury. In many cases there has been a tendency to assume that the number of electrons present in a metal is of the order of the number of atoms present. From the foregoing data, we see that on heating liquid mercury at constant volume the number of carriers increases at least 0.07 per cent. per degree.[2] Such an increase in the number of carriers could not result if the metallic atoms were even approximately completely ionized. We must therefore conclude that *only a small fraction of the atoms of liquid mercury are ionized and furnish carriers for the current.*

The foregoing considerations would appear to demonstrate the necessity for considering the equilibrium between the changed carriers in a metal and the neutral atoms in any theory of metallic conduction which is founded on kinetic considerations. To do this, however, presents great difficulties. It need only be recalled that the corresponding problem is as yet only very incompletely solved in the case of ordinary electrolytes, where the conditions apparently are much simpler than in metals. The whole question of equilibria in systems of charged particles requires fundamental study and there is little hope that the problem of metallic conduction will be solved from a theoretical standpoint until such studies are forthcoming. The lack of tangible results in the problem of metallic conduction and of the metallic state in general is ample proof of the inadequacy of present day conceptions respecting this subject.

BOSTON, March, 1914.

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[1] Kraus, this Journal, *18*, 101 (1904.)

[2] If we assume that the decrease in translatory speed is proportional to the square root of the absolute temperature, the resistance decrease per degree due to this cause is 0.18 per cent. per degree. The increase in the number of carriers on this basis would therefore be 0.25 per cent. per degree.

SOME EFFECTS OF DIFFRACTION ON BRIGHTNESS MEASUREMENTS MADE WITH THE HOLBORN-KURLBAUM OPTICAL PYROMETER.

By A. G. Worthing and W. E. Forsythe.

Introduction.

THE study of the properties of many substances at high temperatures is often conveniently carried out with the substance mounted as a filament in a lamp bulb. Ordinarily such filaments will be small. In such cases particularly, in the opinion of the writers, the only method of optical pyrometry both convenient and theoretically safe which may be employed as an aid, is that one on which the Holborn-Kurlbaum pyrometer is based. Its underlying principles as described by the originators[1] are simple. Essentially as is indicated in Fig. 1, an image of a background *A* is formed by the lens *B* in the plane of a filament *D* which may be heated to incandescence. This filament will be referred to as the pyrometer filament. An eyepiece *F* backed with an approximately monochromatic glass filter *G* is used to view the pyrometer filament thus seen projected against the background. By varying the current through the pyrometer filament, it may under suitable conditions be made to match the background image in brightness, that is to disappear against the background. Evidently various backgrounds may be compared.

In attempting to determine the temperature of a certain tungsten lamp filament, first using an ordinary commercial Holborn-Kurlbaum pyrometer and then a modified one such as will be described in this paper both of which had been calibrated against the same black body, a difference of about 40° in temperature was obtained with the filament at about 2,300° Kelvin. This difference was too much to be ascribed to errors in measurement. An investigation of the causes has led to interesting results.

Arrangement of Apparatus.

The apparatus as arranged is indicated in Fig. 1. Large filament tungsten and carbon lamps served at various times for the background *A*. *B*, the objective lens, was a Zeiss-Tessar lens with a 28 mm. aperture and

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[1] E. Holborn and F. Kurlbaum, Ann. d. Phys., Vol. 10, p. 22, 1903; Ber. d. K. Akad. d. Wiss., Vol. 30, p. 712, 1901.

a 21 cm. focal length. The entrance cone aperture in the diaphragm *C* was variable in size and circular in shape. Its actual location was between the component parts of the lens *B*. As pyrometer filaments at *D* various carbon, tungsten, and platinum filaments of different sizes were used. An ordinary piece of sheet metal with a circular aperature served as the diaphragm *E*. Various telescopes were used for the eyepiece *F*. As monochromatic glass filters, pieces of ruby and of blue uviol glass having maximums of transmissions respectively at about $\lambda = 0.66\mu$ and at $\lambda = 0.46\mu$, were used. The various parts of the apparatus were independently adjustable and during the course of the work many shifts were made. Throughout great care was taken to see that the radiation from the background entirely filled the aperture at *E*, in other words to see that the angle α always exceeded the angle β, and to see that the apparatus was alined axially.

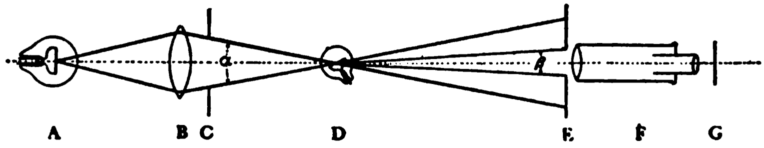

Fig. 1.

Diagram showing arrangement of apparatus. *A*, background; *B*, objective lens; *C*, entrance cone diaphragm; *D*, pyrometer filament; *E*, eyepiece diaphragm; *F*, eyepiece; *G*, monochromatic filter.

EXPERIMENTAL RESULTS.

A large part of the work consisted in determining the apparent variations in the ratio of the brightness of the pyrometer filament to the brightness of the background image formed in its plane as functions of α. In this connection there were determined the various currents through the pyrometer filament at apparent brightness matches, corresponding to the various values of α. By means of similar measurements made with various sectored discs between *C* and *D* and with the aperture at *C* constant, it was possible to calibrate the brightness of the pyrometer filament in terms of the current passing through it, and thus to reduce the previously determined filament currents to filament brightnesses.

The results thus obtained for a particular arrangement of apparatus (AB = 25 cms.; BD = 128 cms.; DE = 185 cms.; diameter of aperture at E = 9 mm.) for a succession of values of α, when various pyrometer filaments were successively substituted at *D*, are presented in Fig. 2. The method of obtaining the actual value of the ratio platted as ordinates will be discussed later. It is sufficient to consider at present the relative variations occurring in the curves for each individual pyrometer filament. In each instance decreasing the entrance cone angle α apparently decreased the brightness of the pyrometer filament or increased the brightness of the background. Evidently no actual changes in the brightness of the pyrometer filament or of the background could be thus produced. There is also a change in this variation depending upon the size of the pyrometer filament but no change to any appreciable extent depending upon the material of the filament. Further, curves *B* and *C* indicate

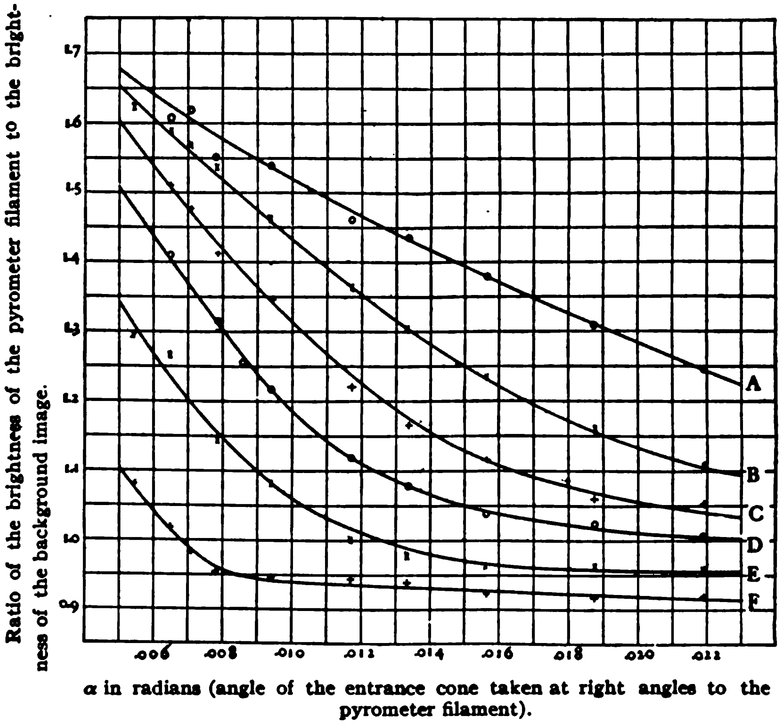

Fig. 2.

Variations in the apparent brightness of the background image with α for various sized pyrometer filaments.

Curve.	Diameter of Pyrometer Filament.	Material of Pyrometer Filament.	Average Wave-length of Light Used.
A	0.016 mm.	tungsten	0.66
B	0.025 mm.	"	0.66
C	0.025 mm.	"	0.46
D	0.045 mm.	carbon	0.66
E	0.050 mm.	tungsten	0.66
F	0.100 mm.	"	0.66

that the variations are different for the same pyrometer filament when different colored lights are used.

SUGGESTED EXPLANATIONS.

Possible explanations which have occurred to the writers which might wholly or in part explain the general phenomena are:

1st. The lens and mirror actions of the pyrometer lamp bulb.
2d. The heating of the pyrometer filament by radiation from the background.
3d. The reflection into the eyepiece of light from the background at the edges of the pyrometer filament.
4th. A possible transmission by the pyrometer filament of light from the background incident on it.
5th. A possible change in the emissive properties of the pyrometer filament due to incident radiation from the background.
6th. The diffraction of light from the background at the edges of the pyrometer filament.

The using of a platinum wire in air free from an enclosing glass bulb as a pyrometer filament, the interposing of a water cell between *A* and *D*, the geometrical consideration of the areas of the pyrometer filament which could serve as surfaces reflecting incident radiation into the eyepiece, the fact that the effect noted was found to be independent of the material and the temperature of the pyrometer filament, and the fact that the effect noted was found with the aid of variable rectangular apertures at *C* to depend upon the angle of the entrance cone of radiation taken in a plane normal to the pyrometer filament and not at all upon the angle taken in a plane parallel to it, respectively showed that the first five suggested explanations could not account for the variation noted. None of these modifications have yielded results inconsistent with diffraction as an explanation, in fact the results obtained with the rectangular apertures at *C* form positive evidence in its favor.

Experiments by Gouy[1] on the diffraction of unpolarized light by a sharp edge, indicated that the light which was bent by diffraction into the geometrical shadow, was partially polarized in the plane parallel to the edge. With this in mind there were made some tests exactly similar to those already described in which polarized light was used. A nicol introduced just in front of the apertures at *E* was first set so that the plane of polarization was parallel to the pyrometer filament and then later at right-angles to it. A large carbon filament whose position was so oriented that the pyrometer filament was always seen projected along

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[1] C. R., Vol. 98, p. 1573, 1884.

the center of and parallel to its image, was used as the background. This insured that the brightness of the background would be the same in the two cases. For a given relatively large aperture at *C*, the larger current was required through the pyrometer filament for an apparent brightness match in the case where the plane of polarization was perpendicular to the pyrometer filament. In this same case when the aperture at *C* was decreased to some relatively small value, the apparent relative change in brightness was less than with light polarized parallel to the filament. This is consistent with the results of Gouy noted above, on the assumption that the diffracted light to which the apparent changes in brightness are ascribed, was for the most part made up of light which had been bent by diffraction into the geometrical shadow. This assumption as will be noted later, was experimentally verified. The differences found in the apparent relative brightnesses of the pyrometer filament of tungsten for the two conditions of polarization noted, indicated an apparent polarization of 20 per cent. in the light from it. Taking into consideration the polarization of the light from the background which is diffracted, this is consistent with the value of 12 per cent. which may be obtained from data which have already been published by one of the writers.[1]

If from the central part of the aperture at *C*, one blocks out a region such that from a consideration of geometrical optics only, none of the light coming from the background in the immediate neighborhood of the place where the pyrometer filament is seen projected can enter the aperture at *E*, and if no current is passed through the pyrometer filament, one can still see the filament apparently glowing where it crosses the background image when looking at it through the eyepiece. In case the resolving power of the eyepiece is sufficiently great, this apparent brightness of the pyrometer filament is seen to consist of two bright streaks along the edges. Whether or not the axes of these bright streaks actually lie within or without the boundaries of the pyrometer filament is very difficult of determination. The fact that with eyepieces of small resolving power they seem to cover the entire filament and more is due largely to irradiation. A photograph of such a case is shown in Fig. 3, *a*. Fig. 3, *b* indicates a similar condition in which case, however, the pyrometer filament was rendered visible by means of a small current. By blocking out first the upper and then the lower portions of the remaining part of the aperture at *C*, it was easily demonstrable that this light consisted largely of light which was bent by diffraction into the geometrical shadow, and but to a comparatively small extent to light which was diffracted in the other direction.

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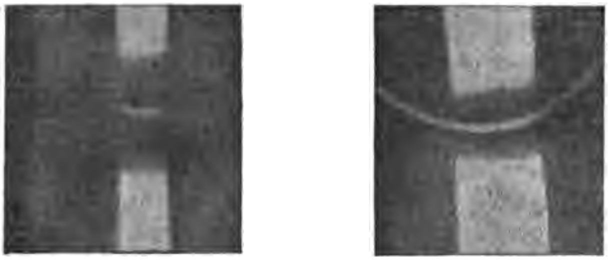
[1] A. G. Worthing, Astrophys. Jour., Vol. 34, p. 345, 1912.

The apparent brightness which these lines of diffracted light gave to the pyrometer filament were found to account in magnitude for the variations noted. By means of an extra lens and an extra pyrometer filament located between *D* and *E* (Fig. 1), there were measured first the apparent brightness of the unheated pyrometer filament at *D* due to this diffracted light when the central part of the aperture at *C* was blocked out as just noted, and then the apparent brightness of the heated pyrometer filament when the aperture at *C* was equal to that portion which previously had been blocked out. The results for three such cases are given in Table I. The agreement in the brightness sums was better than one could reasonably expect.



a b

Fig. 3.

Photographs showing apparent brightness of pyrometer filament due to light from the background diffracted by it. *a*, **pyrometer filament cold;** *b*, **pyrometer filament made luminous by a small current.**

Table I.

Effects of the Different Elements of the Entrance Cone of Radiation on the Apparent Brightness of a 0.050 mm. Tungsten Pyrometer Filament.

Outer Edge of Lens Screened Off.		Central Part of Lens Screened Off.		Brightness Sums.
Angles Subtended by Central Opening.	Relative Brightness of Pyrometer Filament.	Angle Subtended by Central Screen.	Relative Brightness of Pyrometer Filament.	
0.012 radians	0.755	0.012 radians	0.240	0.995
0.028	0.888	0.028	0.117	1.005
0.038	0.936	0.038	0.064	1.000

The results with rectangular apertures at *C* led to the trying of similar apertures at *E*. In this case with the aperture at *C* circular, some small variations were found only when the aperture was varied in a direction parallel to the pyrometer filament. These were accounted for in magnitude and direction by computations based on the fact that as the aperture at *E* was thus enlarged the average value of the entrance cone angle α was diminished.

The results obtained with the rectangular apertures at *C*, the obtaining of results with the aid of polarized light which are consistent with previously obtained results, the resolution of the apparent added brightness of the pyrometer filament in certain cases into two narrow lines of light along its edge, and the accounting in magnitude for the variations in brightness of the pyrometer filament by means of the apparent added brightness due to these narrow lines of light seem to indicate conclusively that the phenomena are wholly due to diffraction.

RELATION BETWEEN THE BRIGHTNESS OF THE PYROMETER FILAMENT AND THAT OF THE BACKGROUND IMAGE.

The condition usually assumed as holding in optical pyrometry is that the ratio of the brightness of the pyrometer filament to that of the background image formed in its plane *is* unity when there is an apparent brightness match. The present work shows this in general to be untrue˙ Actual determinations of this ratio which for brevity's sake we shall denote by k have been made. The method of doing this is theoretically simple. Three lamps whose filaments are represented here by 1, 2 and 3 were used, filament 1 being the pyrometer filament for which for a given entrance cone angle α, it was desirable to find k. With filaments 1 and 2 as pyrometer filament and background respectively, an apparent brightness match was obtained and their currents measured. Then, using these filaments successively as backgrounds, carrying the currents just determined, their brightnesses were compared with the aid of a pyrometer filament (filament 3) whose brightness-current relation had previously been determined as described above. Due consideration was of course necessarily given to the transmissions of the lens and of the individual lamp bulbs. Considerable difficulty is experienced experimentally in case filament l is small. This is due to the fact that it is very difficult to make pyrometer settings when the image of the background is small, as was the case with lamp l in most of our attempts to determine this ratio k. For the lamps used in obtaining the data presented in Fig. 2, we have determined with considerable care the corresponding values of k for an angle α equal to 0.0182 radians. These values of k have been used in locating the curves definitely in Fig. 2. Attention is once more called to the fact that the curves as platted actually represent the ratio of the brightness of the pyrometer filament to the brightness of the background image formed in its plane. In some instances as indicated the pyrometer filament is not as bright as the background image but in general the pyrometer filament is seen to be considerably brighter. For the lamps *A* and *B* that were used the transmission of single thicknesses of the lamp bulbs was approximately 91 per cent. The transmission of the lens used was 77 per cent. It is readily seen that when the lamps *A* and *B* were used as pyrometer filaments for small values of the entrance cone angle α, in the neighborhood of 0.006 radians, the pyrometer filament was brighter than the filament used in the background itself, although the background was viewed through several more thicknesses of glass than was the pyrometer filament.

Some determinations of k made under conditions when angle α was of the order of 0.1 of a radian, indicated that, if the curves in Fig. 2 could be prolonged, they would finally become asymptotic to a value for the relative brightness in the neighborhood of 91 per cent. or 92 per cent. These measurements were attended with considerable difficulty, however, and should not be relied upon too fully. This k which represents the ratio of the brightness of the pyrometer filament to the brightness of the background image when they appear of equal brightness, as may be seen from the curves of Fig. 2, is dependent on the angle α, the diameter of the pyrometer filament and the character of the light used but apparently not to any appreciable extent upon the character of the material of the pyrometer filament.

It was of interest to find out in this connection whether the size of the background had an effect or not. It was thought sufficient to equate by the color match method as described by Hyde[1] the brightnesses of two very different-sized filaments of the same material which were to be used as backgrounds and then to compare their brightnesses with a given pyrometer filament. As two such backgrounds a 0.25 mm. and a 0.016 mm. tungsten filament were used. The two methods gave identical results. A trial with two other filaments not differing as widely in diameter gave like results. From this it may be concluded that k is independent of the size of the background, as one might naturally expect.

APPEARANCE OF A PYROMETER FILAMENT AS SEEN PROJECTED AGAINST THE BACKGROUND.

There are to be distinguished here three cases; 1st. When the entrance cone angle α is relatively large. 2d. When it is relatively small. 3d. When it is very small. In Fig. 4 we have given for such cases schematic diagrams illustrating certain apparent brightness variations in the neighborhood of the intersection of a 0.025 mm. tungsten pyrometer

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[1] Astrophys. Jour., 36, p. 89, 1912.

filament with the background image when the condition is fulfilled that the pyrometer filament shall disappear against the background image, in case the resolving power of the eyepiece is sufficiently small to permit of it. For simplicity the variation from Lambert's cosine law of the radiation from the pyrometer filament is here disregarded. The diagrams themselves, however, indicate what may be seen when, with the pyrometer lamp adjusted for such disappearance, we use an eyepiece arrangement with a considerably higher resolving power so that greater detail may be seen. While the diagrams indicate the bright and dark bands which may be noticed as located just outside the confines of the pyrometer filament, the exactness of this method of representation should not be too strongly insisted upon. When the entrance cone is relatively large (case *A*) the pyrometer filament is less bright than the background image and, in case the resolving power of the eyepiece is sufficient, bright lines may be seen along the edges of the pyrometer filament. Of course these bright lines are quite disconcerting when one is attempting to make pyrometer settings.

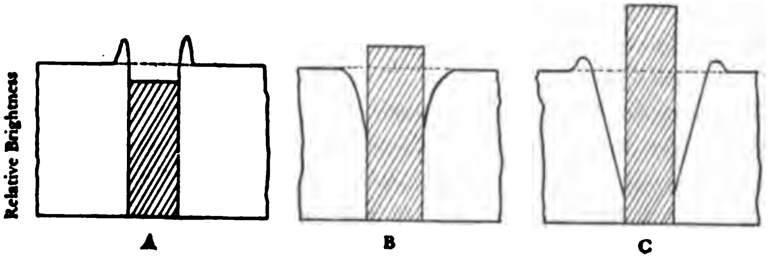

Fig. 4.

Distances along the background image. Schematic diagrams for illustrating certain apparent brightness variations in the neighborhood of the intersection of a 0.025 mm. tungsten pyrometer filament with the background image. Shaded portions represent the pyrometer filament; unshaded portions represent the image of the background. Case A, the entrance cone (angle α) relatively large—pyrometer filament less bright than background image; Case B, the entrance cone (angle α) relatively small—pyrometer filament brighter than background image; Case C, the entrance cone (angle α) relatively very small—pyrometer filament much brighter than background image.

In case a somewhat smaller cone angle α is used, these bright lines disappear so that even with an eyepiece of fairly high resolving power one does not have any trouble in obtaining a perfect disappearance of the pyrometer filament against the background image. The exact explanation of the dark bands noted in cases *B* and *C* and of the extra bright bands in case *C* are not at once apparent though we think they should also be ascribed to diffraction at the pyrometer filament. What has been said thus far as to the appearance of the pyrometer filament, refers strictly to a tungsten pyrometer filament about 0.025 mm. in diameter. The general appearance when using larger pyrometer filaments are exactly similar so far as the eye can discern. However, in view of the results obtained on the ratio of the brightness of the pyrometer filament to the brightness of the background image (Fig. 2), the general description as to relative brightnesses for the three cases must here be modified, for the dark bands along the edges may be seen when the pyrometer filament is not as bright as the background image. The explanation which occurs to the writers is that the condition in such a case is somewhat similar to what has been represented above for case *C*, though the two outer bright bands are not observable. Such a condition may readily be correlated with the relative brightnesses observed. Of course, it is probable that with the 0.025 mm. pyrometer filament in case *B*, the extra bright bands of case *C* are also present though not observable.

This type of diffraction, in which the diffraction fringes are located in the plane of the diffracting edge, so far as the writers know has not been studied heretofore, seems to be an important type and to be well worth further study.

The fact that disappearance can be obtained under such conditions as have been described when an eyepiece with low resolving power is used, is due to the fact that the eye averages the brightness variations over a considerable angular range. Many times in our work we have been able to go from impossible working conditions to fairly satisfactory conditions by substituting for the eyepiece in use, one of lower resolving power.

SOME NECESSARY WORKING PRECAUTIONS.

In view of the results which have been obtained and the difficulties which have been experienced, certain conclusions are definitely reached regarding necessary working precautions. In case one wishes to compare various light sources, excepting possibly where the entrance cone angle α is comparatively large, that is of the order at least of 0.1 of a radian:

1. The sources to be studied should be used as backgrounds and not as pyrometer filaments.

2. A single pyrometer filament should be used throughout the intercomparison.

3. The angles α and β should be definitely fixed. This can be best done by having limiting diaphragms at *C* and *E* which are at fixed distances from the pyrometer filament preferably as far away as possible. A commercial pyrometer of the Holborn-Kurlbaum type which we have in our laboratory possesses fixed diaphragms but they are in no sense limiting diaphragms.

4. The apparatus should be so adjusted that there is approximately axial symmetry.

5. The resolving power of the eyepiece should not be so great as to prevent the practical disappearance of the pyrometer filament against the background image.

6. The image of the background should be large in comparison with the pyrometer filament.

7. The magnifying power of the eyepiece should be sufficiently large so that no difficulty is experienced by the observer in fixing on the intersection and so that effects due to the eye's imperfections are largely eliminated.

It is interesting to note in this connection that the 40° difference in temperature, which as has been noted in the introduction was the starting point of the present paper, was apparently largely due to a failure to appreciate the importance of working precautions Nos. (6) and (7).

Some Desirable Working Conditions.

1. It is obvious that the eyepiece should have as high a resolving power as possible but still with the condition that when the pyrometer lamp is balanced against the background there shall be disappearance. The ideal case is obtained when no matter how great the resolving power of the eyepiece neither bright nor dark bands may be seen at the edge of the pyrometer filament. This condition of disappearance depends upon the size of the pyrometer filament, upon the value of the angle α, and in case the deviations from Lambert's cosine law are not neglected upon the material of the pyrometer filament. Using pyrometer filaments of tungsten, the writers have roughly attempted to find these conditions. For a 0.050 mm. tungsten pyrometer filament using red light, a very satisfactory value for the entrance cone angle α was found to be 0.05 radian. Smaller pyrometer filaments require somewhat larger entrance cone angles, while larger pyrometer filaments require smaller entrance cone angles. This is not entirely in conformity with what would be deduced from Fig. 2, assuming that the most satisfactory value for α is that which corresponds to the condition that the pyrometer filament is very nearly of the same brightness as the background image when there is an apparent brightness match. This has been discussed above under the heading relating to the appearance of the pyrometer filament. It should not be forgotten, however, that for a considerable range on either side of such values, the disappearances will be very satisfactory, and that even when the disappearances are not satisfactory with a certain eyepiece very satisfactory disappearances may be obtained with eyepieces

of smaller resolving power. Sometimes, particularly when it is necessary to use a highly magnified background image, a small pyrometer filament is better than a large one, because for a given large magnification due to the eyepiece the bright or the dark bands will be less apparent for the smaller filament. This is due to the fact that for a given eyepiece resolving power the range of good disappearances for the smaller filament considerably overlaps that of the larger.

2. An eyepiece and eyepiece diaphragm combination which gives comparatively large clear images is of course desirable. A short focus objective lens for the eyepiece and as large an aperture as possible in the diaphragm are necessary to reduce to a minimum the diffraction of the light which occurs at the eyepiece diaphragm. Using light of short wave-length is advantageous here.

3. The pyrometer lamp is important. In general the writers have found filaments in either hairpin loops or single straight filaments mounted in spherical bulbs to be most satisfactory. In general in any particular problem the pyrometer filament should be considerably smaller than the filament of the background. In some cases carbon may be preferable to tungsten for pyrometer filaments. In other cases the reverse is true. Due to the deviations of the emissions from Lambert's cosine law as described by one of the writers[1] one might expect in general to be able to use carbon filaments advantageously when the entrance cone angle α is large and tungsten filaments when this angle is small. So far as the writers can see there is but little advantage of the one over the other based on the supposition that the relative change in brightness for a given relative change in current is greater for one type of filament than for the other. Whatever advantage there may be of this nature is apparently in favor of the untreated carbon filament and is of the order of 5 per cent. In case small angles of α are necessitated, there is no question but that it is much more desirable to have pyrometer filaments of tungsten than of carbon.

4. For the sake of ease of adjustment particularly in obtaining axial symmetry, it is very desirable to have the diaphragm C one with a variable aperture.

5. In cases where high accuracy is demanded, a gain may be made in case the brightnesses permit, by using light of the shorter wave-lengths. Not only are some diffraction effects eliminated but also an actual gain is experienced due to the fact that, while for both red and blue light the least relative change in brightness which is just detectable is about the same, the relative change in brightness for a given small change in temperature is the greater for the shorter wave-length light.

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[1] A. G. Worthing, Astrophys. Jour., 36, 345, 1912.

EFFECTS OF PRESENT RESULTS ON PREVIOUS WORK.

It seems natural in this connection to consider what effect the present work may have on the results of other investigations where the Holborn-Kurlbaum optical pyrometer principle has been used. An inspection of the literature shows that writers in general have not specified carefully the conditions under which their pyrometers have been used. Because of this we restrict our comments to papers with which we have been personally connected and to papers which have been issued from this laboratory.

1st. In a paper on "A Study of the Energy Losses in Electric Incandescent Lamps" by Hyde, Cady and Worthing,[1] the lamps studied were used as pyrometer lamps. Since the results on a single lamp used as a pyrometer lamp were complete in themselves, the method was free from error, and the results are undoubtedly of the right magnitude.

2d. In a paper entitled "Measurements of Intrinsic Brightness by a New Method," by Ives and Luckiesh,[2] the various sources were used as pyrometer filaments. The arrangement of the apparatus and the method employed in obtaining the brightness in which the pyrometer filament and the image of the background were assumed to be the same at a brightness match, were such that certain corrections to their results are necessary. The values obtained in most instances should be reduced by about 7 or 8 per cent. Most of them are probably relatively correct.

3d. A paper on the "Radiant Efficiency of Incandescent Filaments," by W. E. Forsythe,[3] is open to the same objection. The temperatures which were obtained were probably too high by about 1 per cent. A redetermination of these temperatures is to be undertaken in the near future in this laboratory.

4th. In a paper on "On the Deviation from Lambert's Cosine Law of the Emission from Tungsten and Carbon at Glowing Temperatures," by A. G. Worthing,[4] the method used was correct in principle.

5th. A paper "The Relation Between Black Body and True Temperatures for Tungsten, Tantalum, Molybdenum, and Carbon, and the Temperature Variation of their Reflecting Power," by C. E. Mendenhall and W. E. Forsythe;[5] the results are free from serious error due to diffraction.

SUMMARY.

1. Errors due to diffraction at the filament of the pyrometer lamp of the light from the background have been discovered in connection with

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[1] Trans. of Illum. Eng. Soc. (U. S.), 6, p. 238, 1911; Illum. Eng. (London), 4, p. 389, 1911.

[2] Elec. World, 57, p. 438, 1911.

[3] Phys. Rev., 34, p. 333, 1912.

[4] Astro. Jour., 36, p. 345, 1912.

[5] Astro. Jour., 37, p. 380, 1913.

the Holborn-Kurlbaum optical pyrometer. In some cases in measuring brightnesses errors as great as 60 per cent. may occur, though in temperature measurements the corresponding errors would be much less.

2. Determinations have been made of the ratio of the brightness of the pyrometer filament to the brightness of the image of the background formed in the same plane. The use of large filaments as pyrometer filaments and large angles for the entrance cone angle α tend toward making the pyrometer filament less bright than the background image when an apparent match is obtained, conversely small filaments and small angles for the entrance cone angle α tend toward making the pyrometer filament brighter than the background image for an apparent brightness match.

3. Some necessary working precautions and some desirable working conditions for high accuracy have been considered, and may be found incorporated under appropriate subheadings of this paper.

4. The effects of the present results on certain published works have been considered.

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,
NELA PARK, CLEVELAND, OHIO,
March, 1914.

THE EFFECT OF PRESSURE ON THE ABSORPTION OF LIGHT BY BROMINE AND CHLORINE, AND ITS THEORETICAL SIGNIFICANCE.

By David L. Webster.

Part I. Experimental.

Introduction.—In this part of the paper experiments are described showing first how the visible part of the wide absorption band of bromine obeys Beer's law, that the absorption at any wave-length with a given number of molecules per unit cross-section of the beam is independent of the length of the tube through which these molecules are scattered. In the narrow bands of bromine, on the other hand, the experiments show wide deviations from Beer's law, that may be explained by the changes of width of the fine lines of which these bands are composed. In the wide band of chlorine we find a slight deviation, that may be due to chemical changes.

Since this law was first announced by Beer,[1] for solutions, many experiments have been tried to test it as applied to gases. The first exception to be found was in the visible bands of oxygen, in which the absorption by a certain amount of gas was found by Janssen[2] not to be independent of the pressure, but proportional to it.

In the infra-red, an exception was found by Ångström[3] in carbon dioxide, in which the absorption of the total radiation from a Nernst lamp increased rapidly with the pressure. A threefold increase in the absorption by carbon dioxide was found by G. Bolinder,[4] on raising the total pressure from 20 mm. to 1 atm. by adding air.

These phenomena were observed spectroscopically by Rubens and Ladenburg[5] and Ångström[6] who found the absorption at any wave-length by a given mass of carbon dioxide or monoxide dependent only on its total pressure, whether this was changed by changing the length of the tube containing the gas or by admitting air.

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[1] A. Beer, Pogg. Ann., *86*, 78, 1852.

[2] Rep. Brit. Assoc., 547, 1888.

[3] Drud. Ann., *6*, 163, 1901.

[4] K. Angstrom, Arkiv for Matematik, Astronomi och Fysik, Bd. 4, No. 30, 1908.

[5] Vehr. d. D. Phys. Ges., *7*, 170, 1905.

[6] L. c.

These experiments were continued by von Bahr,[1] who extended them to many gases with results similar to those found in carbon dioxide. She found also that as the pressure was raised, the absorption increased rapidly at first, and then more slowly, approaching its maximum at a lower pressure in a heavy gas than in a light one.

In the visible spectrum, the first work after that of Janssen was by Baccei,[2] who verified Beer's law for several weakly absorbing gases, such as hydrogen sulphide and acetylene. Dufour,[3] on the contrary, found that when the total pressure on nitric peroxide or bromine vapor was raised by air, hydrogen or carbon dioxide, the fine absorption lines became hazy and were displaced by amounts proportional to the pressure. Füchtbauer[4] confirmed Dufour's results for bromine, and Wood[5] found an even larger pressure widening in mercury vapor at wave-length, 2536 A. Similar effects observed by Füchtbauer in potassium vapor and by Fredenhagen[6] in sodium were believed by these observers to be in some way connected with the increase of the ionization of the vapors by the transparent gases.

These results were interpreted by Füchtbauer as showing that the effects observed by Ångström and von Bahr were due to a widening of fine lines composing the infra-red bands, and a consequent increase of the absorption observable with the low dispersion of a spectrobolometer. Von Bahr, while agreeing that some of the bands showed evidence of such a discontinuous character, found some increase of absorption in bands that were apparently continuous. This led to the hypothesis that an increase of pressure caused an increase in the number of absorbing molecules.

An especially interesting field for exact work on pressure-absorption effects is presented by bromine vapor, because it has in the visible region a wide and apparently continuous band and also a few dozen narrow bands that may be resolved into fine lines with a moderate dispersion. Thus it gives a chance to study both types of bands in the same substance and the same part of the spectrum.

The wide band begins, according to Kayser[7] at about 3,350 A, and extends to the yellow, or even through the red if enough vapor is present, while the narrow bands occur only between 5,110 and 6,250 A, and are,

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[1] Eva von Bahr, Ann. Phys. *29*, 780–96.

[2] Nuovo Cimento, *9*, 177, 1899, Beibl., *23*, 636. 1899.

[3] C. R., *145*, 173 and 757, 1907.

[4] Phys. Zeit. *12*, 722, 1911.

[5] Phil. Mag. *23*, 689, 1912.

[6] Phys. Zeit., *12*, 909, 1911.

[7] Handbuch der Spectroscopie, Vol. III., paragraph 214.

on the average, about 10 A wide and 20 apart, each one being composed of many fine lines.

Description of Apparatus.—To investigate the effects of pressure on this spectrum, I used the apparatus represented in Fig. 1. It is a modification of that of Janssen, Ångström and von Bahr. *A* is a Schuchert arc lamp, which may be made to give a very steady light. *W* is a water cell, which was sometimes filled with water to absorb merely the infra-red and prevent heating of the nicols, *N* and *N′*, and sometimes with a copper sulphate solution to absorb the red and yellow as far as 5,800 A. *W′*, full of cobalt chloride dissolved in acetone (recommended by Uhler and Wood), was sometimes used to remove the red end as far as 5,450 A.

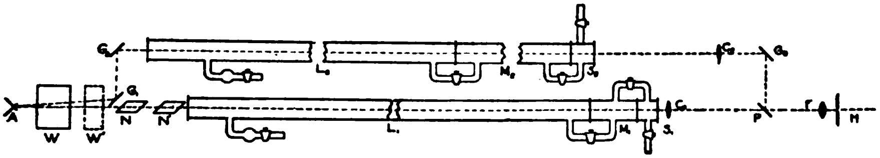

Fig. 1.

These colors were useful in preventing diffuse light when working in the blue and violet, where the spectrum was naturally faint and the absorption strong.

From *W′* one beam of light passed through nicol *N*, which could be rotated to any desired angle by a shaft running to the other end of the apparatus. The angle was determined by a scale on its mounting. From *N* the beam passed through a fixed nicol, *N′* and three tubes, L_1, M_1, and S_1, whose lengths were 185.1, 10.14, and 5.385 cm. respectively, making the expansion ratio

$$(S_1 + M_1)/S_1 = 2.88,$$

and

$$(S_1 + M_1 + L_1)/S_1 = 37.2.$$

Another beam, after reflection from the mirrors G_1 and G_2, passed through the tubes L_2, M_2, and S_2, whose lengths were 141.4, 55.30, and 5.18 cm. respectively, making the expansion ratios 11.7 and 39.0.

Mirror G_3 reflected this second beam to mirror *P* which reflected it parallel to the first beam, that passed over the edge of *P*.

While G_1, G_2, and G_3 were of ordinary silvered plate glass, *P* was a small piece of glass that had been scratched across one surface, covered on the other with a cathode deposit of platinum, and broken along the scratch. This process gives a plane, reflecting surface, with no appreciable rounding off of the edge, though several trials may be necessary to obtain good results. This simple and ingenious photometer, for which

I am indebted to Dr. A. H. Pfund, may also be made of silvered glass, though platinum is better if there are ever any acid fumes in the air.

The two beams, after leaving P passed through the short focus lens, F, which formed an image of the edge of P across the slit, H, of a Hilger wave-length spectrometer. C_1 and C_2 were long focus lenses used to condense the beams slightly, to give more light and avoid trouble from shadows of dust particles on the windows of the tubes.

This arrangement gave two spectra, one from each tube, and one directly over the other. Thus if the edge of P was perfect, one might focus F for any color so as to leave no black line between the spectra. I could then make their intensities equal for that color by turning the nicol, N, and my assistant, Mr. R. S. C. King, could read the angle, θ, from the position of total extinction. Since the intensity of the light transmitted varies as $\sin^2 \theta$, this gives a means of calculating the percentage chance of either beam when the condition of the bromine through which it passed was changed.

Experimental Procedure.—First the tubes were pumped to a pressure of a few cm., and bromine vapor was admitted to drive out any air in them or in the bromine reservoir, which was connected by glass only to the inlets of S_1 and S_2. Then they were pumped to a pressure of .02 or .01 mm. or less by a Gaede mercury pump protected by a hard glass tube full of hot copper gauze.

S_2 was then filled with bromine and the photometer was read. Then S_1 was filled and the photometer was read again. These readings gave the fraction of the light absorbed by S_1. (A reading with no bromine was not taken because of the excessive brightness of the light under those conditions.)

The third and fourth readings were taken with the bromine from S_1 expanded to M_1 and L_1 respectively, and the fifth and sixth with the bromine from S_2 expanded into M_2 and L_2. From the $\sin^2$ law the changes in the fraction of the light transmitted in each case could readily be determined.

The initial pressure of the bromine was determined from the temperature of the bromine reservoir and the data on its vapor tensions in Landolt & Bornstein's stable, with graphical interpolation where it was necessary. The room temperature was between 19 and 21° in nearly all the experiments.

Experimental Errors.—Treating the sources of error in the order in which we meet them in following the beams through the tubes, we have first the chance of a lack of constancy in the ratio of intensities of the beams as they leave the arc, due to a change of the position of the crater of the positive carbon. As the angle between the beams was small, such errors could be only slight ones; and as the changes were slow, the assistant, King, could keep watch of the arc and prevent them.

The scale on the rotating nicol was tested and found accurate.

A possible source of error in photometric work of this sort with nicols is any lack of uniformity in the beam due to inaccuracies in the interface of the rotating nicol, that produce light and dark streaks in the beam that rotate with it. Fortunately, however, such errors could be avoided by inclining the axis of the nicol very slightly to the direction of the beam, though not enough to impair the complete polarization of the light.

Leaks in the stopcocks were very effectively prevented by greasing them with a stiff paste of powdered Acheson graphite moistened with pure concentrated sulphuric acid, and protected from the moisture of the air by vaseline. This combination was found much more effective than phosphoric acid, and less likely to dissolve bromine and release it again when the pressure is reduced.

Any lack of equality in the ratios of the volumes of S_1, M_1 and L_1 or S_2, M_2 and L_2 to their lengths was avoided by weighing the tubes empty and full of distilled water (which gives no air bubbles) and correcting deficiences of volume by bulbs in their outlets or excesses by bits of glass rod. These ratios were made equal to 0.1 per cent.

Except for the uncertainty of all photometric balancing, the largest error was probably due to the reflection of light from the second windows of the short tubes back to the first and then back on its original course. This might change the intensity on expanding the bromine out of the short tubes by 0.6 per cent. at most, and thereby produce an error that is present in every experiment on absorption that involves the use of such windows.

Errors due to the reflection of light from the sides of the tubes were avoided by the use of eight, slightly ground, black glass diaframs in each tube, held in place by glass braces. These of course were in the tubes when the volumes were measured.

The bromine used was obtained from the Baker & Adamson Co., and was free from all volatile impurities except a trace of chlorine, which must certainly have been eliminated by evaporation in the preliminary experiments of getting the apparatus in running order. Traces of air in the vapor, which probably would affect the result but little, were avoided by letting a large amount of vapor pass through the apparatus during the pumping to drive the air out of the reservoir and tubes, and then pumping to about .01 mm. The possibility of impurities from the de Khotinsky cement on the windows was avoided by putting all the cement on the outside of the tubes, and letting none come through the cracks between the tubes and the windows, thus minimizing the area of cement surface exposed to the bromine.

The errors of photometric balancing were minimized by making at least four or five, and often ten, settings of the photometer for each angle to be determined, and making the intensity of the spectra uniform for as large a distance as possible above and below the line between them. A check on all these errors was furnished by the equality of the reading with the bromine in the two long tubes to that with it in the two short tubes, since the beams were absorbed equally in both these cases.

The result of all these possibilities for error is an uncertainty in the intensity of the light transmitted by the bromine which is probably not over 1 per cent. in all parts of the spectrum where the absorption is continuous; in the narrow bands, where some fine lines are visible and the intensity changes rapidly along the spectrum, the errors are much greater, though probably not over 3 or 4 per cent.

Results of Experiments on Bromine.—Satisfactory experiments were performed at ten points in the wide band, most of them being tested two or three times, and all but two of them with bromine enough to transmit less than .15 of the light of the wave-length in question. Diffuse red or yellow light was eliminated by a color screen throughout these experiments.

The wave-lengths in millimicrons, temperatures of the bromine reservoir, and initial pressures in millimeters are given in the following table:

Wave-length:	420,	430,	440,	450,	460,	465,	475,	490,	500,	507.
Temperature:	−30,	−20,	−17,	−18,	−20,	−20,	−10,	−7.3,	0,	0
Pressure:			20,				35,	45,	65,	65.

The pressures not given are for temperatures below the lowest for which Landolt and Börnstein give data. At − 20° the pressure is probably about 15 mm. The room temperature was about 20°.

From these pressures it was found that *at all these wave-lengths, with an expansion in any ratio less than 39 from the pressures given above, no change of intensity could be observed.*

This result may be stated better in terms of the absorption coefficient. If the intensity I of the light after travelling a distance x through the bromine at a density ρ is expressed in terms of the initial intensity I_0 by the equation,

$$I = I_0 e^{-a\rho x},$$

Lambert's law is that a is independent of x for any constant value of ρ, and Beer's is that a is independent of x or ρ for any constant value of the product ρx. Some approximate values of $a\rho$ at 20° and a pressure of 45 mm. (that of the triple point, − 7.3°) are as follows:

Wave-length in $\mu\mu$:	420.	440.	460.	480.	490.	500.	507.
ap in cm.$^{-1}$:	.9.	.85.	.75.	.6.	.55.	.50.	.45.

As these values are calculated from experiments in which the pressures were not known accurately, they are not exact.

Since Lambert's law may be expected to hold for such a continuous band as this, we may express deviations from Beer's law best in terms of the relative change,

$$\frac{\Delta a}{a} = \frac{\Delta I}{I \log I/I_0}.$$

In these experiments, except at 420$\mu\mu$, where $I/I_0 = .56$, and 460, where $I/I_0 = .32$, this ratio was well below .2 and often below .1, so that its logarithm was below − 1.6 or − 2.3. Therefore, since I is measured to 1 per cent., we may safely say that *with expansion ratios up to 39 from the pressures quoted above, any changes in a that occur with ρx constant must be less than ½ per cent.*

A striking contrast to this result is presented by the narrow bands, that lie on the long wave side of 510$\mu\mu$. In this region it is of little use to express the results in terms of a, for when the bands are made up of lines, and only averaged intensities over several lines may be measured, Lambert's law may be expected not to hold.

This assertion was verified by measurements of the absorption produced by different known amounts of bromine, both for a wave-length, 4,900 A, well removed from the narrow bands, and for another, 5,320 A, in a band about 20 A wide, that showed marked deviations from Beer's law. This was done by balancing the photometer first with no bromine in the tubes, again after filling and expanding S_2, a third time after filling and expanding S_1, a fourth after a second filling and expanding of S_2, and so on until the light was too faint to go farther. From the equation

$$I = I_0 e^{-a\rho x},$$

assuming Beer's law and allowing for the amount of bromine that remained in the short tubes after each expansion, the value of a for the bromine at its initial density was calculated from each measurement.

At 4,900 A, starting with a very intense light and an initial pressure of 18 mm. (temperature of reservoir, − 17.5), it was found possible to fill the tubes five times, ending with tube 2 transmitting only .00375 of the light, and tube 1, on account of its slightly greater length, only .00350. At 5,320 A, with an initial pressure of 60 mm. (temperature, − 1°) such faint lights could not be measured as well on account of the rapid variations of the intensity along the spectrum, so the tubes were filled only three times, and the fractions of the light transmitted at the end were .068 and .059.

Averaging the values of $a\rho$ from the two tubes, we have for the five steps at 4,900, the following results:

.217, .219, .221, .223, .225 cm^{-1}.

The three determinations at 5,320 gives the values:

.232, .197, .178 cm^{-1}.

The apparent change of a at 4,900 A is undoubtedly due to a warming of the reservoir, from − 18.0 to − 17.0°, which occurred during the experiment, and which would just account for this change if a is really constant. In the other experiment, no such temperature change occurred, so that the change in a at 5,320 A, which is in the opposite direction to that which would be caused by the deviation from Beer's law, is certainly due to the discontinuous nature of the narrow band.

An important result of this deviation from Lambert's law is the fact that, just as the values of a for any value of ρ depend on the length of the tube used in the experiments, the deviations from Beer's law must likewise depend on this arbitrary length.

The exact values of these deviations are therefore of no real importance, though their order of magnitude is of some theoretical interest. Since they are caused by the changes of width of the fine lines when the pressure changes, the exact widths of these lines under different pressures would be even more interesting. These, however, can be measured only with a high dispersion.

The changes of intensity of these bands often amount to 25 or even 50 per cent., as one may readily see from the graphs in figure 2, where the results of some of these experiments are shown. Other experiments at these wave-lengths and at 5,255, 5,380, 5,565, 5,665, and 5,875 A gave similar results; the general nature of the changes in all these bands is the same, as one may readily see in the spectroscope, where the whole narrow band region becomes brighter and the bands themselves slightly less distinct at every expansion of the bromine.

Experiments on Chlorine.—These experiments were performed with an apparatus similar to that used for bromine, except that the short tubes were 10 cm. long and the expansion was in one stage only, and to obtain more light in the blue and violet a spectrometer of lower dispersion was used.

Since the chlorine band lies mostly in the ultraviolet, Beer's law could be tested for only a very limited part of it. Experiments were tried on the region from 416 to 422$\mu\mu$, where intensity was increased by bright lines in the arc spectrum, that could be made to appear continuous by widening the spectrometer slit, and also for the region 435 to 437$\mu\mu$.

For longer wave-lengths the absorption was too weak to give any accuracy, and for shorter ones the spectrum was too faint. At 420, seven satisfactory experiments were performed, with initial pressures from 21 to 91 cm.; at 436 there were four, with initial pressures from 50 to 79.5 cm.

Beside the expansion experiments, others were tried on the effect of admitting dried air to the tubes containing the chlorine after the expansion. These results were not so accurate as those of the expansion experiments, because the chlorine was driven to the end of the tube by the air, and, being heavy, had a tendency to sink to the bottom of the tube and thus escape from the path of the light. Such experiments could not be performed with bromine, because it would condense on the walls of the tubes and take a long time to evaporate and diffuse through the air.

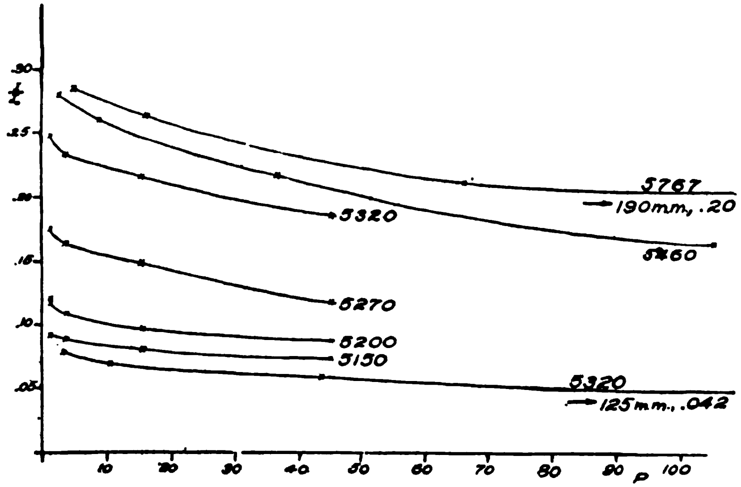

Fig. 2.

At 420, the mean of all the values of $(-\Delta a/a)$ for these expansion experiments is .012, with a mean error of .004, showing an increase of absorptive power with pressure in this case. The mean of the values of $(\Delta a_a/a)$, the relative change on admitting air, is $-$ .005, with a mean error of .01, so that we cannot say that there is any change in a when the total pressure is raised by admitting air.

At 436, the mean of the values of $(-\Delta a/a)$ is $-$ .0005, with a mean error of .004; and the mean of $(\Delta a_a/a)$ is $-$ .004, with a mean error of .002, whose smallness is probably accidental. We cannot therefore say that there is any change at this wave-length in either case.

This difference between the effects at two points so near each other is quite surprising, especially as no such effects were found in bromine.

An explanation, however, is suggested by the experiments of M. Trautz,[1] who finds that light absorbed in chlorine produces not only the well known increase in its chemical activity, and the Budde effect, or sudden expansion of 10 or 12 per cent., but also a sudden drop of several per cent. in its specific heat at constant volume, with a corresponding rise in the specific heat ratio. All these effects are reversed at once when the light is shut off. These phenomena indicate that the light probably produces some allotropic modification of the chlorine, just as light of the Schumann region converts oxygen to ozone.

As this reaction causes a change of volume, and as it is reversible, any expansion must cause a change in the percentage of the unstable form. Thus if the unstable form has an absorption band different from that of the unmodified chlorine, there will be deviations from Beer's law that will be different for different parts of the band.

In this connection it may be noted that the addition of air, unlike compression, does not increase the frequency of collisions of the chlorine molecules, and thus promote the reversal of the reaction caused by the light. It is therefore not surprising if the addition of air does not reverse the optical effect of the expansion.

Summary of Results.—A rapid increase of absorptive power of a given mass of gas with pressure was found by Janssen in the visible bands of oxygen, and by Ångström, von Bahr and others in the infra-red bands of many gases.

In the wide band of bromine, these experiments show that such changes, if they exist, must be extremely small; the slight change found in chlorine, due possibly to chemical effects, is very small compared to those in oxygen or the infra-red bands.

The narrow bands of bromine, like those of the infra-red, show a rapid increase of absorptive power with pressure. The effect in this case seems to be due to the widening of the fine lines composing these bands.

PART II. THEORETICAL.

This part of the paper contains a brief critical review of the development of the theory of absorption to its present state, with special reference to its explanation of pressure-absorption effects and some suggestions as to how the theory may be modified to give a better account of such phenomena. It is then shown how this modification helps in the explanation of some other phenomena of absorption.

The first explanation of the difference between the dielectric constant and the square of the index of refraction of any substance as a result of

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[1] Z. fur Elektrochemie, *18*, 513–20, 1912, or see A. Reis, Die Naturwissenschaften, *1*, 38, 1913.

the inertia of minute charged particles in it was given by Maxwell,[1] but was forgotten until resurrected by Rayleigh in 1899. Meanwhile an elastic solid form of it had been re-invented by Sellmeier and developed by Helmholtz and Ketteler into something like the present electromagnetic form.

The fundamental assumption on which this theory in its present form is based is that the electrons are free to vibrate about certain positions of stable equilibrium. Since the existence of nearly monochromatic radiations suggests that these vibrations are simple harmonic, it is often assumed that the restoring force on the electron is approximately proportional to its displacement from its equilibrium position, and sometimes that it is due to a continuous distribution of positive electricity through which the electron moves.

A well-known result of this hypothesis is that, when subject to a sinusoidal electric force, the electrons will execute steady vibrations about their positions of equilibrium with amplitudes depending not only on the maximum value of the force, but also on the frequencies of the force and of the electrons' own free vibrations. Thus the polarization of the medium, and consequently the index of refraction, μ, will depend on the frequency according to the well-known equation,

$$\frac{4\pi P}{E} = \mu^2 - 1 = \frac{4\pi Ne^2}{m(n_0^2 - n^2)},$$

where P is the polarization, measured in the standard electrostatic system, E the electric force, whose frequency is n, m the mass of the electron, e its charge, and N the number of vibrating electrons per unit volume. n_0, the frequency of the free vibrations of the electrons, is always the frequency of an emission or absorption band. This formula for the dependence of μ on n and n_0 agrees well with experimental results.[2]

The dependence of μ on N, and consequently on the density of the absorbing gas, has been treated very successfully by Lorentz,[3] by a slight modification of this formula, taking account of the influence of surrounding molecules on the vibrations of the one in question, on which they exert an electrostatic force equal to aP, where a is a numerical constant, approximately $4\pi/3$. Since $P = Ner$, this results in a ponderomotive force aNe^2r, acting against the restoring force, and thereby weakening the resultant of the two by a very small amount proportional to N. Thus in the above formula for μ, the frequency n_0 of the free vibration of a single electron must be replaced by the slightly lower frequency,

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[1] Math. Tripos Exam., 1869.

[2] See, for example, D. A. Goldhammer, Dispersion und Absorption des Lichtes, B. G. Teubner, 1913.

[3] H. A. Lorentz, Theory of Electrons, B. G. Teubner, 1909.

$$n_0' = \sqrt{n_0^2 - \frac{aNe^2}{m}},$$

with which a large collection of these electrons would vibrate freely in unison.

For the effect of increasing density on absorption lines, this gives the well-known shift toward the lower frequencies; and for the effect of density changes on μ it gives the formula,

$$\frac{\mu^2 - 1}{\mu^2 + 2} = constant \times \text{density},$$

which was found by Lorentz to agree well with Lorenz's experiments.

This theory would lead one to expect a steady vibration of each electron, in phase with the inducing force if $n < n_0'$, in opposite phase if $n > n_0'$, and of an amplitude becoming infinite as n approaches n_0' from either side. Since this last result is evidently absurd, some additional assumption must be made to avoid it. So far all theories of absorption based on the classical electromagnetics agree, but in this assumption they differ.

A way out of this difficulty appeared to Planck to be the assumption that the electron was restrained enough by the force $(2/3)(e^2/c^3)(d^3r/dt^3)$, which results, along with the force of inertia, $-m(d^2r/dt^2)$, from the effect of the radiations from each part of the electron on other parts, and which accounts for the damping of vibrations by radiation.

This force, however, is so small that when $n = n_0'$, according to Wood, the vibration would attain such an amplitude that with only one electron in a cube a wave-length on an edge the intensity of the light would be reduced to one half in going a single wave-length.

Furthermore, with so few vibrating electrons, we run into still another difficulty, because in deriving the equations of propagation of light through systems of molecules, the electric and magnetic vectors were averaged over volume elements that were small compared to distances in which appreciable changes occur, but large compared to distances between molecules. In this case, such elements do not exist, and the problem must be treated by considering the forces radiated from each electron separately.

In the direction of the beam these forces would result in weakening the original force; and in other directions they would cancel each other only if there were many electrons in each cube a wave-length on an edge and if the intensity changed but little within a wave-length. These two conditions, as we have seen, are mutually inconsistent if the electrons are retarded only by their own radiations. If, nevertheless, these conditions are not satisfied, the radiated forces will not cancel, and there will be a strong resonance radiation, or selective reflection.

Thus it appears that this is not really a theory of absorption at all, but of reflection only, so that we may expect it to approximate to the truth only in cases, such as in mercury vapor at wave-length 2,536 A and the sodium *D* lines as observed by Wood, in the light of comets' tails, at the surfaces of solid aniline dyes, and in other such cases where strong selective reflection occurs. The more completely the light is reflected, rather than absorbed, the larger part of the damping of the vibrations may be assigned to this cause. But where there is any true absorption, there must be some other retarding force as well.

Perhaps the most natural assumption to obtain this force was the one made by Helmholtz, that the electron moves in a viscous medium, with a resistance proportional to its velocity. This hypothesis, nevertheless, has many serious disadvantages, the greatest of which is that it neglects the most obvious experimental fact in the whole subject of absorption, namely the heating of absorbing substances by light. For in ordinary bodies, friction, which we explain by molecular collisions, always transfers energy from the larger bodies to the molecules, whose heat motions are thereby strengthened; but no amount of friction within an isolated mechanical system ever produces motion of the system as a whole. Similarly this electronic friction would transfer energy from the electrons to some sort of sub-electronic bodies, from which it could not all be recovered, and thus it would violate the law of the conservation of observable energy; likewise it would never transfer energy to the molecules, which do, as a matter of fact, acquire it.

Lorentz, recognizing this defect in Helmholtz's hypothesis, and also wishing to account for the widening of absorption lines by pressure, assumed that there was no damping, but that the electron could continue to vibrate further and further, restrained only by radiation, until the molecule collides with another. Then, he assumed, all the energy of the vibration would be transferred to energy of translation of the molecules. The effect of this he proved equivalent to a damping force, $-g(dr/dt)$, where $g = (2m/\tau)$, τ being the mean time between collisions of a molecule. Thus, defining the absorption coefficient k by the equation

$$I = I_0 e^{-kz}$$

and denoting its maximum value by k_m, one may prove that

$$k = \frac{4\pi n^2 g N e^2/c}{m^2(n_0'^2 - n^2)^2 + n^2 g^2}, \quad k_m = \frac{4\pi N e^2}{cg}, \quad \frac{k}{k_m} = \frac{1}{1 + \frac{m^2}{n^2 g^2}(n_0'^2 - n^2)^2}.$$

Thus one may evaluate τ from data on the width of an absorption line, which Lorentz defines as the frequency interval from n_0' to a point on either side where $k/k_m = 1/\kappa$, κ being some small number, say 10 or 20. For this position $\tau(n_0 - n) = \sqrt{\kappa - 1}$. Hence as the pressure increases, and τ decreases, a line must widen, though the absorption at the center must decrease, these effects being just what Dufour[1] found in nitric peroxide and bromine on adding air.

On evaluating τ from this equation, with Hallo's statement that the D lines in a certain sodium flame were an Ångström in width, Lorentz found τ of the order of 1 or 2 times 10^{-11} sec., whereas the kinetic theory would give 10^{-10}. Yet these lines were narrow compared to the oxygen and halogen bands, and to the ultra-violet bands of all gases.

As a better explanation of the wide bands, Livens[2] called attention to an omission in Lorentz's treatment of the influence of the surrounding molecules on the vibrations of an electron. In evaluating the factor a of the electric force aP, Lorentz divided these electrons into two classes, the first including all that lie outside of a sphere drawn around the electron in question with a radius large compared to the distance between molecules, but small compared to the wave-length, and the second including all within this sphere. The force due to the first class was proved to be $(4\pi/3)P$, while that due to the second would be zero if the molecules were in a regular cubical arrangement, so it was taken as zero for the haphazard arrangements found in a gas. But as Livens pointed out, even though the mean of all its values for all the molecules in a gas is zero, it is rarely zero for any individual molecule, because of the lack of symmetry of the surrounding ones, though it may be either positive or negative. Hence the modified frequencies of the electrons would not all equal n_0', but some would be greater and some less. Thus as the line shifted with pressure, it would also widen.

As Livens said, this would account not only for pressure widening of lines in gases, but also for the comparatively great width of bands in solids and liquids.

A critical examination nevertheless shows some serious defects in Livens's theory. In the first place, he abandons entirely Lorentz's assumption of damping by collisions, which is necessary in some form to account for heating by light without ionization or chemical change; and he assumes Helmholtz's internal friction, which is impossible, not only for the reasons noted above, but also for many others, such as the fact that it would not allow resonance radiation in extremely rarified gases, nor ionization in the ultra-violet by feeble intensities of light.

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[1] L. c.

[2] Phil. Mag., *24*, 268–93 and 523–8, 1912.

Furthermore, on Livens's theory the width of a band in a liquid or solid might be expected to increase with the number and electric moment of the molecules vibrating in phase with the one in question. Thus we should expect wide deviations from Beer's law in solutions, though it is well-known that in the absence of chemical changes such deviations do not occur.

Finally, for similar reasons, we should not expect much difference in the widths of bands of about the same intensity in different solutions, though those in crystals might differ. Livens's effect therefore can account only for the width of the narrowest bands observed in solutions, that is, for widths of a few Ångströms observed in the salts of the rare earths. Wider bands, in solutions, and especially in gases, are still unexplained.

Even the Doppler effect, which has often been invoked to explain part of the width of lines in gases, has been found guilty of only a small fraction of an Ångström.

Unequal Frequencies of Free Vibration in a Band.—Collecting the possible sources of width in a band, and the widths for which they can account, in gases, we have: first the damping by radiation, giving possibly a few thousandths of an Ångström; second, damping by collisions, giving perhaps a few tenths; third, the Doppler effect, ordinarily negligible; and fourth, Livens's effect, giving perhaps a few tenths. All these causes together would probably not give over an Ångström under the most favorable conditions, and the bands to be explained range from a few thousandths of an Ångström to hundreds in width.

We are therefore driven to the assumption that *the natural frequencies of the electrons, when free from all electrostatic influences of their neighbors, are not so nearly equal as they are ordinarily supposed to be.*

This assumption is not entirely new, since the statement that some bands are made up of fine lines is a form of it in which the different frequencies are separated by finite intervals. This suggests the possibility that all bands over about an Ångström in width may be composed of fine lines, just as the narrow bands of bromine are.

In deciding this point, tests of Beer's law are of especial value. For unless the distances between the centers of the lines are much less than the widths of the lines themselves, they ought to give deviations from Beer's law with any dispersion too low to resolve them. Therefore, although other causes, such as changes in the number of absorbing molecules, may also given deviations from Beer's law, a band that obeys it may probably be assumed to be continuous.

From the experiments described in the first part of this paper, *that* *show great exactness for Beer's law as applied to the wide band of bromine,* and deviations in chlorine that are very small and that may be due to chemical changes, we may now conclude that *the width of these bands is probably due to a continuous distribution of frequencies of undisturbed vibrations.*

In sharp contrast to these results are those of Janssen in the visible oxygen bands and Ångström and von Bahr in the infra-red bands of many gases, where large deviations from Beer's law were found. In some of the bands investigated by von Bahr, signs of discontinuity were discovered, both during these experiments and later[1] with a higher dispersion. In some cases, however, the absorption seemed to increase with pressure even after the lines had widened into a perfectly continuous band, as though the increased pressure kept a larger fraction of the molecules in a condition for absorption.

Consequences of the Hypothesis of a Continuous Distribution of Free Frequencies.—Inquiring as to the cause of this distribution of free frequencies, we are met by the question of the origin of the restoring force that was assumed in explaining the elementary facts of absorption. Except in the infra-red, where the vibrations are probably those of charged atoms, it may, as has usually been supposed, be due to a sphere of positive electricity in which the electron moves, or it may be explainable only by the quantum hypothesis. Since, however, the latter hypothesis gives as yet no explanation of pressure-absorption effects, we may disregard it here.

On the positive sphere hypothesis, if there are several electrons in an atom, the force on any one of them when it is displaced a given distance from its position of equilibrium will be modified by any change in the positions of the others, or by any compression or distortion of the sphere such as chemical and cohesive forces would produce. It would therefore be really quite surprising if the frequency of any electron or group of electrons were always exactly its normal value, and it is reasonable to suppose that it may often be subject to great deviations from this value. Similarly in the infra-red, the frequencies of vibrating atoms must be modified by any intra-atomic motions such as would be caused by collisions.

From these considerations it is evident that an increase in the violence of the collisions will increase such disturbances and therefore widen the lines or bands. This is confirmed experimentally by the widening of the infra-red bands examined by von Bahr, and also of many finer visible lines, by a rise of temperature.

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[1] E. v. Bahr, Deutsch. Phys. Ges., Verh. *15*, 16, pp. 710–737, and *15*, 21, 1150–58; A. Eucken, ibid., pp. 1159–62.

Similarly the strong and very irregular cohesive forces and pressures that are put upon the atoms in liquefaction may be expected to produce the wide and perfectly continuous bands that are observed in most liquids and solids.

Since the wide bands of bromine and chlorine do not seem to be affected much by a change in the frequency of collisions, it appears that they, and probably the similar bands found in other gases in the ultra-violet, must owe their width to intra-molecular motions that are not much affected by collisions.

Heating, Ionizing and Chemical Effects of Absorption.—The fact that these bands do not behave like those of the infra-red may, in the light of our new assumption, be closely related to the fact that absorption of light of low frequencies results only in heat, while that of high frequencies is much more likely to produce ionization or chemical effects. For now that we are free to follow Lorentz in ascribing the damping of the vibrations to collisions, we may well inquire why such a damping should occur. Since the collisions at ordinary temperatures do not set electrons of visible frequencies into an appreciable state of vibration, they cannot be sudden shocks, but must involve gradually increasing and decreasing forces, that last for several periods of such vibrations. Conversely, since the maximum energy density in the black body spectrum at ordinary temperatures is near 8 or 9 μ, we may expect the times of collisions to be of the order of magnitude of a single period of vibration of such light, that is, about 10 or 20 periods of visible light.

If now two molecules collide with a low frequency electron or atom in a state of violent vibration, it may be expected to transfer some part, though not necessarily all, of its energy to the translatory motion of the molecules. If, on the other hand, the frequency is high, it will execute many vibrations during the collision, and any forces on the other atom due to it will alternate so many times during their approach and recession that they will produce no change in the energy of translation. Thus we may expect the production of heat readily by low frequencies, but not so readily by high.

This being the case, the energy of high frequency vibrations will accumulate without much molestation by collisions even in a solid; likewise, if the collisions occur very seldom, as in the tail of a comet, where the molecules are about a meter apart, we must have an undisturbed accumulation of energy of any frequency.

This process may give various results. First, if the electron is bound firmly enough, it will accumulate energy until it reaches the limit set by the damping by radiation, and then it will continue to emit the strong selective reflection observed in comets' tails and in sodium and mercury vapors.

If however such an amplitude of vibration would cause the electron to escape from the atom, then ions will be produced in a gas or photoelectrons from a solid. Here it may be noted that, in the absence of any considerable damping, such ionization will occur even with very low intensities, though greater intensities will produce ions quicker, and consequently more often. Likewise, since ionization occurs as soon as a certain amplitude is reached, the subsequent velocity of the electron will not depend at all on the intensity of the light.

Still another possibility in such cases is that the electron will be thrown into some other situation in the atom by its vibration, and thereby change its chemical behavior, thus producing the well-known stimulation of reactions, especially of decomposition, by light.

SUMMARY.

1. A critical analysis of Helmholtz's assumption of viscous resistance to the motion of a vibrating electron shows that it is inconsistent with experimental facts, and that Lorentz's hypothesis of damping by collisions is necessary to account for the production of heat by light of low frequencies.

2. A compilation of the possible causes of width of spectrum lines then shows a deficiency in the present theories that can be remedied best, as shown by the experiments on chlorine and bromine, by the assumption that the frequencies of free vibration of electrons producing a band are not all equal, but are often continuously distributed about their normal value.

3. This assumption is found to lead to an explanation of the production of heat by low frequencies, of ionization and chemical effects by high, and of the strong selective reflection observed in comets' tails and other extremely rarified gases. Finally, the laws by which chemical effects and ionization depend on the intensity of the light are explained.

ON THE ASYMMETRIC EMISSION OF PHOTO-ELECTRONS FROM THIN FILMS OF PLATINUM. I.

By Otto Stuhlmann, Jr.

Introduction.

As early as 1889, W. Hallwachs[1] showed that a photo-electric effect could be obtained from a silvered quartz plate when light was incident through the quartz.

Later Rubens and E. Ladenburg[2] took up the work and observed a similar effect for gold leaf. They found the ratio of the photo-electric effect for incident and emergent light to be 100 to unity while the fraction of the ultra-violet light transmitted, photo-electrically measured, was one one-thousandth of the incident.

The work was taken up by the writer[3] who showed that the photo-electric effect of thin cathode films of different metals deposited on quartz depends on the thickness of the film and whether the film is on the side of the quartz facing the light (incident effect = I) or on the side away from the light (emergent effect = E). The ionization currents were measured in air. It was shown that when the films are thin enough, about 10^{-7} cm. or less, that the ratio emergent to incident currents (E/I) was greater than unity and for thicker films less than unity. This ratio was constant for thin films and was shown to be equal to 1.17 for platinum.

Independent experiments carried out by R. D. Kleeman[4] both in hydrogen and in vacuo led to the same conclusion as to the asymmetry of the photo-electric effect for emergent and incident light.

Recently J. Robinson[5] showed that the above observed dissymmetry of emission should be separated into two quite distinct effects; the actual number of electrons emitted and the velocities of the electrons emitted.

As to the Actual Number of the Electrons Emitted.

It was shown by Robinson[6] that the magnitude of the ratio of emergent to incident current depended upon the source of illumination. When the

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[1] W. Hallwachs, Tagebl. d. Heidelberger Nat.-Vers., S. 24, 1889.

[2] Rubens and E. Ladenburg, Ber. d. D. Ges., 24, p. 749, 1907.

[3] O. Stuhlmann, Jr., Phil. Mag., 20, p. 854, 1911.

[4] R. D. Kleeman, Proc. Roy. Soc., 84, p. 92, 1910.

[5] J. Robinson, Phil. Mag., 23, p. 542, 1912. Pt. II., Phil. Mag., 25, p. 115, 1913.

[6] Loc. cit., Pt. II., p. 125.

films were illuminated by a spark between brass terminals the ratio of emergent to incident effect was equal to 1.27, while a mercury vapor arc in quartz changed the value to 1.16. An iron arc as originally used by the writer gave 1.17 for this ratio. More recent results show a change in this value in a decreasing order of magnitude as follows: iron arc, quartz mercury vapor lamp, iron spark, zinc spark and cadmium spark in air.

The current curves plotted against film thickness show a maximum at about 10^{-7} cm. For thin films the emergent being greater than the incident effect. The rate of increase, however, does not take place so suddenly as Robinson's curves lead one to think. A more careful study of the change of current with film thickness as shown by the writer and K. T. Compton,[1] attributes the sudden tremendous increase in current, to method of making contact between film and holder when the plates are sputtered in vacuo.

Recently Partzsh and W. Hallwachs[2] became interested in this question from an optical point of view. They investigated a series of cathode deposited platinum films ranging in thickness from 1 to $50\mu\mu$. The incident, reflected and transmitted beams were measured both for emergent and incident positions. If the film were illuminated so that the transition layer was quartz-platinum, 40 per cent. more light was absorbed by the film than when the illumination fell directly on the metal side. They further showed that the reflection coefficient for the transition layer quartz-platinum, at first decreased with increasing thickness of film to about $3\mu\mu$ where a minimum occurred. Then increased until at $7\mu\mu$ the original value recurred. A further increase in thickness was accompanied by an increase in the reflection coefficient until it approached the value for platinum. From these data in connection with the values of the variations in transmission with change in thickness of film one may plot the absorption curve.

Since absorption of light and photo-electric current are proportional, one would expect these curves to be identical with the ones obtained by measuring the photo-electric current with change in thickness.[3] This, however, is not quite true, because the photo-electric effect shows a peculiar sudden[4] increase in current and then a well-defined minimum, which the curves by Partzsh and Hallwachs do not account for.

They further show that thin platinum films on quartz, whose thickness is less than 10^{-6} cm., absorb, when illuminated through the quartz plate,

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[1] O. Stuhlmann, Jr., and K. T. Compton, PHYS. REV., S. 2, Vol. II., p. 205, 1913. Pt. II., PHYS. REV., S. 2, Vol. II., p. 327, 1913.

[2] Partzsch and W. Hallwachs, Ann. d. Phys., 41, p. 247, 1913.

[3] Stuhlmann and Compton, PHYS. REV., S. 2, Vol. II., p. 208, 1913.

[4] For a possible explanation of this see J. Robinson, Phil. Mag., 25, p. 127, 1913.

1.5 to 1.4 times as much of the incident light as when the illumination falls directly on the platinum. These differences, the authors conclude, are great enough to account for the ratio of the emergent to the incident effect as photo-electrically measured by Kleeman, Robinson and the writer.

It was thought by some writers, since the ratio of the emergent to the incident effect was greater than unity, that the emitted electrons received a velocity component in the direction of the propagation of the light. A longitudinal action of the light, if present, is evidently of a smaller order of magnitude than the above photo-electric ratios, of emergent to incident currents, seem to indicate.

As to the Velocities of the Electrons Emitted.

The maximum velocities of emission, for platinum cathode films deposited on quartz, were also investigated by J. Robinson.[1] He found that for very thin films (less than 10^{-7} cm.) the maximum velocity of emission due to the emergent light was greater than that due to the incident light; for thicker films the reverse was found to be true. In his first paper the maximum value for the ratio of emergent to incident velocities, as obtained by projecting his curve backwards, gives 1.24, while his second paper shows only 1.12 for this maximum value, although the same apparatus and source of illumination were used.

Since the maximum energies of emission are independent of the intensity of the source of illumination, Partzsh and Hallwachs[2] conclude that the variation in absorption of the two positions of the films would be no adequate explanation of this observed change in velocity.

Robinson also determined the distribution of velocity curves for four samples of platinum films of different thickness. Partzsch and Hallwachs believe that objections might be raised to these results, in so far as values for the velocities were obtained only at two volt intervals, hence making any conclusions that might be drawn from the curves doubtful.

Concerning the Velocities with which Photo-electrons are Emitted from Matter.

An interesting and valuable suggestion was recently made by Mr. A. Ll. Hughes[3] concerning the velocities with which photo-electrons are emitted from matter. In his review of the literature on the subject of emission of photo-electrons from metallic surfaces, he emphasizes the

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[1] Loc. cit.

[2] Loc. cit., p. 267.

[3] A. Ll. Hughes, Phil. Mag., 25, p. 683 (1913).

fact that the maximum velocities investigated were always those of electrons emitted from the side of the plate on which the light was incident. Under these conditions the Plank constant (*h*) as experimentally determined by the application of the Einstein-Richardson[1] equation, for the photo-electric effect, was always found to be less than 6.55×10^{-27} ergs sec., by an amount ranging from 10 to 25 per cent. Robinson[2] in his experiments on very thin platinum films shows that the velocities of the electrons for emergent light are greater than those for incident light, for thicker films the reverse is the case. For a certain thickness of film the emergent velocity is equal to the incident velocity and special experiments showed that for the same thickness of film the emergent current is equal to the incident current. The orientation of the plane of polarization of the light did not have any influence on the dissymmetry.

It may be inferred that originally all the electrons are, perhaps, emitted more or less in the direction in which the light is traveling and that the energy lost by a photo-electron in swinging round from its original direction of emission is at least comparable to the per cent. increase of the emergent over the incident effect. For an infinitely thin film this difference as shown by Robinson is about 12 per cent. Introducing the correction suggested by the asymmetric emission, Hughes concludes that the experimental and theoretical results may be made to check in the case of platinum, and presumably in other cases also, if we keep in mind the electrons which emerge in the direction of the incident light.

H. S. Allen[3] suggests that if this be borne out by further experiments, we reach the important conclusion that for light of a given frequency, electrons acquire the same energy in the case of all metals, though the work done in escaping is a quantity characteristic of the metal.

No experimental results as to the ratios of the emergent to the incident velocities for other metals are available and so it is not possible, at present, to test this view further. More precise information would be obtained if the ratios were investigated using monochromatic light. In view of these suggestions a further study of the velocities of emission from thin metal films was warranted.

PART I.

The first part of this paper is devoted to a solution of some of these questions. With this object in view Robinson's design of apparatus was practically adopted, in order, first to determine if his results could be

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[1] O. W. Richardson, PHYS. REV., Vol. 34, p. 146 (1912); Phil. Mag., 23, p. 615 (1912); Science, 36, p. 57 (1912).

[2] Loc. cit.

[3] H. S. Allen, Photo-Electricity, Longmans, Green & Co., 1913.

duplicated, second to get the distribution of velocity curves for emergent and incident effect with change in thickness of the films.

Apparatus.

The apparatus as shown in Fig. 1 consists of a heavy brass outer cylinder put to earth. Coaxial with it is a light brass cylindrical box (*C*) supported by a rod passing through a ground glass stopper *B*. A circular quartz plate *Q*, 1 mm. thick and 2 cm. in diameter, passing through the ground glass stopper *R* and insulated from it with hard sealing wax, projects into the inner cylinder. This quartz plate is covered with the platinum deposit in the form of a semi-transparent film and is attached to the rod by means of a screw clamp. The ground glass joint served to rotate the quartz plate through any angle so that the film may be made to face the light or be turned away from the light, in order to measure the photo-electric effect in these two positions. The inner cylindrical box (*C*) is 7.5 × 3.5 cm. over all, with circular openings at both ends, 2 cm. in diameter. The cylinder was connected to a potentiometer circuit *P* containing a Weston standard voltmeter reading to one one-thousandth of a volt. The electrometer, connected to the quartz plate carrying the platinum film, gave a deflection of 765 divisions per volt at a scale distance of one meter.

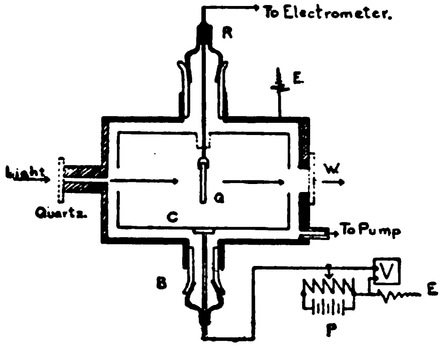

Fig. 1.

All the internal metal surfaces were covered with camphor soot to prevent effects due to stray reflected light.

A narrow beam of light from a quartz mercury vapor lamp stopped down to 3 mm., passed through a quartz lens (f 15 cm.) which focused the source on the platinum film. The beam entered the apparatus through a quartz window 1 mm. thick and passed out through a plate glass window at *W*. The quartz plate *Q*, carrying the metal film, the quartz window and the glass window at *W*, were all parallel and normal to the incident beam, so that any reflected light passed back along its entering path. This was obtained by rotating the plate *Q* until the reflected light passed through the forward opening.

The source of light was a quartz Cooper Hewitt mercury vapor lamp (Type *Z*) using 4 amperes direct current at 110 volts. It was found to be a reliable, very constant and brilliant source of illumination.

The platinum films were deposited from a circular platinum disk 5 cm. in diameter, in a separate glass chamber. From here they were introduced rapidly into the testing vessel. Previous experiences with these metal films having shown that such handling does not effect their photo-electric properties. Experiments by the writer and K. T. Compton[1] have shown that a cathode discharge, through the apparatus in which velocity measurements are to be made, may deposit films giving off electrons with spurious high velocities. Under these conditions the films may also show effects depending on the method of making contact between them and the holder, such results having previously been interpreted as properties of these cathode films. To insure against these effects the films were deposited in a separate chamber and mounted for examination in identical ways Through the control of the cathode fall and current Tyndall and H. G. Hughes[2] have conclusively shown that the thickness of the film is a linear function of the time.

A Gaede rotary mercury pump was used to furnish the desired vacuum. A gold leaf mercury vapor condenser was inserted just after the pump to prevent the mercury vapor from amalgamating with the platinum films. The results were all compiled from data taken at a pressure of 2×10^{-5} cm. or less.

VELOCITY MEASUREMENTS OF THE RATIOS E/I.

The maximum initial kinetic energy of the emitted electrons, expressed in equivalent volts was determined by charging the cylinder C to such a negative potential that the electrometer connected to the platinum coated plate Q, showed no deflection. The velocity v of an electron is connected with this maximum potential V by the relation $\frac{1}{2}mv^2 = eV$. The equivalent maximum potential taken up by the film was measured, when the film was turned to face the light (I) and when it was turned away from the light (E).

This maximum potential was corrected for contact difference of potential, since under the above conditions the electrons are driven back into the metal by a difference of potential equivalent to the contact difference of potential between the platinum and the surrounding metal case, plus the potential impressed by the outside circuit on the cylinder. The contact difference of potential was measured in air by means of the ionization method. For this purpose a tube containing 0.89 mg. of radium chloride was fastened against the window W and the potential measured to which the cylinder (C) had to be charged to produce no deflection. This method is very sensitive to changes in contact differ-

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[1] Loc. cit.

[2] Tyndall and H. G. Hughes, Phil. Mag., 27, p. 415, 1914.

ence of potential and has been very successively used by K. T. Compton[1] and N. Shaw.[2]

It was found that very erratic results were obtained upon first introducing the films into the testing chamber. Separate experiments showed a gradual change to a steady state. This was shown to be due to the gradual drift of the contact difference of potential to its final value. For thin films this effect was more pronounced than for thicker films. The contact difference of potential rose rapidly to a maximum of several volts, from where, after several hours, it exponentially dropped to its final constant value. For very thick films or a metal plate no such maximum appeared, the curves rapidly rising to their normal value, giving identical results as those obtained by N. Shaw.[3] The following data were obtained after the steady state had been reached.

TABLE I.

Measurements of the Maximum Energy of the Electrons Produced by Incident and Emergent Light.

Relative Thickness of Film.	Emergent Light.		Incident Light.		Ratio of Emergent to Incident Energy Corrected for C.D.P.
	V. Max.	C.D.P.	V. Max.	C.D.P.	
1	.428	+.295	.212	+.298	1.42
2	1.000	.300	.565	.303	1.49
3	1.630	.254	1.667	.202	1.01
4	1.094	.322	1.224	.302	.927
5	.600	.356	.775	.337	.860
Pt. plate.	....	....	1.438	.353	....

Table I. gives the ratios of emergent to incident kinetic energies corrected for contact difference of potential as obtained above. It was assumed that the measurements of contact difference in air would not differ appreciably from those measured in vacuo, under the same conditions. Characteristic results are only given, showing the variation in the ratio E/I with change in thickness. Numbers 1 and 2 are characteristic results from very thin transparent films whose thicknesses are less than 10^{-7} cm. Number 3 is quoted because of its unique value showing a film where the emergent and incident effects are practically the same. Numbers 4 and 5 show the change in the values of E and I to thick films, while the value for a freshly scraped platinum plate is given as a comparison standard.

It is evident that the films are of platinum since they possess practically the same contact difference of potential as the platinum plate.

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[1] K. T. Compton, Phil. Mag., 23, p. 579, 1912.

[2] N. Shaw, Phil. Mag., 25, p. 241, 1913.

[3] N. Shaw, loc. cit.

They are consistent with previous results obtained along a different line[1] and agree with those cited by Robinson[2] except in so far as the ratios given above are a little larger. This is perhaps due to the fact that each of the above values is the mean of several determinations obtained by means of a null method of observation. They differ from Robinson's results in so far that these and previous results by the writer and K. T. Compton[3] show that a distinct maximum exists both for incident and emergent energies at a thickness comparable to 10^{-7} cm., where Robinson found his maximum for the current-thickness curves. After this maximum the emergent energy diminishes as the thickness of the film increases while the incident energy gradually increases to the normal value for platinum[4] as the thickness of the film increases. From the data of Partzsh and Hallwachs[5] one may plot the absorption curve for changes in thickness. No such pronounced maximum exists, as would account either for the sudden increase in current or velocity. The hypothesis of a longitudinal component in the light producing the asymmetry is highly improbable. The theory advanced by Robinson of a secondary effect coexisting with the liberation of an electron is one which has to be reckoned with. At any rate further information is desirable before advancing even a tentative hypothesis to explain the asymmetry in the velocity of emission.

VELOCITY DISTRIBUTION CURVES.

A curve giving the relation between the photo-electric current and the potential difference between the illuminated film and the surrounding case is called a velocity distribution curve.[6] When the potential difference is such as to retard the electrons then the photo-electric current is usually taken to be the measure of the number of electrons possessing velocities greater than that implied by the potential difference. If no contact difference of potential existed the ideal curve would saturate at zero potential.

It is well known that these velocity distribution curves are liable to distortion by reflection of electrons, but the value of the maximum velocity is not effected by them. If reflection of electrons by the surrounding case takes place the curve will be modified and depressed along

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[1] O. Stuhlmann and K. T. Compton, PHYS. REV., S. 2, Vol. II., p. 205, 1913.

[2] J. Robinson, loc. cit.

[3] Loc. cit.

[4] Loc. cit.

[5] Loc. cit.

[6] For the theory of the "Velocity Distribution Curves" see A. Ll. Hughes, Phil. Trans. Roy. Soc., Lond., *A*, Vol. 212, pp. 205–226, 1912.

the current axis in proportion to this reflection, hence giving a slight negative current for potentials greater than the equivalent maximum velocity. Reflection of light also alters the shape of the curve, but this has been taken care of by blackening the interval surfaces with camphor soot. The photo-electric sensitiveness of this effect falls within the experimental errors. The effect of a stray magnetic field or of the approaching of the electrons to the surface in an oblique direction both tend to shift the curve.

With the existing apparatus experiments were made to determine how the velocity distribution curve normally obtained from solid metals under the above conditions was effected by the change in direction of the illumination on the metal film. Second to see if a change in thickness of the film was followed by a corresponding change in the slope of the curves analogous to the change in the absorption curves of Partzsch and Hallwachs. Third to determine whether the asymmetry of the maximum velocity could be substantiated by the relative magnitudes of the curves.

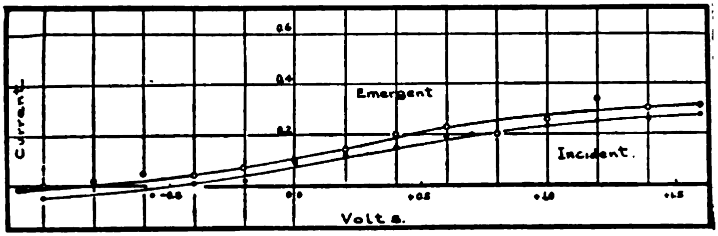

Fig. 2.

The films used were the same as those shown in Table I., for which the maximum velocities were determined. The observations were made at the same time as the previous values for the maximum velocity. The film was rotated to face the light when a definite potential difference was maintained between it and the cylinder and the speed of the electrometer index noted. The plate was then rotated through 180° so that the film was turned away from the light and the reading repeated. These readings for alternate emergent and incident effect were taken at 0.2 volt intervals. They could be checked by taking each curve separately.

The curves of Figs. 2 and 3 show this distribution of velocity among the electrons of films 1 and 2 in Table I., whose thicknesses are less than 10^{-7} cm. The ratio of the emergent to the incident effect throughout the whole range of potential is constant. This is significant in so far as it shows that the asymmetry for the maximum velocity can not be a spurious effect. This ratio can not be affected by any of the errors discussed above. If the emergent and incident curves are modified by the errors, they will be equally affected, so that they will not influence the ratio E/I. The measurable currents are small so that the individual readings are susceptible to errors. Several readings were taken, for each point on the curves in order to reduce the average deviation to about 3 per cent. At any potential the number of electrons emitted for the emergent position are greater then the number from the incident position. This can be accounted for by the relative larger absorption of the film when facing away from the light than when turned towards the light.

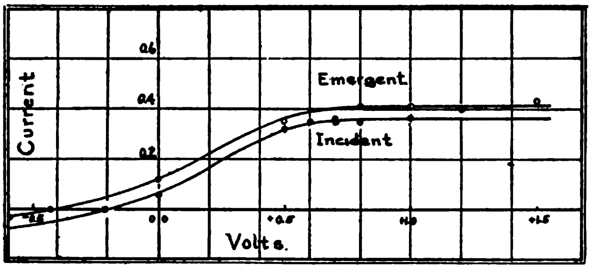

Fig. 3.

The distribution of velocity curve for film 3 is given in Table II. The emergent and incident curves are too near alike to warrant a diagram to show the difference between the effects.

TABLE II.

Distribution of Velocity Curve for Film 3.

Difference of Potential in Volts.	Emergent Current. Divisions per Second.	Incident Current. Divisions per Second.
−2.0	−0.27	−0.16
−1.2	+2.5	+1.9
−1.0	4.8	4.0
− .8	6.4	6.3
− .6	15.0	12.0
− .4	19.4	20.0
− .32	25.0	19.0
− .2	30.0	25.4
∓ .0	38.0	33.7
+ .2	42.9	40.0
+ .32	45.4	44.1
+ .4	48.4	45.4
+ .6	50.0	48.0
+ .8	57.5	52.5
+1.0	65.9	57.5
+1.2	67.1	63.4
+1.6	71.8	69.5

$$\frac{E}{I} = \frac{1.884}{1.869} = 1.01$$

The number of electrons liberated are more than one hundred times as large as those from films 1 and 2. The saturation value of the current places this film just beyond the first maximum in the current-thickness curve.[1] Robinson showed that by plotting the ratio E/I for the maximum potential against E/I for the saturation current a film was found whose thickness was such that both ratios were equal to unity. The nearest experimental approach to this critical thickness are the values cited in the above table.

Fig. 4 represents the conditions for film 4 in Table I. The units of the current axis are ten times larger than those for curves Figs. 2 and 3.

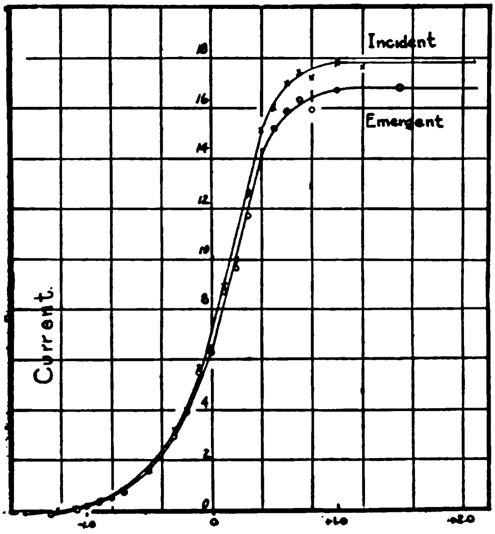

Fig. 4.

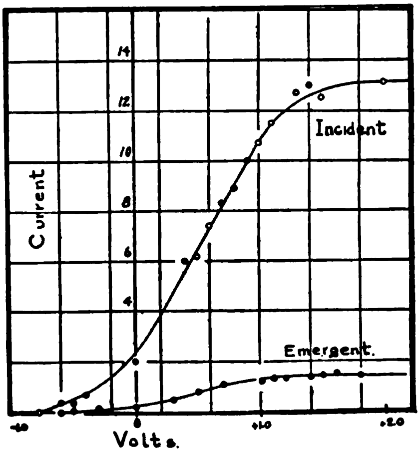

Fig. 5.

The incident velocities are now slightly larger than the emergent velocities. After this thickness the emergent velocities drop off faster than the incident, as seen in Fig. 5. This curve represents the conditions in a thick film. The emergent velocity curve is rapidly disappearing while the incident attains the normal value for platinum, equivalent to a film of infinite thickness.

DISCUSSION.

Evidently the most important results obtained are those from films whose thicknesses are less than 10^{-7} cm. Here both the ratios of emergent to incident velocity and current are greater than unity.

The current phenomena we have seen can partly be explained by Partzsch and Hallwachs' absorption curves, showing that the absorption for the emergent effect is greater than for incident effect, although the

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[1] Loc. cit.

photo-electric current seems to be out of proportion to the absorption curves.

If the current effect were due to a layer of gas or double layer at the surface of the film, then this would effect thin films and thick films alike.[1]

Due to some secondary effect there may be a change in the coefficient of absorption of these thin films large enough to account for this sudden increase in current.

A view more in agreement with recent developments on the presence of "reactionsfähige gase" affecting the photo-electric current, would be the following. The relative difference between the observed photo-electric currents and the absorption curves as determined optically by Partzsch and Hallwachs, can be accounted for by the tremendous influence that various impurities in the residual gases, in which they were deposited, have on these films. S. Werner[2] has conclusively shown that platinum films deposited in various gases, although of equal thickness, possess very different photo-electric sensitiveness. They are also more sensitive than the metal itself.

At a thickness of 10^{-7} cm. the film may have special properties of occluding or absorbing the residual gases in which they are sputtered The photo-electric effect evidently depends upon these absorbed or occluded gases carried down with the particles of metal as they pass through the residual gases. For Hallwachs and Wiedman[3] have recently shown that the photo-electric effect of potassium could be obliterated by distilling the metal in vacuo, thus removing all intermolecular occluded or absorbed gases, while Kustner,[4] Fredenhagen[5] and Paech[6] have shown the importance of the presence of such gases to produce the well-known photo-electric effect of the various metals.

Since the photo-electric current is so largely influenced by the absorbed or occluded gases it might reasonably be concluded that the velocity of emission of the electrons is affected by these same conclusions. Because the above results show that a large increase in the photo-electric current is accompanied by a relative increase in the maximum velocity of emission.

Further experiments on other metals, using monochromatic light, are now under way. These we hope will throw more light on the diffi-

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[1] O. Stuhlmann and K. T. Compton, loc. cit.

[2] S. Werner, Dissert. Uppsala, 1913; also Ark för Mat., Astron. Och. Fysik, 8, No. 27, S. 7, 1913.

[3] Wm. Hallwachs and G. Wiedmann, Ber. D. Phys. Ges., Jan., 1914, p. 107.

[4] H. Kustner, PHYS. Zs., XV (Jan.), p. 68, 1914.

[5] G. Paech., Ann. d. Phys., 43, p. 35, 1914.

[6] K. Fredenhagen, PHYS. Zs., XV (Jan.), 1914, p. 65.

culties underlying an explanation for the cause of the asymmetry in the velocities of emission.

I take great pleasure in acknowledging my indebtedness to Prof. A. W. Goodspeed, the director of the laboratory, for the generosity with which many special pieces of apparatus were purchased to complete this work.

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(To be continued.)

THE FACTORS WHICH DETERMINE THE QUANTITY OF ROENTGEN RADIATION GIVEN OFF BY AN X-RAY TUBE.

BY WHEELER P. DAVEY.

SINCE the discovery of Röntgen rays in 1895 there have been many methods proposed by which the quantity of this radiation might be measured. Physicians have almost without exception based their measurements upon the dehydrating, chemical, or photographic effects of the rays. If Röntgen rays are allowed to fall upon certain salts of the alkali metals, especially barium platino-cyanide, the crystals become dehydrated and their color is in consequence changed. The radiometers of Holzknecht (1902, 1910), Sabouraud-Noiré (1904), and Bordier (1906, 1910) are based upon this principle. They are subject to the criticism that the error of reading is great because the color change is small, that heat tends to discolor the crystals in the same manner as Röntgen rays, and that the very light which is used to detect the color change has a tendency to restore the original color. In 1903 Freund proposed a radiometer based upon the fact that Röntgen rays precipitated iodine from a mixture of iodoform and chloroform. The amount of iodine precipitated was determined by the color which it imparted to the liquid, but ordinary light was found to have the same action in discoloring the liquid as Röntgen rays. This method was improved in 1906 by Bordier and Galimard. In the same year Schwartz discovered that when a mixture of ammonium oxalate and bichloride of mercury was exposed to the action of the rays, a chemical reaction took place yielding calomel. He measured the amount of calomel precipitated by means of the amount of whitening it produced in the liquid.

In 1905 Kienböck brought out his "quantimeter" which consisted of strips of standardized photographic paper which were to be developed, after exposure to the rays, in a standard developer for a standard length of time at 18° C. By means of aluminum strips, an idea as to the penetrating ability of the rays could be gained at the same time. The quantity of Röntgen rays is determined by the amount of blackening of the paper. This method was perhaps an improvement in respect to the degree of accuracy obtainable. It is objectionable from the viewpoint of the Röntgenologist in that the development of the paper must be made under very exact conditions, and that in cases where no more than a certain quantity of radiation must be used (as in therapeutic work), the rays must be cut off while a trial reading is taken. All six of the above schemes for measuring quantity of Röntgen rays have this objectionable feature,—the accurate matching of colors or tints is difficult, and in the case of some observers impossible.

Physicists have, in the main, confined themselves to two methods of measuring Röntgen rays. If the quantities to be dealt with were rather large, they have used the Kienböck papers. If the quantities to be measured were small, or if extreme accuracy were desired, they have resorted to the more delicate method of measuring the ionization produced by the rays. In the latter case it has been customary to employ either some sort of electroscope or an ionizing chamber connected to a quadrant electrometer. The chief experimental error in the ionization method has been due to "natural leak," *i. e.*, spontaneous ionization in the air. At certain seasons of the year, this source of error becomes so large that ionization measurements are almost impossible.

It would simplify the measurement for many purposes if the quantity of Röntgen rays could be measured in terms of the current through the tube, the potential difference across it, and the inverse square of the distance from the tube. The advantage of such a method would be that readings would be unaffected by climatic conditions, by the temperature or humidity of the room, by the kind or intensity of the illumination in the room, or by the sensitiveness of the observer's eye for small changes of tint or color. The accuracy of reading would be high. Further, readings could be taken while the tube is running, so that a definite quantity of rays could be obtained without the inconvenience and delay required by "trial readings." It would be impossible to compare, *in terms of voltmeter and millimeter readings*, results obtained on different outfits. This could be overcome by calibrating each outfit in terms of Kienböck or other units which can be measured by the direct action of the rays themselves. This result could then be reduced by means of the inverse square law to the quantity obtainable at a meter's distance.

In the early days such a method was proposed, but the current to be measured from a static machine or from induction coils of the size then in use, was very small, and the order of accuracy was therefore small. This coupled with the imperfections of the tubes of the time caused the method to fall into disuse. In 1904, Cardini[1] investigated this method of measurement and found, using a Holtz machine and a system of con-

[1] P. Cardini, N. Cimento, 7, pp. 241–259, Apr. 1904.

densers in series, that the quantity of Röntgen rays, as measured by ionization, was proportional to the energy expended between the terminals of the tube. In 1909, DeBeaujeu[1] reported a similar result, using currents not to exceed 1.5 M.A. Since the advent of modern high power generating outfits for X-ray work there has been considerable doubt as to whether Cardini's law is of universal application or whether it is merely a close approximation for a tube at small power input. The present work was undertaken to investigate this point, and, in addition, to find out whether the state of vacuum inside the tube was able to affect the quantity of Röntgen rays given off except in so far as it might help to determine the current at a given potential difference.

APPARATUS.

The generating plant consisted of a commutator transformer machine ("interrupterless machine") made by the Kny Scheerer Co. The secondary had a resistance of 113,000 ohms. The primary had a resistance of .265 ohm, and was operated on a 220-volt circuit. The E.M.F. of the secondary was controlled by inserting known resistances in the primary circuit by means of a rheostat. The commutator made use of 140° out of each 180° of the electric wave from the secondary of the transformer.

Two tubes were used, both of which had been pumped for use on a transformer by Green and Bauer. The first was a platinum target tube of standard type which had been in use for some time. It was in a nearly "stable" condition, softening a little on running and hardening after running. The other was a standard type of tube having a tungsten target and G & B cathode. It was in such a condition as to harden while running for all exposures made, except those above 20 M.A. In all cases, the tube hardened after running.

Current was measured by a moving coil milliammeter. Potential difference across the tube was measured by radiographs of a Benoist penetrometer[2] on Schleussner X-ray plates and reduced to readings on an electrostatic voltmeter, and to approximate readings on the equivalent spark gap between cones whose bases were 9 mm. in diameter and whose altitudes were 12 mm.

The quantity of X-rays was measured by the ionization in a zero-reading electroscope especially constructed so as to compensate in large measure for "natural leak." It consisted of a block of amber A sup-

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[1] J. De Beaujeu, Ann. d' Electrobiol., Nov. 1909.

[2] The penetrating ability of an X-ray beam is expressed on the Benoist scale in terms of the thickness in millimeters of aluminium which offers the same opacity to the rays as is offered by a sheet of silver .11 mm. thick.

porting four squares of aluminum, B_1, B_2, B_3. B_4, B_1 and B_4 were permanently connected to a storage battery, giving a potential of 400 volts. D was a gold leaf which was permanently grounded throughout the experiment. A charge upon the gold leaf increased or decreased the sensitiveness of the instrument, depending upon the sign of that charge, so that if it had been desirable, the sensitiveness of the electroscope could have been altered by connecting D to some source of potential. The whole was completely screened from electrostatic influences and was behind a lead screen of sufficient thickness to cut out all the rays from the tube. The only radiations reaching the electroscope were the parallel rays entering as shown in Fig. 1 through a lead pipe, having a cylindrical hole $\frac{3}{8}$ inch in diameter. The distance from the target of the tube to the ionizing chamber was 19 inches.

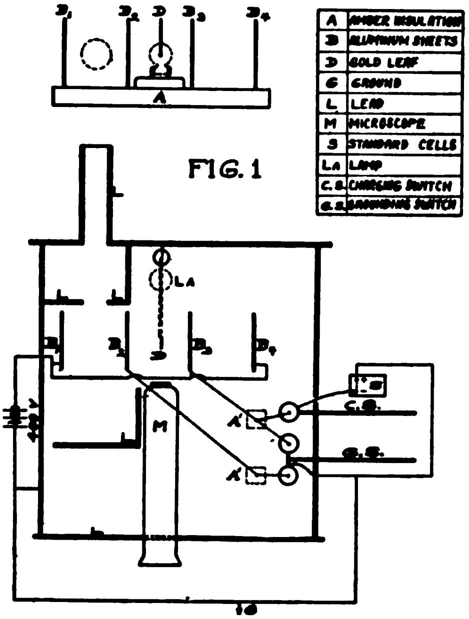

Fig. 1.

Procedure.

The X-ray tube was brought into such a condition that, when run with 35.60 ohms in series with the primary of the transformer, the milliammeter showed a current of some predetermined amount. The rheostat was then adjusted so as to put whatever resistance was desired in series with the primary. Plates B_2 and B_3 were grounded and immediately insulated from each other and the ground. The tube was operated for a predetermined interval of time. Plate B_3 was then charged by means of the standard cells until the gold leaf D came back to its original position. The amount of ionization produced by the rays was thus expressed in terms of the potential on B_3 necessary to cause a zero reading of the gold leaf. Any leak from B_1 to B_2 due to spontaneous ionization was almost entirely balanced by the corresponding leak from B_4 to B_3. Except for the few seconds after the tube was started in operation and before B_3 was charged, the leak from B_2 to D was almost entirely balanced by the leak from B_3 to D. The entire suppression of "natural leak" is impossible, due to unavoidable slight asymmetry of the apparatus. But it was reduced to an amount so small as to be entirely negligible especially if the time of exposure to the rays is less than five

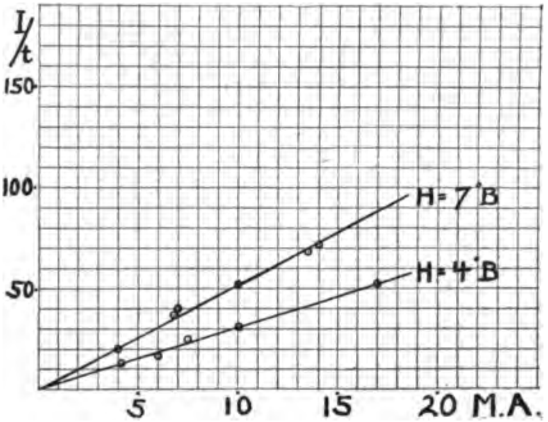

Fig. 2.

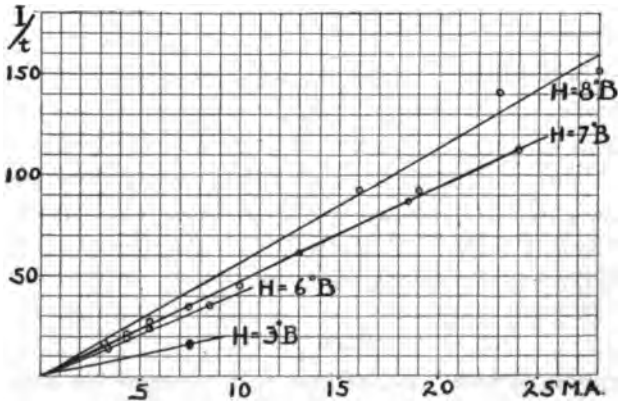

Fig. 3.

minutes. For exposures up to half an hour, the error due to natural leak was found to be so small as to be negligible for most purposes.

It was found that a potential difference of 300 V was necessary between B_1 and B_2, so as to insure saturation current from the strongest source of ionization used in the work. Care was therefore taken to so limit the time of exposure, that B_2 was never charged up to 100 V, thus leaving potential difference between B_1 and B_2 always in excess of 300 V. For this reason, the maximum current used through the tube was 28 M.A., as the time of exposure for larger currents was too short to make accurate readings on the milliammeter.

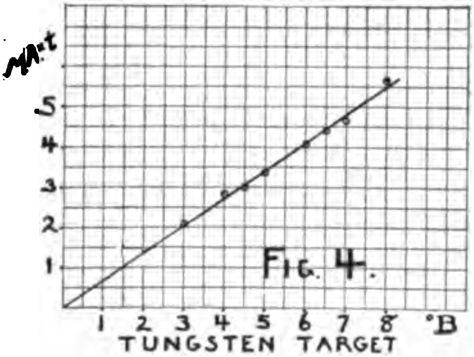

Fig. 4.

In the case of the platinum target tube, times were measured by means of an automatic time switch connected to the primary circuit of the transformer. This time switch gave only arbitrary time-units, and worked fairly well during the time of the experiment on the platinum

TABLE I.

Hardness in °B.	P.D. as Measured by Electrostatic Voltmeter.	Cm. Parallel Spark-Gap.	Condition of Tube.	Ohms Resistance in Series with Primary of Transformer.	Current in M.A.	Ionization.	Time in Arbitrary Units.	Ionization per Unit of Time.	Ionization per Unit of Time per M.A.	Average.	Ionization per Unit of Time per M.A. per °B.
4	25,000	6.6	B	35.60	4.1	90	7	12.8	3.1		
			B	29.10	6.0	85	5	17.0	2.8		
			B	23.70	7.5	85	3½	24.3	3.2		
			B	18.90	10.0	86	2¾	31.2	3.1		
			B	11.97	17.0	80	1½	53.3	3.1	3.1	.77
6	43,000	9.9	B	35.60	3.0	95	7	13.6	4.5		
			B	23.70	5.5	90	3½	25.7	4.7	4.6	.76
7	52,000	11.6	A	29.10	4.0	90	4½	20.0	5.0		
			A	18.90	6.8	92	2½	36.8	5.4		
			A	18.90	7.0	90	2¼	40.0	5.7		
			A	15.10	10.0	92	1¾	52.6	5.2		
			A	11.97	13.5	86	1¼	68.8	5.1		
			A	11.97	14.0	90	1¼	72.0	5.1	5.2	.74

Average ionization per unit of time per M.A. per ° B...... .76
Mean deviation from average.................. 2 per cent.

target tube. But shortly afterwards, it became irregular in action, so that time on it was only accurate to 10 per cent. For this reason, when using the tungsten target tube, times were measured on a chronograph, whose circuit was operated by the same switch that controlled the primary current of the transformer.

RESULTS.

Platinum Target Tube.—Before each reading was taken, the tube was under one of two conditions of vacuum so as to take (*A*) 3.4 M.A. and (*B*) 4.2 M.A. when 35.60 ohms were in series with the primary of the transformer.

Tungsten Target Tube.—Before each reading was taken, the tube was

TABLE II.

Hardness in °B.	P.D. Across Tube Measured by E.S. Voltmeter.	Cm. Parallel Spark-Gap.	Condition of Tube.	Ohms Resistance in Series with Primary of Transformer.	Current in M.A.	Ionization.	Time in Seconds.	Ionization per Second.	Ionization per Second per M.A.	Average.	Ionization per Second per M.A. per °B.
3	16,000	5.0	E	23.70	7.5	80	4.90	16.3	2.1		
			E	23.70	7.5	80	5.00	16.0	2.1	2.1	.70
4	25,000	6.6	D	35.60	3.8	81	7.40	10.9	2.8	2.8	.70
4½	29,500	7.4	D	29.10	3.8	80	6.95	11.5	3.0	3.0	.67
5	34,000	8.3	D	29.10	4.5	85	5.45	15.5	3.4	3.4	.68
6	43,000	9.9	C	35.60	3.4	85	6.20	13.7	4.0		
			C	29.10	4.3	74	4.10	18.0	4.2		
			C	29.10	4.3	80	4.40	18.2	4.2		
			C	23.70	5.5	80	3.40	23.5	4.3		
			D	18.90	8.5	70	2.00	35.0	4.1		
			D	18.90	8.5	92	2.65	34.7	4.1	4.15	.68
7	52,000	11.6	C	35.60	3.2	75	4.90	15.3	4.8		
			C	29.10	4.3	71	3.40	20.8	4.8		
			C	29.10	4.3	71	3.50	20.3	4.7		
			C	23.70	5.5	85	3.10	27.4	4.9		
			C	18.90	7.5	85	2.45	34.7	4.6		
			C	15.10	10.0	90	2.00	45.0	4.5		
			C	11.97	13.0	92	1.50	61.3	4.7		
			D	9.40	18.5	87	1.00	87.0	4.7		
			D	9.40	19.0	72	.77	93.5	4.9		
			D	7.16	24.0	80	.71	112.6	4.7	4.7	.67
8	61,000	13.2	C	9.40	16.0	85	.90	93.4	5.8		
			C	7.16	23.0	96	.68	141.2	5.9		
			C	5.40	28.0	91	.60	151.6	5.4	5.7	.71

Average ionization per second per M.A. per ° B. .... .69
Mean deviation from average .... 2 per cent.

put under one of three conditions of vacuum, so as to take (*C*) 3.4 M.A., (*D*) 3.8 M.A., (*E*) 4.4 M.A. when 35.60 ohms were in series with the primary of the transformer.

The last column of both tables might equally as well have been expressed in terms of $\frac{\text{ionization}}{\text{time} \times \text{M.A.} \times \text{voltage}}$ or $\frac{\text{ionization}}{\text{time} \times \text{M.A.} \times \text{spark gap}}$ but the three are in direct proportion within the range used. In any case the quotients have no absolute value, as the measurements of ionization are only relative, the constant of the electroscope entering into all the results. An attempt was made to calibrate the electroscope by means of the γ-rays from a known quantity of radium, but was unsuccessful due to the difficulty of completely screening the plates *E* and *F* from the very penetrating γ-rays.

SUMMARY AND CONCLUSIONS.

1. An electroscope designed as shown above is practically free from the error due to natural leak when used under ordinary laboratory conditions. If occasion should warrant, it would seem possible to develop such an instrument for use under less favorable conditions than are usually found in laboratory practice.

2. With a constant potential difference across the terminals of the tube (*i. e.*, constant "hardness") the ionization per second (*i. e.*, the quantity of X-rays per second) is directly proportional to the first power of the current through the tube.

3. Except in so far as the state of vacuum in the tube helps to determine the current at a given voltage drop, it has no effect upon the quantity of radiation given off, at least within the ordinary working limits of a standard tube.

4. For rays from a given tube, operated by a given source of E.M.F., the quantity of radiation is directly proportional to the energy consumed in the tube. It is therefore only necessary for any given generating outfit to measure once for all by means of Kienböck paper, or other fairly reliable method, the quantity of Röntgen rays delivered at a meter's distance per kilovolt per milliampere per second. Thereafter all measurements of Röntgen-rays produced with that outfit may be made in terms of the energy (kilovolt-milliampere-seconds) per (meter),^{2} sent through the tube, and translated in terms of Kienböck, Sabouraud or other accepted untis for purposes of comparison with data obtained on other outfits.

A part of the expense of this work was paid from a grant from the

Huntington fund to Prof. J. S. Shearer to further the work of standardization of Röntgen ray measurements for therapeutic purposes.

It is a pleasure to acknowledge my obligation to this source and to express my deepest thanks to Prof. E. L. Nichols and Prof. J. S. Shearer for their interest and hearty cooperation, and to my wife for her constant aid during the course of the work.

DEPARTMENT OF PHYSICS,
CORNELL UNIVERSITY.

THE MEAN DEPTH OF FORMATION OF X-RAYS IN A PLATINUM TARGET.

By Laura G. Davey.

W. R. HAM has shown[1] that X-rays are formed on an average, not on the surface of a target, but some little distance inside, this distance being called "the mean depth of formation" of X-rays. He was the first to measure this distance for any kind of target, using for his measurements a lead target.

The theory was as follows: If the normal to the target is not midway between the electroscopes (Fig. 1) the rays from a given point in the target have to pass through a greater thickness of the target in going to one electroscope than to the other. Let O (Fig. 1) be the source of an ether-pulse so situated as to be at the mean depth at which the X-rays originate. Let $l_1 - l_2 = x =$ the excess of target the pulse has to traverse in going to R over that which it traverses in going to L; $d =$ the distance from O to the point at which the cathode ray particle enters the target; $h =$ the perpendicular distance from O to the surface of the target; $\theta =$ the angle which the normal to the target makes with the cathode stream; $\theta_0 =$ the angle which the lines from the electroscopes to the target make with the cathode stream. Then

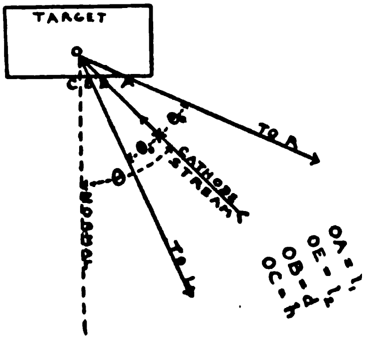

Fig. 1.

$$x = h\left\{\frac{1}{\cos(\theta + \theta_0)} - \frac{1}{\cos(\theta - \theta_0)}\right\}.$$

But

$$h = d \cos\theta.$$

Therefore

$$x = d\cos\theta\left\{\frac{1}{\cos(\theta + \theta_0)} - \frac{1}{\cos(\theta - \theta_0)}\right\}$$

or

$$x = d \cdot F(\theta, \theta_0).$$

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[1] W. R. Ham, Phys. Rev., XXX., 1, Jan., 1910.

Let I_L = intensity of X-rays directed towards L,
I_R = intensity of X-rays directed towards R.
Then

$$I_R = I_L \epsilon^{-\lambda x}$$

where x is defined as above and λ is the coefficient of absorption found experimentally for the voltage used.

Ham has found that d for lead is 4.2×10^{-5} cm. at 14,000 volts and 6.4×10^{-5} cm. at 21,300 volts, thus proving that the mean depth of formation of X-rays is directly proportional to the potential difference across the tube. Ham's work was done with a specially constructed tube made from a bell-jar with windows of a uniform thickness inserted at the proper angles. The target was pivoted and was moved magnetically from the outside. The tube was attached to a Holtz pump during the entire time of taking readings so that there was always mercury vapor present. The means of excitation was a Holtz machine.

About a year later with the same apparatus W. P. Davey[1] measured the mean depth of formation in a silver target and found it to be 5.4×10^{-5} cm. at 10,000 volts and 9.2×10^{-5} cm. at 17,000 volts, thus confirming the fact that the mean depth of formation of X-rays is directly proportional to the potential difference across the tube.

It seemed that it would prove interesting to repeat the experiment with a standard platinum-target tube excited by a commercial "interrupterless" (transformer) machine. The thickness of the glass of various X-ray tubes was investigated and as long as the measurements were made on an arc whose plane was perpendicular to the axis of the tube the variations in thickness were negligible. This was to have been expected from the mechanics of glass blowing.

An attempt was first made to use ionizing chambers connected to a quadrant electrometer, thus using a null method much like the one of Ham, Lassalle and Smith[2] but this proved unsuccessful because of the disturbances due to the powerful variable static field of the transformer circuit. An arrangement was then set up as follows:

T (Fig. 2) is a platinum-target tube mounted so that it can be rotated through 180° about the axis *OO*. The target is set at an angle of 45° to the cathode stream.

SS and *KK* are thick lead screens.

CC are lead curtains over the apertures in *KK* which can be lowered to cut off direct radiation or raised to allow radiation to pass through.

BB are lead boxes serving to cut off secondary rays from the electro-

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[1] W. P. Davey, Journal of the Franklin Inst., March, 1911.

[2] Ham, Lassalle and Smith, Jour. Franklin Inst., July, 1911.

scopes L and R, and (being permanently grounded) also acting as static shields.

L and R are set at an angle of 20° to the cathode stream. The amount of discharge of the electroscope is read by the microscopes MM.

The target was first turned so as to have its plane vertical and facing towards the electroscope L. Then the tube was brought into a definite condition (6° Benoist[1]) such that the electrostatic voltmeter V registered 43,000 volts. The electroscopes were charged to 200 volts. Then the tube was run approximately four seconds with the holes in KK closed by the curtains CC, thus giving the amount of discharge caused by spontaneous ionization and secondary radiation. Then the curtains were raised and the amount of discharge due to spontaneous ionization, secondary radiation, and primary radiation was observed. From these two readings the amount of primary radiation was computed. Then the tube was rotated 180° and readings were again taken both with curtains down and up. Lastly a piece of platinum .00269 cm. thick was inserted in the path of the rays going toward R and the above observations were repeated.

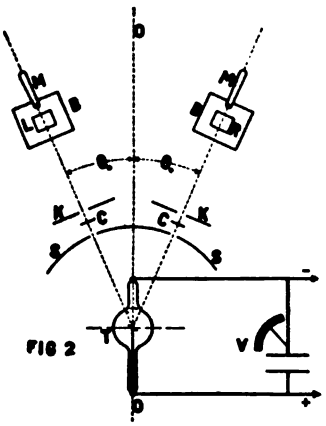

Fig. 2.

Table I.

To Find λ at 6° B.

	R	L	R Corrected for Natural Leak and Secondary Radiation.	L Corrected for Natural Leak and Secondary Radiation.
Without Pt....	4.40	16.9	3.89	14.9
With Pt.......	1.51	16.0	1.00	14.0

In order to avoid inaccuracy due to inequalities in times of exposure the following method of treating the data was used. Without any platinum,

$$Lm = R,$$

where m is the factor of proportionality. From the above data,

$$m = \frac{389}{1490}.$$

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[1] The penetrating ability of an X-ray beam is expressed in the Benoist scale in terms of the thickness in millimeters of Al which offers the same opacity to the rays as is offered by a sheet of Ag .11 mm. thick.

Therefore during the time the run was made with the platinum, R would have deflected

$$14m = \frac{339 \times 14}{1490} = 3.65$$

if there had been no platinum. But the platinum reduced the reading to 1.00.

Now if

$$I_0 = \text{intensity of X-rays without Pt.} = 3.65,$$

and if

$$I = \text{intensity of X-rays with Pt.} = 1.00,$$

and if

$$x = \text{thickness of the Pt.} = .00269 \text{ cm.},$$

then

$$I = I_0 \epsilon^{-x\lambda}$$

becomes

$$1.00 = 3.65 \epsilon^{-.00269\lambda}$$

at 6° Benoist, and

$$\lambda = 481.$$

TO FIND THE MEAN DEPTH OF FORMATION OF X-RAYS INSIDE THE PLATINUM TARGET.

R was calibrated in terms of L as follows: If R and L were exposed to exactly the same radiation then the deflection of R might be expressed in terms of the deflection of L thus:

$$Rk = L.$$

Let R_1 = the reading of R when the target faces R,
L_1 = the corresponding reading of L,
R_2 = the reading of R when the target faces L,
L_2 = the corresponding reading of L.

Then,

$$\frac{R_1 k}{L_1} = \frac{L_2}{R_2 k},$$

from which

$$k = \sqrt{\frac{L_1 L_2}{R_1 R_2}}.$$

TABLE II.

	R	L	R Corrected for Natural Leak and Secondary Radiation.	L Corrected for Natural Leak and Secondary Radiation.
Target facing L...	4.05	20.1	$3.79 = R_2$	$17.7 = L_2$
Target facing R...	4.40	16.9	$3.89 = R_1$	$14.9 = L_1$

Whence

$$k = 4.23.$$

Therefore,

$$kR_1 = 4.23 \times 3.89 = 16.45$$

and

$$kR_2 = 4.23 \times 3.79 = 16.03.$$

We may therefore write,

TABLE III.

	I_L	$I_R = kR_2$	$I_R = kR_1$
When target faces L......	17.7	16.03	
When target faces R......	14.9		16.45

Now when the target faces L,

$$I_R = I_L\epsilon^{-\lambda x}$$

and when the target faces R,

$$I_L = I_R\epsilon^{-\lambda x}.$$

Hence,

$$\epsilon^{\lambda x} = \frac{17.7}{16.03} = \frac{16.45}{14.9} = 1.10$$

and

$$x = .00020- \text{ cm.},$$

$$d = \frac{x}{.891} = .00022 \text{ cm. at } 6^\circ \text{ Benoist.}$$

Since it has been shown[1] that the depth of formation of X-rays in a target is directly proportional to the P.D. across the tube, and since this is directly proportional to the "hardness" of the tube in ° Benoist between the limits of 3° B. and 8° B., it follows that the depth of formation of X-rays in a platinum target in a tube of hardness H° B. is

$$\frac{H}{6} \times .00022 \text{ cm.} = .00004\, H \text{ cm. nearly.}$$

I wish to express my appreciation to Prof. E. L. Nichols and to Prof. J. S. Shearer for their many kindnesses throughout the course of the work and to my husband for aid in taking data.

DEPARTMENT OF PHYSICS,
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[1] W. R. Ham, loc. cit.; W. P. Davey, loc. cit.

A NEW DESIGN OF FLICKER PHOTOMETER FOR LABORATORY COLORED LIGHT PHOTOMETRY.

BY HERBERT E. IVES AND EDW. J. BRADY.

IT is now rather generally agreed that the difficulties of colored light photometry are best met in practice by eliminating the color differences through the medium of colored absorbing screens or auxiliary standards of different colors. Given the colored absorbing screens or other means the next matter is to determine their photometric transmissions or candle power values. In this determination all the difficulties of heterochromatic photometry must be faced, and a choice of instruments and photometric methods must be made. Later such choice must be made the subject of general agreement among the standardizing laboratories in whose hands such calibrating work should rest.

From a previous study of the subject[1] the conclusion has been drawn that the most desirable instrument for this purpose is the flicker photometer, but with the reservation that certain conditions of illumination and field-size must be adhered to and that the average results of many observers must be taken. Reference should be made to the original papers for the details of the work leading to these conclusions. The present paper describes a design of flicker photometer intended for the class of work mentioned and planned to be used in the manner specified.

Since a number of points of importance in the design of any flicker photometer were learned in the previous study, attention is here at the outset given to an account of these, as preliminary to the description of how these points have been provided for in the new instrument.

The attainment of high sensibility in a flicker photometer is dependent upon careful attention to several factors:

First, the dividing line between the two fields which are alternated should be a line without breadth,

Second, the speed of alternation should be accurately adjustable, and

Third, it should be possible to vary the relative brightness of the two fields quickly over a wide range with a minimum of physical effort.

The first point is quite as important in the flicker photometer as in the

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[1] Philosophical Magazine, July, Sept., Nov., Dec., 1912, "Studies in the Photometry of Lights of Different Colors," Herbert E. Ives.

ordinary equality or contrast type. A thick dividing line in the latter instrument reduces the sensibility. In the flicker photometer a black line produces flicker of itself, necessitating a certain speed of alternation merely to make this mechanical imperfection disappear. As noted below, an unnecessary increase of speed decreases sensibility.

The question of speed in a flicker photometer is important, although the emphasis has often been laid on a mistaken idea. The speed determines the sensibility, and the speed necessary in any particular case is determined by the color difference. The speed does not affect the absolute values obtained, as has been thought by some, but it does change the accuracy of setting. For illustration, in Fig. 1 let AB be the photometer bar on which a flicker head is movable, having at A and B lights of equal candle power. If the speed of alternation is high enough (say, 70 or 80 alternations per second) the photometer head may be placed anywhere on the bar and no flicker can be perceived; this condition is represented by the line *a–a*. If the speed is now dropped to a certain lower value, it is found that at the ends of the bar flicker is visible, but for a long space in the center one point of setting is as good as another, (*b–b*). At each successive decrease of speed the region of uncertainty is decreased, hence the sensibility is increased. If the two lights are different in color this decrease of speed may be carried only so far as no perception of color alternation appears. The greater the color difference the higher must be the speed to prevent "color flicker."

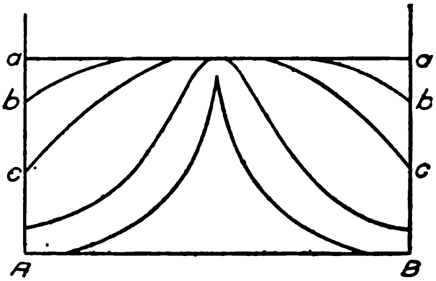

Fig. 1.

Graphical Representation of Speed-Sensibility Relations in the Flicker Photometer.

An interesting consequence follows from these considerations, namely, that with no color difference the speed demanded is zero, in other words, that the flicker photometer degenerates into an equality photometer in its limiting case where no color differences exist to call for the fusion due to alternation. Instructions which are sometimes given to the effect that the "correct" speed is to be determined with two lights of the same color and this speed then used with differently colored lights, are misleading, unless the speed called for by a bad dividing line is higher than that demanded by the color difference.

With regard to the third point, it is to be remembered that the position of match is that of minimum of flicker. Were the eye to maintain its capacity to perceive flicker absolutely constant from moment to moment, to remain absolutely fixed in position, and were it possible to maintain the speed of alternation exactly the same, then the point of disappearance of flicker might be sought and worked up to slowly from each side. But in practice it is always best to adjust the speed until a very sharp minimum of flicker is produced. In finding a maximum or minimum point it is useless to approach from one side. Instead a series of oscillations about this point is necessary, the oscillations constantly decreasing in amplitude until the slightest touch either way increases the flicker. In order to produce these oscillations of brightness best, they should be producible by the easy motion of the hand or fingers. If a motion of the body is necessary, the eye also is apt to be moved from the photometer eye-piece and this movement, even when small, is seriously disturbing. It causes, beside the disturbance of attention, an increase of flicker just as a quickly moving object reveals the fluctuating character of an otherwise apparently steady light. Obviously a flicker photometer head should be of the stationary and not of the movable type.

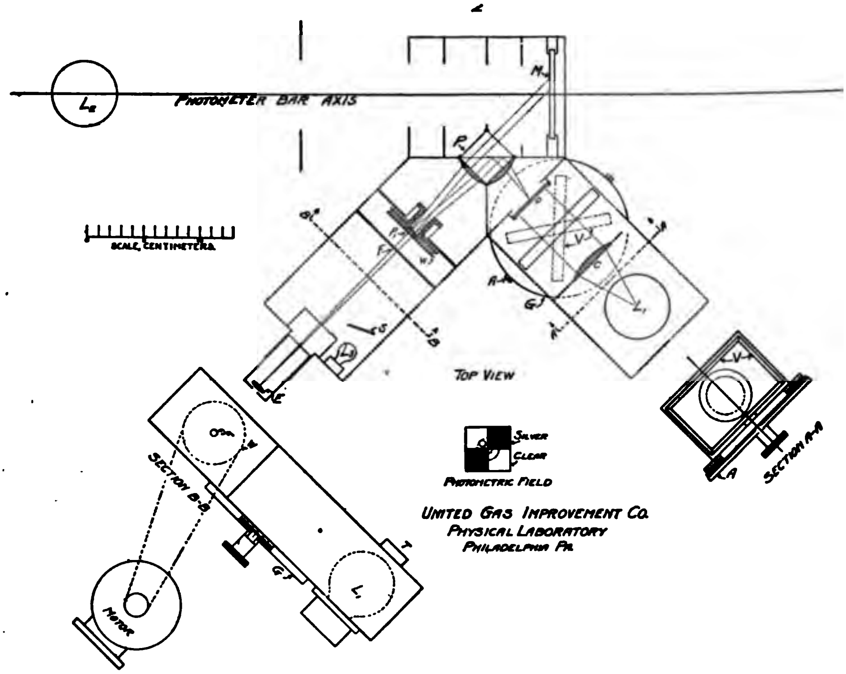

Fig. 2.

The instrument is best described by reference to Fig. 2, which gives plan and sections. The photometer is planned to be used solely by the substitution method, so that while the test and standard light sources are carried successively at appropriate places on the photometer bar, the comparison lamp is held at a fixed position in the photometer body. Tracing the paths of the beams of light which ultimately reach the eye will be the easiest way to understand the photometer's behavior. The light from the standard lamp (L_2) falls upon the matt white surface (M); thence a portion is reflected through the cube (P) and the small prism (P_1) to the eye. The light from the comparison lamp (L_1) passes through the lens (C); thence through the variable neutral-tint screen (V) to the flashed opal glass (O). Part of the diffused light of the latter enters the cube (P), is reflected through the small prism (P_1) to the eye-piece and eye at E.

The mechanism for producing the alternation of the two illuminations is comprised in the cube (P) and the prism (P_1). The cube is a modified Lummer-Brodhun. One of the component prisms is left untouched; the other is silvered and the silvering cut away in the manner shown in the separate sketch of the photometric field. In the present case the silver was cut away with a sand blast, after which the two prisms were cemented together with Canada Balsam, which practically eliminated the diffusing roughness of its surface originally caused by the sand-blast. The slight selective coloring, due to the silver, is of course of no importance, because of the substitution method employed. The small prism (P_1), having an angle of 10°, is mounted in a collar (W), which is rotated by the motor to which it is connected by a belt. The plate (F), perforated by a central round opening of 2° diameter, as viewed from the eye-piece, limits the actual area of the prism used. When the motor is running the beam of light which reaches the eye is, as it were, whirled around the axis of the instrument, alternating passing over the clear and the silvered parts of the cube (P). The successive appearance of the openings at F are readily seen from the separate sketch of the photometric field, where the small circle represents the section of the beam coming to the eye from F. The practically invisible dividing line between the silvered and the clear portion of the cube and the smooth continuous motion of the prism (P_1) insures the first condition considered above.

The second condition above dwelt upon is secured by using a series wound motor with a sliding variable resistance. In conjunction with the latter, sockets are provided in the base of the instrument for the insertion of various fixed resistances; these, together with the variable resistance, provide for any desired speed.

The third desirable condition is that in regard to easy variation of the relative illumination. This is here secured by the use of the variable neutral tint screen (V). This consists of two opaque line gratings on glass slightly separated from each other. When rotated about an axis parallel to the lines the transmission[1] varies with a rapidity determined by the separation of the glasses. This screen is rotated by a milled head which also carries the pointer (A). The size of the milled head is such that it may be turned easily by the fingers. Small lights to illuminate the scale are also provided, but are not shown in the sketch, and the scale may be read either from the observer's side or else by an assistant viewing the other end of the pointer. The lens (C) is intended to help make the illumination on (O) uniform, and as a further refinement the cube (P) is furnished with a convex lens focused on one spot of the opal glass (O). A calibration curve of the screen in this particular instrument is shown in Fig. 3. It is practically a straight line over all the region which would be used, remembering that it is intended to have all measurements made at a single brightness (corresponding to 25 meter candles on a white surface), depending upon the variable screen merely for small ranges.

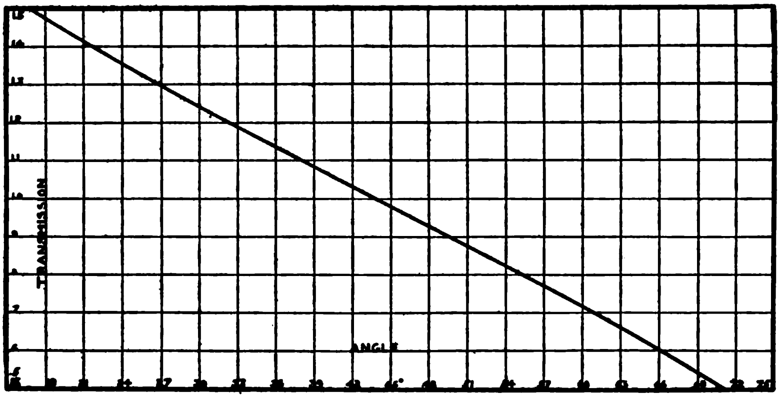

Fig. 3.

Calibration Curve of Variable Neutral Tint Screen.

A novelty in this instrument is the means for illuminating the surroundings of the photometric field to nearly the same brightness, furnished by the small lamp (L_3) which throws its light on the white walls of the compartment facing F. S is a small translucent screen for securing uniform illumination. This bright surrounding field materially contributes to the comfort of reading.

Mechanical features of importance are the ventilating openings, of which one is illustrated at T, and the care taken to make every optical part immediately removable for cleaning. This latter is a point often neglected by instrument makers.

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[1] "A Variable Neutral Tint Screen," H. E. Ives, Electrical World, Mar. 16, 1912.

The photometer in use thus far has proved itself admirably sensitive. A recent set of readings by two observers furnished the following figures, which are representative: With a very slight color difference between the comparison and test (incandescent electric) lamp the mean variation from the mean of ten settings by observer "A" was .38 per cent. and by observer "B" .45 per cent. With the test light a nearly monochromatic green (equivalent wave-length .546μ), the corresponding figures were .84 per cent. and .67 per cent. A set of seven readings on the same light source, the apparatus having been disturbed for other readings in between, showed an extreme variation from the mean of less than one per cent.

The instrument is now being used to calibrate certain absorbing solutions which have been developed to eliminate the color difference between the more important artificial illuminants and the ordinary carbon lamp standards.

Physical Laboratory, The United Gas Improvement Company,
May, 1912.

A STUDY OF CONTACT POTENTIALS AND PHOTO-ELECTRIC PROPERTIES OF METALS IN VACUO: AND THE MUTUAL RELATION BETWEEN THESE PHENOMENA.

By Albert E. Hennings.

THREE years ago Professor Millikan suggested as a promising subject for research, the careful study of the contact potential differences between metals in vacuo under conditions which would eliminate the effects of surface films. He considered it imperative that such a study be undertaken both because of the light which it ought to throw on the true nature of contact potentials and because his own photo-electric studies had produced evidence that the positive potentials acquired by metals under the influence of ultra-violet light were very probably related in some definite, but not yet fully understood way, to the contact potential differences between the metals. After carrying on preliminary experiments for a year or more with different types of tubes, the writer decided upon the experimental arrangement and designed the apparatus which have been used in the work herein reported.

The plan in brief was to test, simultaneously and in vacuo, the contact potentials and photo-electric properties of eight of the ordinary metals not only under conditions which have usually obtained in the study of either of these phenomena, viz., with surfaces prepared in air and subsequently surrounded by a vacuum, but also with surfaces mechanically prepared in a vacuum which was the best obtainable and which was maintained throughout all operations, and finally to observe the time changes in contact potential differences and photosensitiveness of photo-electrically or mechanically treated surfaces.

So far as I am aware no experiments have been made heretofore on the contact potential differences between metal surfaces freed from gas or oxide films. The chemical theory of contact potentials makes the whole effect depend essentially upon the existence of such films. Hence the significance of experiments of this kind upon the theory of contact potentials. Photo-electric experiments on clean surfaces in vacuo, however, have already been made and described by Pohl and Pringsheim,[1]

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[1] Verh. d. Deutch Phys. Ges., 13, 474, 1911; 14, 46, 506, 546, 1912.

and Hughes,[1] using freshly distilled surfaces and by Herrmann,[2] Richardson and Compton,[3] and Page,[4] using mechanically cleaned surfaces.

The Results.

The facts which may be considered as having been definitely established by the present investigation are the following:

1. The contact potentials of the metals are not noticeably changed by differences in pressure ranging from atmospheric pressure to that of the best vacuum obtained.

2. All the metals are rendered more electro-positive when polished in vacuo and gradually become more electro-negative just as do freshly polished surfaces in air.

3. Prolonged illumination with ultra-violet light greatly modifies the contact potentials of film coated surfaces, and whatever the direction or magnitude of the change may be each metal exhibits a tendency to recover its original potential when the action of the light has ceased. With newly prepared surfaces these changes are negligible or inappreciable.

4. On the photo-electric side the results as a whole confirm the point of view adopted by Richardson and Compton in regard to the relation between photo- and contact potentials.

5. The photo-sensitiveness of all the metals is very largely increased when the surface films, which in general always exist even in the best vacuum unless the metals have been subjected to special treatment as described below, are removed.

6. Although the order of the metals when arranged according to their photosensitiveness in vacuo is usually not at all that of the Volta contact series, the order assumed by freshly cleaned metals approaches very closely that of this series.

Description of Apparatus and Outline of Experimental Procedure.

The apparatus designed for this study is enclosed in a glass bulb about 13 cm. in diameter with five projecting arms—four being in the horizontal plane and the fifth occupying a vertical position—the axes of all meeting at the center of the bulb. Fig. 1 represents a horizontal section in the plane of the intersecting axes of the two pairs of projecting arms and Fig. 2 represents a vertical section in the plane including the vertical

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[1] Roy. Soc., Phil. Trans., 212, pp. 205–226.

[2] Verh. d. Deutch Phys. Ges., 14, 557, 1912.

[3] Phil. Mag., 24, pp. 575–594.

[4] Amer. Jour. Sci., 36, 501, 1913.

axis and that of one of the pairs of projecting arms. The horizontal arms are 42 to 45 mm. in diameter.

Eight metals, magnesium, aluminum, zinc, tin, iron, brass, copper, silver, in the form of disks 2 cm. in diameter are mounted upon an amber hub 38 mm. in diameter with the planes of their faces parallel to the axis of the wheel so formed. To one end of the rod passing through the hub and supporting it by means of suitable bearings *b*, *b*, in brass tubes

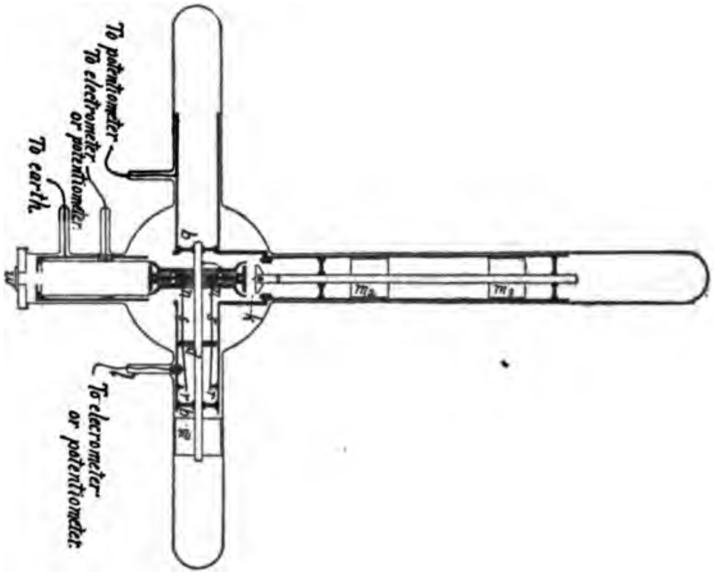

Fig. 1.

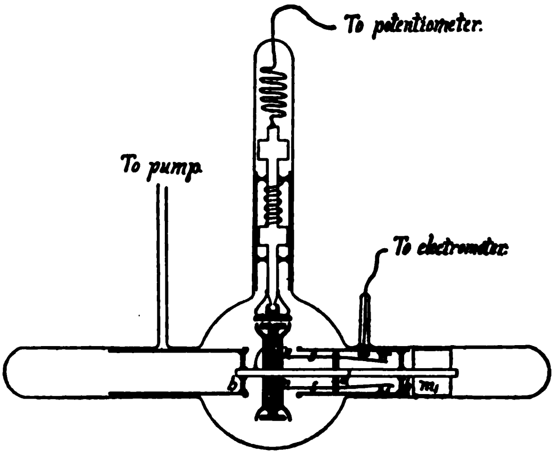

Fig. 2.

Fig. 2*a*.

ALBERT E. HENNINGS.

joined rigidly together and fitted firmly in the projecting arms of the bulb is attached a double anchor-shaped piece of iron, m_1. By means of an external electro-magnet the wheel may be rotated and if necessary moved a short distance along the axis, making it possible to adjust very accurately the position of any of the eight metal disks.

The longest of the projecting arms is fitted with a brass tube which is attached to, and holds rigidly together, the brass tubes which support the axle of the wheel. This tube serves also to give support to the bearings for a rod carrying at its inner end a well-tempered wedge-shaped steel tool, *k*, 2 cm. wide, and at two other points the iron armatures, m_2 and m_3. The latter by means of a second external electro-magnet may be rotated and moved along the axis. The knife-edge is thus made to rotate and simultaneously to press against the surface of one of the metal disks brought into juxtaposition. By this device all of the eight metal surfaces were effectively scraped off in vacuo. With the softer metals precaution had to be taken so as not to cut away too much. To insure the successful operation of this device the knife carries at its middle point a thin conical projection which fits into a hole in the center of the disk. In order that the knife may always bear upon the whole surface the rod is jointed very near its inner end.

The shortest of the projecting arms carries a brass cylinder with an inner concentric one of a double layer of fine meshed wire gauze. Both cylinders were blackened by oxidation and are insulated from each other with amber. The further end of the double cylinder so formed is closed save for a central circular opening 11 mm. in diameter through which ultra-violet light may be directed upon one of the metal disks when brought into such a position as to practically close the double cylinder at the inner end which is open and as near the wheel as possible without interfering with its motion. The ultra-violet light is admitted through the thin quartz window, *w*, which closes a circular opening 9 mm. in diameter in a heavy glass plate. Both the quartz and glass plates are sealed on with de Khotinsky cement. The outer brass cylinder was earthed in this work and the inner one of gauze was used as the receiving electrode for the measurements of the photo-currents.

The metal under investigation is insulated from all the rest by means of the commutator arrangement now to be described. Leading from the brass nuts, *n*, into which the rods attached to the metal disks are screwed, are the heavy steel wires, *s*, one for each of the disks. These wires are held apart by the amber disk, *d*, which is fixed upon the axle of the wheel, and which causes the wires to press against an immovable ring, *r*, inserted inside the brass supporting tube. When the wheel rotates each wire slides over and remains in contact with the inner surface of this ring except at two points in a complete revolution, one when the disk to which the wire is attached is in position for ultra-violet light illumination and the other when it is in position for the measurement of the contact potential. (This latter position will be indicated and the manner in which the measurement is made will be described presently.) At these two points the steel wire is forced to break contact with the ring by means of a curved piece of brass which is insulated by means of amber. The platinum wire, *l*, leading to the brass is then in communication with the now insulated disk. To insure the best possible contact the curved piece of brass and the brass ring are faced with platinum and each of the wires at the two points where contact is made is likewise covered with platinum. There is a second amber disk (omitted from the diagram to avoid confusion) placed near the end of the wires, which is slotted radially and which serves to prevent each wire from being carried around by the friction between it and the surface with which it is making contact. The plate under illumination may be charged to various potentials and the distribution of velocity curve determined by observing the rate with which the wire gauze cylinder used as receiving electrode charges up. This procedure was the one adopted for the major portion of the photo-electric data recorded in this paper. Measurements of the positive potentials acquired by the insulated plate when exposed to ultra-violet light were also made by putting the plate in communication with the electrometer and applying the accelerating or retarding potentials to the wire gauze cylinder.

The vertical arm supports by means of an inserted brass frame a spool-shaped piece of iron which carries at its lower extremity a brass plate 3½ cm. in diameter. A solenoid outside the tube is used to raise or to lower the brass plate, which with each of the metal surfaces in turn constitutes the condenser system employed in determining the contact potentials. The metal disk which is to be examined when brought into position directly beneath the vertical tube is insulated by the commutator device already described. The two surfaces are brought near together by lowering the brass plate upon the three projecting arms which surround, but are insulated from, the disk. The plane of the points of these three arms was made as nearly parallel as possible to that of the disk and about .12 mm. (before the surfaces were scraped) above it. It is not essential that the two planes be absolutely parallel if for each combination the relative position of each of the surfaces when near together is always exactly the same. The brass plate must therefore never fail to rest upon all three points when lowered. This condition is met by having the brass plate upon a ball and socket joint. Between the rod above the joint and the back of the plate are attached also three brass springs, the purpose of which is to make certain that the movable plate is kept horizontal when it is being moved away from the disk beneath. A portion of the weight of the iron spool is born by a spiral spring attached between it and the supporting frame in order that the solenoid need not be inconveniently large.

Theoretically the simplest way to determine the contact potential between the brass plate and the metal disk underneath would be to place such a potential upon the brass plate and all the metal parts of the apparatus except the metal disk under observation which is in communication with the electrometer and which is first grounded and then insulated, that no deflection of the electrometer needle is observed upon raising the brass plate, but practically this procedure is not sufficiently expeditious. It proved far more satisfactory to apply to the movable plate and the framework some convenient positive potential and then with a reversing key, the same negative potential, observing in each case the electrometer deflection when the upper plate was raised. Proceeding in this manner the contact potential difference is determined by the relationship

$$E = PD\frac{d_1 + d_2}{d_1 - d_2},$$

where

E represents the contact potential difference.

PD represents the potential positively or negatively applied to movable plate.

d_1 represents deflection when $+ PD$ is applied.

d_2 represents deflection when $- PD$ is applied.

The deflections must of course be taken with their sign. The mean of these two deflections is the deflection observed with no charge applied to the movable plate. The several metals are thus compared with the brass plate. The contact potential difference between any pair may be taken as the difference between the contact potential differences of each of the pair with brass.

PRELIMINARY MEASUREMENTS OF THE CONTACT POTENTIALS.

When the parts of the apparatus had been assembled in the bulb, the contact potentials were measured before sealing up the tubes through which the various parts had been introduced. The numbers in the first column in Table I. represent the values in volts observed for each of the metals when compared with the movable brass plate. The second column gives a series of values obtained with the closed tube after remaining for several days filled with ordinary air at atmospheric pressure. The third column gives the value for dry air, *i. e.*, the air had been pumped out and dry air introduced, the process being repeated a number of times and the apparatus then left to stand several days after sealing up with drying tubes of phosphorus pentoxide. The bulb was then again exhausted and left so for three days, when it was again filled and exhausted. The results given in columns III., IV., V. and VI. exhibit nothing which can be attributed to changes in pressure. The seventh column gives the values obtained with a very low vacuum which had been reached after several days pumping with a Töpler pump and the use of cocoanut charcoal. The remaining columns give the values obtained in succeeding intervals during all of which time the best possible vacuum was maintained. Mercury vapor was prevented from entering the bulb by means of gold foil which filled the glass tube leading to the pump for 40 cm. of its length.

TABLE I.

Metal.	I. Mar. 17, Open Tube Atmos. Pressure.	II. Apr. 25, Sealed Tube Atmos. Pressure.	III. May 16, Atmos. Pressure.	IV. May 18, Vacuum.	V. May 19, Vacuum.	VI. May 20, Atmos. Pressure.	VII. June 3, Vacuum.	VIII. July 1, Vacuum.	IX. July 17, Vacuum.	X. July 24, Vacuum.
Silver	−.16	−.11	−.076	−.076	−.080	−.081	−.073	−.081	−.079	−.084
Tin	+.29	+.31	+.296	+.291	+.285	+.281	+.260	+.248	+.230	+.232
Magnesium	+.98	+.95	+.918	+.911	+.904	+.894	+.890	+.87	+.88	+.86
Zinc	+.35	+.32	+.300	+.295	+.296	+.289	+.285	+.267	+.254	+.248
Aluminum	+.50	+.45	+.428	+.419	+.417	+.410	+.422	+.394	+ 382	+.373
Iron	−.017	+.02	+.031	+.031	+.026	+.027	+.030	+.027	+.038	+.030
Copper	−.08	−.06	−.054	−.062	−.048	−.057	−.059	−.067	−.067	−.075
Brass	−.03	−.02	−.017	−.015	−.022	−.026	−.028	−.034	−.032	−.032

A survey of all the results in Table I. shows that the contact potentials must be independent of pressure. None of the changes are abrupt, indicating that they are all due to the mere ageing with time. The metals on the whole have become more electro-negative. This does not appear to be true in all cases, but the apparent discrepancy lies in the fact that the brass plate with which these metals have been compared is also subject to changes.

THE EFFECT OF ILLUMINATION WITH ULTRA-VIOLET LIGHT UPON THE CONTACT POTENTIALS OF FILM COATED SURFACES.

The illumination of the surfaces of the metals with ultra-violet light was effected through the quartz window from the mercury-in-quartz arc which was fed by a current of 1.85 amperes. The arc was placed about 20 cm. from the disk to be illuminated. By means of diaphragms with circular openings of various sizes it was arranged that the *whole* surface of the disk was acted upon by the light. (It is essential that the whole surface be illuminated, for if the light modifies the surface, the face of the disk remains no longer homogeneous. It is not possible to determine the exact change effected in the contact potential by the light unless it be known just what fraction of the surface was illuminated. The contact potential of such a composite surface would be the average of the modified and unmodified portions of the surface.) Tables II. and III. show not only the manner in which contact potentials of film-coated surfaces are modified under the influence of ultra-violet light but also the manner in which the metals recover from the effect.

TABLE II.

Metal.	Contact P.D.'s Before Illumination.	Contact P.D.'s After Illumination for 10 Min.	2 Hrs. Later.	6 Hrs.	20 Hrs.	30 Hrs.	42 Hrs.
Silver	−.087	−.082	−.078	−.084	−.084	−.087	−.085
Tin	+.225	+.326	+.318	+.306	+.263	+.240	+.229
Magnesium	+.87	+.79	+.84	+.85	+.87	+.85	+.86
Zinc	+.244	+.352	+.347	+.325	+.288	+.252	+.238
Aluminum	+.367	+.333	+.350	+.355	+.362	+.373	+.370
Iron	+.033	+.075	+.072	+.067	+.064	+.056	+.053
Copper	−.070	−.166	−.125	−.117	−.097	−.096	−.096
Brass	−.034	−.137	−.083	−.070	−.047	−.044	−.041

TABLE III.

Metal.	Contact P.D.'s Before Illumination.	Contact P.D.'s After Illumination for 2 Hours.	4 Hrs. later.	10 Hrs.	1 Day.	2 Days.	3 Days.	8 Days.
Silver	−.086	−.076	−.077	−.080	−.077	−.083	−.080	−.095
Tin	+.225	+.357	+.345	+.276	+.263	+.244	+.242	+.254
Magnesium	+.87	+.57	+.67	+.74	+.79	+.81	+.83	+.848
Zinc	+.264	+.540	+.452	+.362	+.293	+.287	+.287	+.237
Aluminum	+.362	+.193	+.286	+.333	+.346	+.353	+.343	+.372
Iron	+.042	+.093	+.090	+.087	+.084	+.080	+.087	+.082
Copper	−.083	−.195	−.142	−.113	−.105	−.096	−.095	−.100
Brass	−.045	−.190	−.111	−.088	−.080	−.063	−.072	−.072

Several similar series of measurements were made and in all cases magnesium, aluminum, copper, and brass became more electro-negative, while silver, tin, zinc, and iron became more electro-positive under the action of the light. With prolonged illumination the changes brought about in the contact potentials are greater and the recovery of the metals is in general less rapid and less complete than when the surfaces are exposed for but a short time, for it is seen that with ten minutes' exposure, Table II., to ultra-violet light, the recovery is on the whole more complete than with two hours' exposure, Table III., indicating that in the latter case the surfaces have been more or less permanently modified. By using much intenser illumination than that used in these cases, it was possible to produce even greater changes. Thus, zinc and magnesium, the one becoming more electro-positive and the other more electro-negative, could be made to change their order in the contact potential series.

There is no doubt that film coated surfaces are modified by ultra-violet light, but the fact that both positive and negative effects may be produced finds no ready explanation. If all the surfaces had become more electro-positive it might be supposed that they had been to a certain extent freed from gas or oxide films. On the other hand if all had become more electro-negative, it might be supposed either that a film or some modification equivalent to one had formed or that electrons freed from the metal had imbedded themselves in a film already existing. It is probable that something of the nature of both of these possibilities exists and what is observed is a resultant effect which depends upon the individual characteristics of the film and the metal. In any case it is almost certain that the surface becomes a non-homogeneous one, if not indeed already such, so that the measurements give merely the average effect of the constituents of the composite surface.

THE RELATION BETWEEN CONTACT AND PHOTO-POTENTIALS WITH FILM-COATED SURFACES.

Compton has shown[1] that the contact potentials of photo-electrically excited metals must be considered in conjunction with the applied accelerating potentials in measurements on photo-currents. By using different receiving electrodes he found that the distribution of velocity curves were all of practically the same shape but merely separated with reference to each other along the voltage axis by amounts equal to the contact differences of potential between the electrodes. Richardson and Compton[2] using analyzed light, found that for all wave-lengths and for all the metals, the maximum currents were reached when the potential

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[1] Phil. Mag., 23, pp. 579–593, Apr., 1912.

[2] Phil. Mag., 24, pp. 575–594.

difference between the excited plate and the receiving electrode was zero volts, proper account of course being taken of the contact potential differences. These contact potential differences were not measured under the conditions which obtained when the metals were under examination but were assumed to be the same as those for undisturbed surfaces in air.[1] This assumption may not always be correct, although it seems to have been so in their experiments. Their point of view, however, in regard to the manner in which contact potential differences are to be taken into account is supported by this work.

TABLE IV.

Metal.	Contact P.D.'s Against Brass Plate.	Accelerating P.D.'s for ¾ Saturation.	Contact P.D.'s with Copper.	Differences in Accelerating P.D.'s.
Silver	−.08	−.25	+.11	+.07
Magnesium	+.53	−.89	+.72	+.71
Zinc	+.49	−.91	+.68	+.73
Aluminum	+.23	−.53	+.42	+.35
Tin	+.28	−.69	+.47	+.51
Iron	+.09	−.45	+.28	+.27
Brass	−.18	−.16	+.01	−.02
Copper	−.19	−.18	00	00

Owing to the fact that film coated surfaces suffer changes when excited, it was difficult to duplicate distribution of velocity curves or to determine accurately the point at which the saturation current is reached. Therefore, instead of plotting the complete curve or locating the point at which saturation occurs, merely the saturation current was found and then such a potential was applied to the excited plate that the current was reduced to three fourths of the saturation value. This was indeed an arbitrary proceeding but nevertheless one giving reliable and definite information in regard to the general way in which the contact potentials and accelerating potentials are related. Table IV. gives a sample series of results obtained when the plates had been previously strongly illuminated in order to bring them into a modified condition which could be approximately maintained. The last two columns indicate a fair agreement between the differences in the contact potential differences and those in the accelerating potentials. Exact agreement is not to be expected for the applied potential necessary to produce the saturation current from a given metal is a variable quantity depending on the condition of the surface. In fact the complete velocity curve may be greatly altered in its general shape and inclination as well as shifted bodily along the voltage axis by changes in the surface.

The apparent initial velocities of emission of the electrons from illuminated surfaces are thus also dependent upon surface conditions. Nor is this dependence such as always to make the modifications in the contact potentials responsible for some of the changes observed to take place. Thus in the case of magnesium, which was found to become more electro-negative with illumination, the initial velocities of emission became greater just as though the surface had become more electro-positive rather than more electro-negative. In seeking an explanation of this fact, it may be supposed that the film covering the surface is, if not neutral, neither strongly positive nor negative, and in parts at least not of sufficient thickness to screen off the intrinsic contact effects of the metal itself. Then it is possible also to conceive of this film as being broken up by illumination in such a way as either to present more relatively inactive surface or to increase its screening effect while at the same time the broken film allows the electrons to escape more readily than before. Such an explanation is strengthened by the additional fact that the magnesium surface after a considerable period of rest, thus permitting the formation of a film, would become increasingly photo-sensitive with a somewhat prolonged exposure to ultra-violet light.

CONTACT POTENTIALS WITH NEWLY PREPARED SURFACES IN VACUO.

The only way to prepare a pure and approximately film-free surface of any of the ordinary metals is to cut away the metal. If this is done in a good vacuum, the best possible conditions have been established for the study of the properties of clean metallic surfaces. It was for this reason that provision was made to scrape off the old surfaces in vacuo. As soon after the scraping was done as adjustments could be conveniently made, the contact potentials against the movable brass plate were measured. The best obtainable vacuum was maintained

TABLE V.

Metal.	Contact P.D's Before Scraping.	Contact P.D's After Scraping.	24 Hrs. Later.	2 Days.	4 Days.
Aluminum........	+.293	+1.04	+.94	+.895	+.874
Iron.............	+.053	+ .24	+.217	+.205	+.192
Copper...........	−.106	+ .10	+.060	−.068	−.110
Silver............	−.097	+ .05	+.021	−.003	.000
Tin..............	+.216	+ .32	+.330	+.324	+.317
Magnesium.......	+.825	+1.47	+.81	+.724	+.713
Zinc.............	+.193	+ .64	+.56	+.512	+.496
Brass............	−.123	+ .26	+.243	+.227	+.228

throughout. In Table V. the first column gives the values observed just before, and the second column those just after, polishing. The metals are all more electro-positive as a result of polishing, just as would have been the case had the operations taken place in air. As the last three columns show, the metals tend to become more electro-negative with time, the rate of change being relatively quite different for the different metals. It is interesting to note that after four days copper and magnesium have become more electro-negative than they had been at any time previous to the scraping except when recently subjected to the action of the ultra-violet light.

Table VI.

Metal.	Contact P.D.'s Before Illumination.	Contact P.D.'s After Illumination.
Silver	+.010	+.014
Magnesium	+.753	+.760
Zinc	+.513	+.498
Aluminum	+.953	+.961
Tin	+.305	+.312
Iron	+.189	+.200
Brass	+.222	+.215
Copper	−.068	−.077

Agreement has not yet been reached in regard to a theory of the true nature of the phenomenon. These results, however, do not strengthen an electrolytic theory, but tend rather to support the view that the pure metals exhibit definite and characteristic potentials because of intrinsic properties which manifest themselves in this manner at the surface. Chemical action is to be considered not so much because of itself as because of the products resulting from it. When a metallic surface has become oxidized or is covered with any sort of a film, the effect observed is essentially that of a composite surface, both film and metal contributing to the general effect except when the film is of such thickness or so disposed as to screen off entirely the effects of the metal.

This point of view is further supported by the fact that with clean surfaces in vacuo, exposure to ultra-violet light produces a little or no change in the contact potentials. Table VI. gives the values obtained before and after 15 minutes' illumination with surfaces which had been prepared about 24 hours previously. It was decided to examine them at this stage in order that the ageing effect which proceeds more rapidly shortly after the scraping of the surface has taken place than it does later, would not mask such changes as might be produced by the action of the light. The changes observed are not great enough to make it possible to assert positively that they are brought about by the illumination. Even if they are due wholly to the action of the light, they are so small relatively that, for the short intervals of illumination which usually obtain in photo-electric studies, it may be safely assumed that no appreciable changes in the contact potentials of surfaces recently prepared in vacuo take place as a result of exposure to the light. It is certain, however, that even with surfaces prepared in vacuo, the presence of films is not completely eliminated. They must begin to form as soon as polishing ceases but it may be taken for granted that they are very much thinner and cover more imperfectly the metals underneath than the old films which were removed. The metals are therefore exposed more directly to the light and hence the results in Table VI. show that the metals themselves are not modified so far as their contact potentials are concerned. On the other hand, since with old films great differences are produced it is more than probable that all the changes which are observed in contact potentials as a result of photo-electric treatment are due to modifications brought about in these films.

PHOTO-CURRENTS FROM SURFACES RECENTLY PREPARED IN VACUO.

The distribution of velocity curves with surfaces recently prepared in vacuo were plotted and are shown in Figs. 3 to 8. They are arranged in pairs and some are repeated in order to show more clearly the relations existing between them. All have not been shown in a single figure in order to avoid confusion. When −8 volts were applied to the illuminated disk, the photo-current was approximately a maximum in all cases. In plotting the curves this current is taken as unity. There is a slight

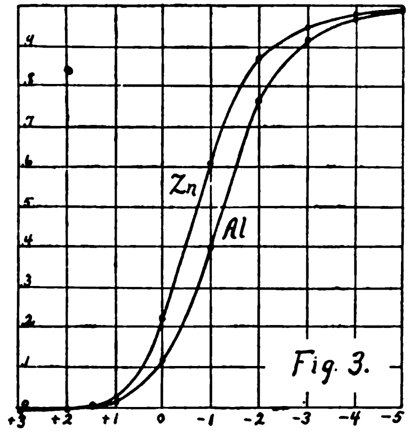

Fig. 3.

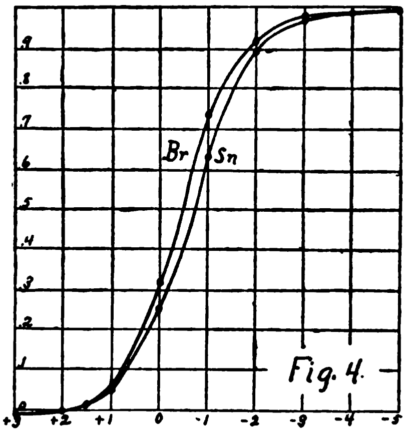

Fig. 4.

effect due to reflected light as indicated by the fact that the velocity curves cross the axis of zero current. This makes it impossible to determine the true positive potentials acquired by the metals when illuminated with unanalyzed ultra-violet light. It is interesting to note, however, that except in the case of magnesium, the point at which the axis is crossed by these curves is approximately the same for all the metals, lying between +1.7 and +1.9 volts. The true positive potential is undoubtedly somewhat greater and it would seem that with perfectly clean surfaces it is the same for all the metals. If this is the actual state of affairs, the more electro-positive a metal is, the greater is the retarding potential, taking into account of course the contact potential of the metal, necessary to keep the electrons from escaping. For magnesium this point, in the case of the curve plotted, corresponds to about +.7 volt. This does not mean that magnesium differs fundamentally from the other metals. It must be mentioned that these curves were not plotted until a few hours after scraping for the reason that the photo-sensitiveness of newly formed surfaces decreases very rapidly with time. This is particularly true of magnesium. It was necessary, therefore, that the surface be old enough that its photo-sensitiveness remain practically unchanged during the interval necessary to make the observations for a complete curve. Consequently the surfaces cannot be considered absolutely free from films, for these are sure to form even in the best obtainable vacuum. The magnesium surface, as would be expected from the readiness with which it oxidizes, merely shows in a more striking manner any modification that may be taking place. With magnesium surfaces newer than the one for which the curves are shown, the current

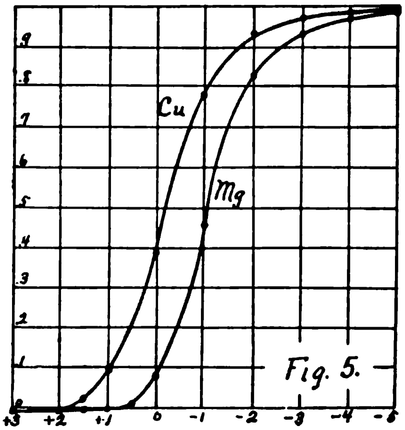

Fig. 5.

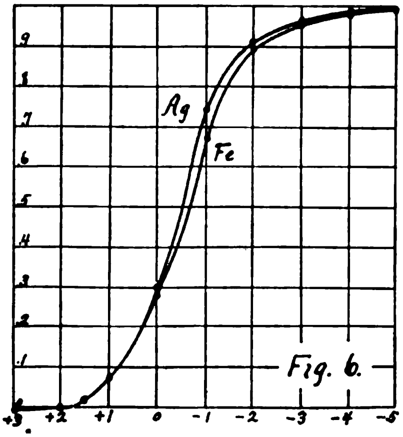

Fig. 6.

axis is crossed at points farther to the left, *i. e.*, the more or less common point for the other metals is approached nearer and nearer the newer the magnesium surface. With older surfaces this point shifts to the right and in fact, with the previously undisturbed surface, the current axis was crossed at −.3 volt. The same effect may be observed with the other metals, but it is so small as to be practically negligible. Hence it may be stated as a general fact that the apparent initial velocity of emission of electrons is less for old, than it is for new, surfaces in spite of the fact that the new surfaces are more electro-positive than the old. The effect is due in part to the electrons liberated from the receiving electrode by light of the higher frequencies reflected from the illuminated surface. This is because the old surfaces are less photo-sensitive than the new, while the number of electrons due to reflected light is practically the same for both and hence the point at which the axis of zero current is crossed by the distribution of velocity curves is further to the right with old than with new surfaces. It seems certain, however, that the film acts also as a mechanical barrier to the passage of electrons from the metal, for the shift of the curve, in the case of magnesium at least, is greater than can be accounted for by the action of electrons liberated from the receiving electrode.

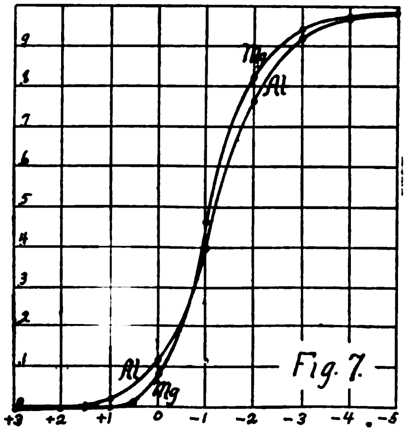

Fig. 7.

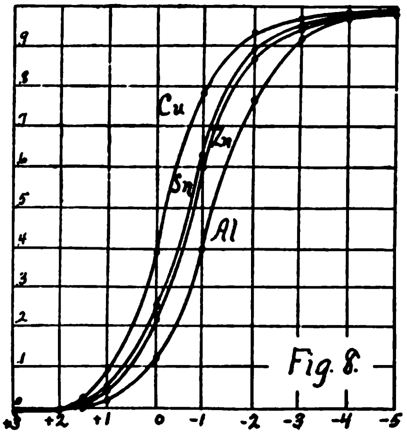

Fig. 8.

Pushing further the comparison of the magnesium curve with those of the other metals, it will be seen from Fig. 7 that the magnesium curve rises more abruptly than the aluminum curve, crosses the latter and again meets it by turning more abruptly upon approaching the maximum. As stated before, this was a surface a few hours old. With a newer surface, the curve may be wholly to the right of, and approach more nearly the same general shape as, that for aluminum. On the other hand, with an older aluminum surface the curve approaches the general shape of that for magnesium here shown. These facts also indicate that the film must be considered as a sort of barrier to the passage of electrons from the metal and as exhibiting a contact potential and photo-electric properties of its own which are superimposed upon, or screen off in part, those of the metal beneath.

TABLE VII.

Metal.	Contact P. D.'s.	Apparent Positive Potentials.
Silver	+.024	1.92
Magnesium	+.782	.82
Zinc	+.543	1.75
Aluminum	+.967	1.83
Tin	+.320	1.74
Iron	+.225	1.90
Brass	+.222	1.75
Copper	−.042	1.87

Table VII. gives in the first column the contact potentials of the several metal surfaces against the movable brass plate when the observations for these curves were made. The second column gives the values of the apparent positive potentials acquired by the metals when the disk to be examined was placed in communication with the electrometer and when the wire gauze cylinder previously used as receiving electrode was earthed. These apparent positive potentials are in fair agreement with those indicated by the curves in Figs. 3–8.

Considering all the curves together, it is seen that they tend to emerge from a more or less common point spreading out fan-like at first and later assuming approximately the same general shape, and that the order thus assumed by the metals is that of their contact potentials. The latter fact furnishes substantial evidence that the differences in the applied potentials necessary to produce the saturation current coincide in a general way with those in the observed contact potential differences. That this should be so is more readily explained by assuming that inherent physical properties of the metals rather than chemical action are coming into play.

THE RELATIVE PHOTO-SENSITIVENESS OF METALLIC SURFACES.

It has been stated that the photo-sensitiveness of the newly prepared metallic surfaces is greater than it is for older surfaces. Observations on the photo-sensitiveness of the metals were made both before and after scraping in order to determine, not only the difference in sensitiveness of new and old surfaces but also the relative sensitiveness of the series of metals, with a view to ascertain if it be related to the contact series. The first column of results in Table VIII. gives the relative photo-sensitiveness of the old unscraped surfaces. It is seen that silver was the most and magnesium the least sensitive of the eight metals, and that the order is not at all that of their contact potentials. (See first column, Table II. or III., for contact potential differences of the old undisturbed surfaces.) In the second and fourth columns of Table VIII. are given two series of results showing the relative sensitiveness of comparatively new surfaces, two days and one day respectively after scraping, and the third and fifth columns give the relative sensitiveness of the metal surfaces just as soon after scraping as observations could be made.

Table VIII.

Metal.	Relative Photosensitiveness.				
	Old Original Surfaces.	New Surfaces.			
		I.		II.	
		Before Scraping.	After Scraping.	Before Scraping.	After Scraping.
Silver	28	34	51	32	45
Magnesium	4	28	1000	42	1000
Zinc	5	49	78	53	87
Aluminum	7	190	470	260	560
Tin	19	42	69	45	72
Iron	10	38	65	46	67
Brass	17	34	46	30	45
Copper	13	40	62	40	58

(See second column, Table V.—contact potential differences of the metals just after scraping, and the third and fourth columns, those one day and two days later.) With the newly scraped surfaces magnesium is by far the most photo-sensitive just as it is the most electro-positive. Its photo-sensitiveness is taken as the base for comparison and is for convenience given a value of 1,000. With the newly scraped surfaces the order of the metals, with the single exception of brass, is seen to be exactly that of the contact series. Any given order does not long persist, however, because the aging of the various surfaces proceeds at relatively widely different rates. Thus magnesium, which is at the head of the list directly after scraping, finds its place at the bottom in two days.

It is known that the highly electro-positive metals, sodium and potassium, are exceedingly sensitive to light. There is not wanting evidence, therefore, that the photo-sensitiveness of clean metallic surfaces is related to their contact potentials.

CONCLUSIONS.

The point of view that the so-called contact potentials are due to intrinsic properties of the metals finds substantial support on the basis of the results of this investigation. The considerations leading to this conclusion may be briefly stated as follows:

1. The metals become more electro-positive when new surfaces are prepared in the best obtainable vacuum.

2. The changes in contact potential differences which are observed to take place in film-coated surfaces subjected to photo-electric treatment are such as to indicate that whatever the modifications are, they take place in the film and not in the metal.

3. The observed initial velocities of emission of electrons from recently prepared surfaces are nearly the same for all metals, suggesting that the more electro-positive a metal is, the greater the actual velocity of emission of electrons from its surface.

4. The apparent initial velocities of electrons are less for film coated than for clean surfaces and not in a manner to be accounted for by accompanying changes observed in the contact potentials. To the extent that this is not explained by the differences between the relative effects of the direct illumination and the electrons set free by reflected light for the new and old surfaces, the film acts as a mere mechanical obstruction in addition to serving as an electric screen or contributing to the electrical field.

5. The fact that the differences in the applied accelerating potentials necessary to produce the saturation currents from clean metallic surfaces under illumination coincide in a general way with those in the observed contact potential differences would not be cited as evidence in favor of an electrolytic or chemical theory of contact potentials.

6. Since the photo-sensitiveness of the metals is much greater with new than with old surfaces, and since the order assumed by metals with clean surfaces is practically that of the contact series, it appears that the electro-positive character of a pure metal is an index of the readiness with which it gives up electrons.

A comprehensive view of the above considerations shows that the facts not only warrant the suggestion that the contact potentials are manifestations of inherent properties of the metals, but also throw some light on what the nature of these properties may be. None of the facts are contradicted by supposing that the electro-positive or electro-negative character of the metal has to do with its ability to retain electrons or its tendency to discharge them. This is essentially the view held by Helmholtz, who conceived of the metals as having specific attractions for positive and negative electricities, the nature and intensity of the electrification displayed by a given metal representing the difference between its attractions for the two kinds.

I take this opportunity to thank Professor Millikan for his counsel and encouragement at all stages of this research and to acknowledge my indebtedness to Mr. Julius Pearson, the mechanician at the Ryerson Physical Laboratory, whose ingenuity and skill made it possible for me to realize and to put into successful operation the apparatus designed for this work.

Ryerson Laboratory,
The University of Chicago,
December, 1913.

ON THE NATURE OF THE VOLTA EFFECT: A REJOINDER.

By Albert E. Hennings.

THE only issue which is involved in "A Critique and Study on the Nature of the Volta Effect,"[1] so far as the results of Sanford's experiment[2] are concerned, is whether the contact charge observed on an insulated conductor, such as a ball, when it is moved from a position in which it makes contact with the inside of a hollow earthed conductor, such as a beaker, to a point wholly outside the latter depends upon the outside or the inside surface of the beaker. Sanford believes he has shown that this charge depends on the nature of the *outside* surface of the beaker. The results presented in the "critique" furnish abundant evidence that the charge observed under these conditions depends in no way on the outside surface, but only on the *inside* surface of the beaker. It was pointed out that in Sanford's arrangement the inductive effects of a static charge on the silk cord used were so great as to mask in a large measure the Volta effect itself. His results are therefore such as to invalidate any conclusion which may be arrived at by using them as a basis for argument. In a "reply"[3] to the "critique" Sanford, by refusing to admit that his results are untrustworthy and asserting that the essential elements of the experiment described by him were not reproduced in my work, implies that my results rather than his are to be considered spurious and that my experimental arrangement is incapable of giving the information desired. The situation thus created is not a happy one, for it leaves a fundamental issue apparently undecided. It seems necessary therefore that the evidence in support of my contentions be more convincingly brought out and that the results and arguments about which question has arisen be examined more carefully than was considered desirable in the "critique."

Two alternative hypotheses in regard to the nature of the Volta effect are offered by Sanford in his original paper, and a "still more crucial experiment" is announced by him which will make it easy to choose between them. The experiment consists in observing the charge dis-

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[1] Phys. Rev., N. S., Vol. II., p. 1, July, 1913.

[2] Phys. Rev., XXXV., p. 484, December, 1912.

[3] Phys. Rev., N. S., Vol. II., p. 495, December, 1913.

played by an insulated zinc ball after it has made contact with the inside of an earthed copper or aluminum beaker. The conditions were varied by placing discs of tinfoil in the bottom and by completely covering the outside with tinfoil. The attempts made by Sanford to observe the charge taken from the beaker by the ball were made solely to determine whether it is the *outside*, or whether it is the *inside* surface of the beaker upon which this charge depends. Herein then, lies the real issue not only because Sanford's choice between his two hypotheses had to be made on the basis of the observed results, but also, and chiefly, because we fail to agree on what the *true* results are. That we should differ so widely as we do makes it imperative that we inquire whether our experiments as designed and performed are essentially different or inherently incapable of furnishing the information desired and whether the results as observed may not be rendered untrustworthy by causes of an extraneous nature.

My final experimental arrangement, it is true, differed somewhat from that adopted by Sanford, but in its initial stage my experiment took the *exact* form that Sanford's did. A few details only were modified. The single feature added was that of enclosing the whole apparatus in a cage of wire netting. As I reported in the "critique," I did not succeed, when using Sanford's identical arrangement, in confirming his results and found moreover that the silk thread used to insulate and operate the ball was the seat of an electrical charge the inductive effects of which were so great as to mask in large measure the Volta effect itself. That the existence of such a charge is an annoying fact may be demonstrated to the complete satisfaction of any one who may have entertained the idea that it is purely hypothetical by bringing a bit of silk or any of the ordinary insulators on the end of an earthed wire toward an insulated conductor which is in communication with an electrometer. The magnitude of the induced charge which may be thus almost invariably observed is surprising as well as convincing. The persistence of such charges and their inductive effects rendered Sanford's particular arrangement incapable of yielding correct or consistent results.

While the slight modification in the experimental proceedure then adopted requires no justification, it may be instructive nevertheless to consider briefly the *essential elements* of the experiment. Which of two conductors, *A*, and *B*, is the more electropositive may be determined as follows: Let *A* be placed upon an insulating stand and let *B*, to which is attached an earthed wire, be brought near and in contact with *A*. Both *A* and *B* are now in communication with the earth. If *B* is then removed by means of the earthed wire attached to it, *A* will display either a positive or a negative charge. Suppose this charge is negative. Again let A and B be in contact with each other but communication with the earth being this time effected by a wire attached to A. If now B is removed by means of an uncharged insulator (such as a silk thread is not likely to be) attached to it, the charge on B will be positive if the one previously displayed by A was negative. In the absence of disturbing causes there is no choice to be made between the two procedures. They are essentially the same and each gives equally well the relative position of the metals in the contact series. To decide the issue about which the discussion centers it is necessary to ascertain merely the character of the electrical transfer between ball and beaker. The fundamental conditions are the same as for the conductors A and B, above. If the ball exhibits a *positive* charge when removed from contact with the inside of the earthed beaker, then the beaker, placed upon an insulating stand, displays a *negative* charge when the earthed ball is moved away from its inner surface. The electrical transfer which takes place is relatively the same in each case. Since the two procedures are therefore virtually identical, the assertion that in one there are reproduced none of the essential elements of the other is an ill-advised one.

In his criticism of my arrangement of a metallic beaker connected to earth by the contact between it and the ball to which is attached an earthed wire, Sanford raises the objection that this wire leading as it does from the inside of the beaker into the space surrounding it outside, becomes itself exposed, and consequently exposes the inside of the beaker also, to the induction of surrounding charges. It is to be remembered that my apparatus was completely enclosed in a large cage of earthed wire netting so that any possible disturbing charges must have been inside the cage. Moreover the insulating parts of the apparatus were also carefully screened by earthed conductors, so that it is impossible that any such charges could create disturbances of so pronounced a character as to make it doubtful whether or not it was the Volta effect which was really observed. The only possible objection to the wire attached to the ball is that it does contribute something to the transfer of electricity between ball and beaker by reason of its own contact potential with reference to the inside of the beaker, but its contribution to the general effect is negligible compared to that attributable to the ball not only because the area of that portion of its surface exposed to and within range of the surface of the beaker is relatively very small but also because the distance between the wire surface and the beaker surface is very great compared to the average of that between the mutually exposed parts of ball and beaker. A steel wire was used in my work however so that, since the ball was one of steel, even this objectionable feature was practically eliminated. Thus, in every conceivable way, the endeavor was made to remove or to reduce to a minimum all disturbing elements so that it was the true Volta effect that was actually observed. Hence it may be legitimately claimed that the results reported in the "critique" are incontestible and therefore establish firmly some of the significant facts in regard to the nature of the Volta effect.

A critical examination of Sanford's results as he has recorded them shows that they cannot be accepted as representing observations on the true Volta effect. In the first of the tables which he gives, the mean deflection of the electroscope for two series of ten observations each, when the zinc ball shares its charge with the electroscope after having made contact with the inside of the earthed beaker, is 14.2 scale divisions in the case of the copper beaker and 35.0 scale divisions in the case of the aluminum beaker. Since the sensitiveness of the electroscope was 10 divisions per volt this gives an apparent contact difference of two volts between aluminum and copper. However, this does not represent the true contact difference nor did Sanford wish it to be considered so. But obviously these mean deflections represent the relative intensities of charges of the same sign. One would expect however the contact charges in the two cases to be of opposite sign since zinc is ordinarily electropositive to copper and electronegative to aluminum. It is not impossible for zinc to be electropositive to aluminum, but in that case the contact charge should have been less in the case of the aluminum beaker than in the case of the copper beaker, for it would be a most unusual situation in which copper is electropositive with reference to either aluminum or zinc. The illogical nature of these results can be explained however by the presence of a static charge of positive sign with reference to the metals upon the silk cord. The effect of this would be to give the ball a negative charge by induction. This induced charge is likely also to be large compared to the true contact charges. The negative charge observed on the ball is the resultant of this induced charge and that due to the contact effect. Now if the zinc is electropositive to copper and electronegative to aluminum, the resultant negative charge on the ball when contact is made with the beakers is less with the copper than with aluminum. It is seen therefore that the "hypothetical" charge on the silk cord explains in part at least the illogical nature of the results presented by Sanford. Of course other disturbing causes may have been present also for he states that he made no attempt whatever to eliminate any of the electrostatic disturbances which are always to be contended with in work of this kind.

The same sort of inconsistencies appear in the results in the other tables. Disturbing causes were therefore undeniably present so that it is not surprising that the results should lead to erroneous conclusions if they be interpreted as representing only the Volta effect.

Sanford considers himself fortified against an attack on the validity of his conclusions because his final statement of what he conceives to be the teachings of his experiment is in apparent agreement with one of the statements which he quotes from the "critique." Now his conclusion was reached by choosing between the two hypotheses which he offered, but the erroneous character of his results led him to reject the one which the true results would have forced him, if his reasoning were logical, to accept. Furthermore, in laying the foundation for the second of his hypotheses, the one he accepted, Sanford explicitly assumes that the potential of the inside of a hollow conductor is the same as that of the outside. This may not be true except when the outside and inside surfaces are of the same metal, for in the case of an earthed copper beaker covered on the outside with tinfoil, the copper and the tinfoil surfaces act as though they each have their own characteristic potentials with reference to the earth. In making this assumption Sanford has committed the error of setting up a condition in his hypothesis which contradicts the very conclusion which he reaches by means of it. Thus it is that erroneous results and an unsound hypothesis have led to what seems to be the correct conclusion.

We are here concerned not with an ultimate theory of the Volta effect but with one of the fundamental facts which must be reckoned with in the formulation of such a theory. My experiments show—and in this they are in direct opposition to Sanford's—that "the Volta effect depends under all circumstances on the nature of the *opposing* surfaces of conductors which have been brought near, or in contact with, each other. Indeed it depends *only* on the surfaces exposed to, and within range of, each other. All, and only, such surfaces, or portions of surfaces, of conductors contribute to the general effect."[1]

University of Saskatchewan,
April, 1914.

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[1] Phys. Rev., N. S., Vol. II., p. 15, July, 1913.

PHASE RELATIONS IN THE ACOUSTIC SHADOW OF A RIGID SPHERE; PHASE DIFFERENCE AT THE EARS.

BY G. W. STEWART.

SATISFACTORY explanations of the many phenomena in sound localization can be realized only after the bases are laid for a successful discrimination between physical and psychological factors. The experiment in localization which seems to be the most difficult of explanation is that first noted by Rostosky,[1] independently reported by Rayleigh[2] and afterward studied by Rayleigh,[3] More and Fry,[4] Myers and Wilson[5] and More.[6] These experiments show that there is an illusion of localization if there are produced at the ears various differences in phase obtained from the same source or from sources of the same frequency. If the difference in phase is 0°, the sound appears directly in front. If the difference is 90°, and the phase of the right ear is leading, the sound appears to be on the right side. When the difference becomes 180°, the sound appears in front or in the rear, and when the difference is 270° the sound is at the left. Finally, at 0° phase difference the sound appears in front. These points are not the only ones, for, as a matter of fact, the sound seems to pass continually about the head, internally or externally, from one of the points to the other. These results are only a portion of those obtained with varying phase differences, but they are sufficient to show that an understanding of the varying phase differences produced at the ears by a rotation of the head, the source of sound being at a given distance, is needed, for it would be a distinct assistance in a discussion of the influence of a difference in phase at the ears.

A brief extension of a theory already published[7] will give the desired information. Let the source be confined to a small area on the face of a rigid sphere, and let the velocity of this source region be harmonic with the same magnitude throughout.

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[1] Philosophische Studien, 19, 1902, p. 557.

[2] Phil. Mag., 13, 1907, p. 214.

[3] Phil. Mag., 13, 1907, p. 316, and Proc. Roy. Soc., London, *A* 83, 1909, p. 61.

[4] Phil. Mag., 13, 1907, p. 452.

[5] Proc. Roy. Soc., 80, 1908, p. 260, and Brit. Jl. of Psych., 11, 1908, p. 363.

[6] Phil. Mag., 18, 1909, p. 308.

[7] G. W. Stewart, PHYS. REV., XXXIII, No. 6, 1911, p. 467.

ψ represents the velocity potential.
a represents the velocity of sound.
r represents the distance from center of sphere.
c represents the radius of the sphere.
dS represents an element of surface.

$$k = \frac{2\pi}{\text{wave length}},$$

$$\gamma = k(at - r + c),$$

$$F = \Sigma \frac{2n + 1}{2} P_n(\mu) \frac{\alpha\alpha' + \beta\beta'}{\alpha^2 + \beta^2},$$

$$G = \Sigma \frac{2n + 1}{2} P_n(\mu) \frac{\alpha\beta' - \alpha'\beta}{\alpha^2 + \beta^2}.$$

α, α', β and β' are defined in the article cited. Then

$$\psi = \frac{ka}{2\pi r} (F \sin \gamma + G \cos \gamma) \int\int Uds. \tag{1}$$

The condensation s is related to ψ by the following

$$s = -\frac{\dot{\psi}}{a^2}, \tag{2}$$

where s is related to the densities by the equation $\rho = \rho_0(1 + s)$.

Substituting (1) in (2) we have

$$s = -\left[\frac{k}{2\pi ra} \int\int Uds\right] \sqrt{F^2 + G^2} \cos (\gamma - \epsilon), \tag{3}$$

if $\epsilon = \tan^{-1} F/G$.

$\int\int UdS$ is the surface integral over the source region of the maximum value of the velocity U, and is therefore independent of r, t, and μ, or the cosine of the angle between lines joining the center of the sphere with the source of sound and the selected point. F and G vary with r and μ. Given a value r, *i. e.*, points on a concentric sphere, the maximum values of s will change with values of $\sqrt{F^2 + G^2}$, and at any given time the phase of s will vary from point to point on this sphere with values of ϵ, γ varying only with the time. Differences of phase can therefore be readily found by obtaining the different values of ϵ.

When these phase differences are ascertained, it is not difficult to obtain the phase differences on the sphere with the source at a distance r. For according to the Helmholtz reciprocal theorem,[1] if the source is at A, the resulting velocity potential at a second point B is the same both in *magnitude* and *phase* as it would have been at A, had the source

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[1] Rayleigh, Theory of Sound, Vol. II., p. 145.

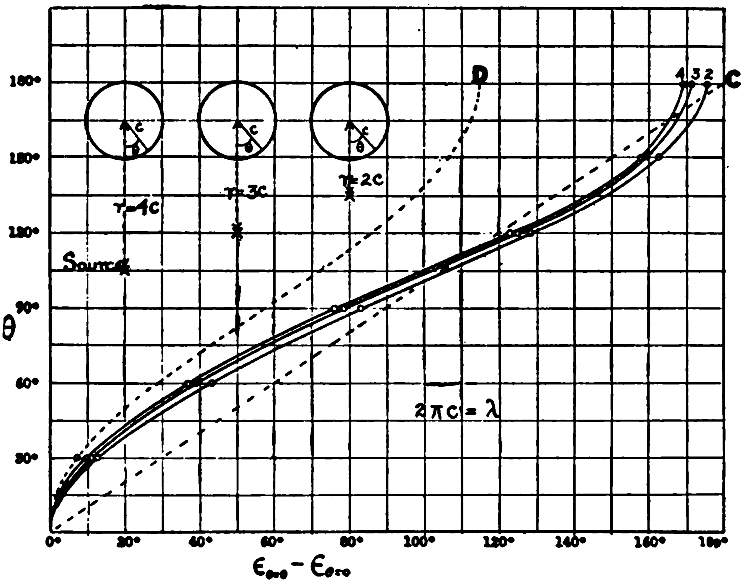
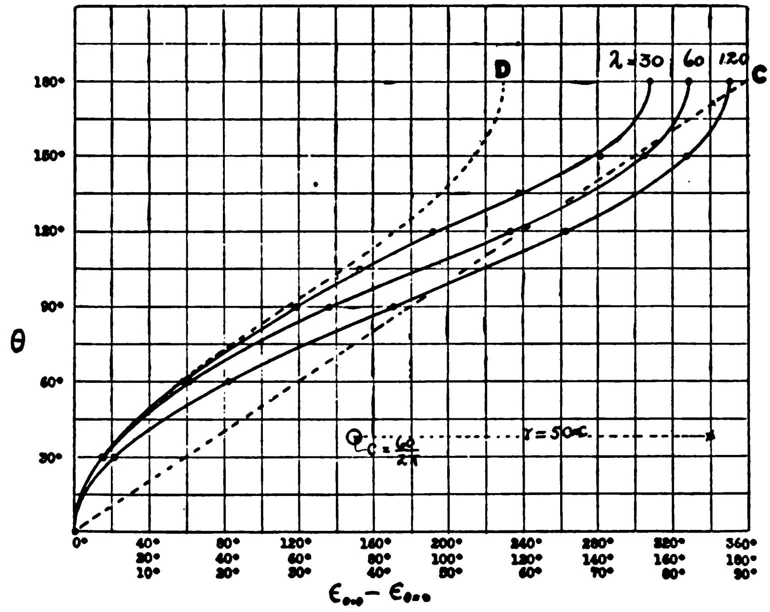

180°
150°
120°
90°
60°
30°
θ
0°
20°
40°
60°
80°
100°
120°
140°
160°
180°
$\epsilon_{\theta=\theta} - \epsilon_{\theta=0}$
D
C
4 3 2
c
θ
r=4c
r=3c
r=2c
Source
2πc = λ

Fig. 1.


180°
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80° 40° 20°
120° 60° 30°
160° 80° 40°
200° 100° 50°
240° 120° 60°
280° 140° 70°
320° 160° 80°
360° 180° 90°
$\epsilon_{\theta=\theta} - \epsilon_{\theta=0}$
D
C
λ=30
60
120
$c = \frac{60}{2\pi}$
r = 50c

Fig. 2.

of sound been at B. According to (2), the same reciprocal relation holds for the condensation s. Hence (3) will be used for the case where the source is at a distance r and the phase relations are desired on the rigid sphere itself.

Results.

We desire to know the variations of phase for positions on the sphere from 0° to 180° where these are taken on any great circle with 0° at the point nearest to the source which is at a distance r from the center of the sphere. The necessary computations for values of F and G are very laborious and consequently the conditions chosen were determined by values that had already been obtained in other work. These are, however, sufficient to indicate what is to be anticipated in the case of various frequencies and distances from the sphere. The accompanying tables, I. and II., give the numerical values. The accompanying curves, Figs. 1 and 2, present the results with sufficient clearness, but the tables are necessary in order to make a more traceable record. In Table I. and Fig. 1 the distance is varied. The results are obviously applicable for any value of c, the wave-length always being equal to the circum-

Table I.

Wave-length $= 2\pi c$.

θ	Distance $r=2c$.		$r=3c$.		$\gamma=4c$.	
	F and G.	$\epsilon_{\theta=\theta}-\epsilon_{\theta=0}$	F and G.	$\epsilon_{\theta=\theta}-\epsilon_{\theta=0}$	F and G.	$\epsilon_{\theta=\theta}-\epsilon_{\theta=0}$
0°	+1.4646 +0.2571	0.0°	+1.0573 +0.2566	0.0°	+0.9232 +0.2506	0.0°
30°	+1.1123 −0.0496	12.51°	+0.9349 +0.0451	10.88°	+0.8556 +0.0778	9.99°
60°	+0.5389 −0.3531	43.19°	+0.5909 −0.2796	38.97°	+0.6018 −0.2376	36.72°
90°	+0.1111 −0.3648	83.03°	+0.1816 −0.3878	78.45°	+0.2184 −0.3912	76.00°
120°	−0.1315 −0.2401	128.64°	−0.1193 −0.3040	125.07°	−0.1086 −0.3374	123.02°
150°	−0.2399 −0.1244	162.54°	−0.2708 −0.1823	159.69°	−0.2880 −0.2190	157.83°
180°	−0.2629 −0.0717	174.70°	−0.3131 −0.1300	171.10°	−0.3402 −0.1652	169.28°

ference of the sphere. In the case of the head, the circumference may be taken as 60 cm. In Table II. and Fig. 2 the numerical value for c is chosen with special reference to the head as the sphere, but the results would hold for any sphere if the relative values of wave-length, circumference and distance are retained.

TABLE II.

$r = 477$ cm. $c = 60/2\pi$

θ	Wave-Length, 120 cm.		60 cm.		30 cm.	
	F and *G*.	$\epsilon_{\theta=\theta}-\epsilon_{\theta=0}$	*F* and *G*.	$\epsilon_{\theta=\theta}-\epsilon_{\theta=0}$	*F* and *G*.	$\epsilon_{\theta=\theta}-\epsilon_{\theta=0}$
0°	+0.5371		+0.6830		+0.7737	0.0°
	+0.1499	0.0°	+0.2320	0.0°	+0.2016	
30°	+0.5346		+0.6800	7.87°	+0.7803	15.06°
	+0.0966	5.34°	+0.1308		−0.0062	
60°	+0.5077		+0.5899		+0.5097	58.04°
	−0.0447	20.60°	−0.1215	30.42°	−0.4826	
990°	+0.4280	42.81°	+0.3149		−0.1478	119.37°
	−0.2201		−0.3665	68.10°	−0.5609	
105°					−0.3638	152.27°
					−0.3314	
120°	+0.3060	65.62°	−0.0599		−0.3879	191.75°
	−0.3650		−0.4307	116.68°	−0.0194	
135°					−0.2378	237.19°
					+0.1359	
150°	+0.1955	81.96°	−0.3367		−0.0235	281.27°
	−0.4467		−0.3522	152.37°	+0.4026	
180°	+0.1517	87.73°	−0.4297		+0.2131	308.14°
	−0.4708		−0.2959	164.21°	+0.4892	

In both figures $\epsilon_{\theta=\theta} - \epsilon_{\theta=0}$ represents the phase difference between the point (c, $\theta = 0$) and all points (c, $\theta = \theta$), the straight line C gives the differences of phase that would occur if the condensation travelled around the circumference with the velocity of sound, and the curve D indicates the variation in phase that would occur along the diameter if the sphere were absent. The same accuracy does not hold for all the results. Those for a wave-length of 120 cm. in Fig. 2 and those of Curves 2 and 3 in Fig. 1 are very accurate, but the computations for the three other curves have not been so carefully checked. Values for 105°

and 135° were introduced to assist in locating the correct curve with the wave-length 30 cm.

Attention might be called to several points of interest.

1. The effect of increasing distance is small.

2. The phase changes the most rapidly in the neighborhood of the portion designated by 90°–120° in Fig. 1.

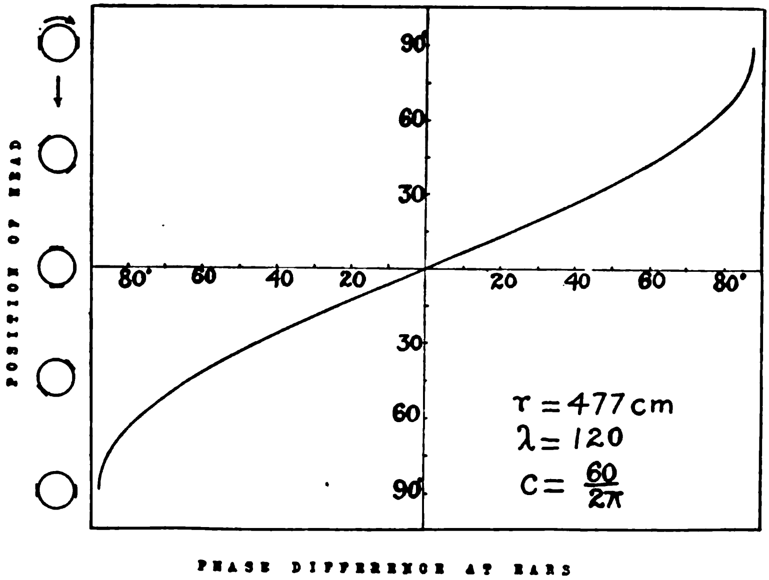

Fig. 3.

3. In most of the curves shown, in a portion of the region designated by 120° < θ < 180° in Fig. 1, the phase differs from that at 0° by an amount in *excess* of what would occur if the sound travelled directly up to the point $\theta = 0°$ and then followed the surface of the sphere with the velocity of sound. This is a surprising result, if one does not give thought to the fact that the condensation is a resultant.

4. The interesting result just stated does not exist with the wave length 30 cm. That this should be the case appears reasonable when one considers that the effective wave front is not so large with the smaller wave-lengths.

5. The difference of phase is relatively less with the decreasing wave length.

Difference in Phase at the Ears.

The difference in phase at the ears can readily be obtained from the preceding results if one supposes the head to be a rigid sphere with the ears diametrically opposite. The results for the case of wave length 120 cm., sphere circumference 60 cm. and distance of sound 477 cm. from the center of the sphere, are given in accompanying Fig. 3. The phase difference changes the most rapidly when the source of sound is in front or behind the hearer. These results give an estimate of the physical aspect of phase differences at the ears.

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THE TIME FACTOR IN SELENIUM RESISTANCE.

By G. E. Grantham.

EARLY observers of the light action of selenium discovered that the electrical resistance of selenium does not reach an equilibrium value instantaneously after exposure to light, and that the rate of change of resistance on exposure is much greater than the recovery rate.

A large amount of material has been published showing the relation between time of exposure and the resistance of the selenium cell;[1] and the effects of various agencies, such as pressure,[2] wave length of light to which the selenium was exposed,[3] temperature of the cell,[4] Röntgen radiation,[5] etc. A few observers have investigated the recovery of the selenium cell during the fractional part of a second following exposure. Since this field has not been entirely covered by experiment the present work was undertaken.

The problem suggested the use of a rotating disk from which sectors had been cut to allow the light to fall upon the cell, the cell to be in one arm of a Wheatstone bridge with some device for connecting a galvanometer in the bridge circuit at a known time interval after exposure. This method introduced the necessity of a preliminary experiment to determine the effects of different speeds of the disk on the change of resistance of the cell.

Preliminary Experiment.

A selenium cell, connected in one arm of a Wheatstone bridge, was placed in a light-tight box, the inside of which was coated with lampblack and shellac. A piece of iron pipe of 5 cm. diameter and 60 cm. long was fitted closely in a hole in one end of the box. A cap which had an opening in it the exact size of one of the sectors was placed over the outer end of the pipe. The rotating disk of 45 cm. diameter was mounted at right angles to the pipe on a shaft to which was attached a speed counter and a small electric motor. Two sectors diametrically opposite were cut from the disk permitting the cell to be exposed twice each

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[1] F. C. Brown, Phys. Rev., XXXII., p. 252; XXXIII., p. 406, 1911.

[2] F. C. Brown and Joel Stebbins, Phys. Rev., XXVI., p. 273, 1908.

[3] Pfund, Phys. Rev., XXVIII., p. 324, 1909.

[4] Miss Louise MacDowell, Phys. Rev., XXXI., p. 529, 1910.

[5] G. Athanasaidis, Ann. d. Physik, 27.4, p. 890.

revolution. The disk was rotated at 215, 690 and 1,200 R.P.M. A one candle power incandescent lamp was placed near the disk and used as the source. The result of these observations is shown graphically in Fig. 1.

The preliminary experiment showed that there was a variation in the resistance of the cell with change of speed of the disk, although the quantity of light falling upon the cell was the same at all speeds. However, this change of resistance was so slow that it could not interfere essentially with the observations taken by the proposed method.

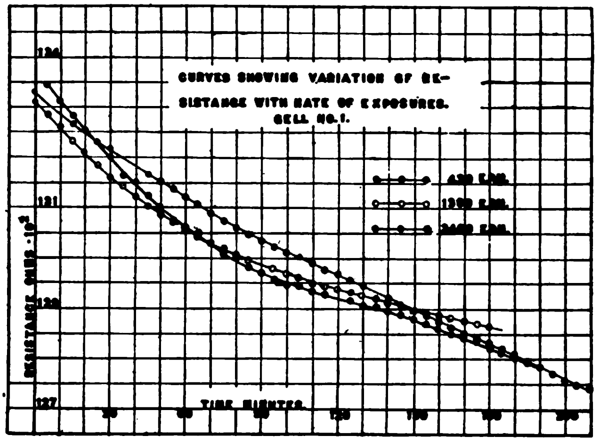

Fig. 1.

Exposures made to a 1 c. p. incandescent lamp.

The apparatus constructed for the study of the resistance change during a short interval was as follows: A small hard rubber disk of 5 cm. diameter and a metal disk of 45 cm. diameter were mounted on the same shaft of a motor equipped with a speed counter. In the circumference of the rubber disk, 90 degrees apart, were inlaid four brass cylinders of 0.8 cm. diameter. The diametrically opposite cylinders were connected in pairs by brass strips on the side of the disk. Two brushes were mounted on a wooden block which was separate from the remainder of the apparatus and yet free to rotate about the same axis as the disk. A pointer attached to the axis of the brushes moved over a fixed graduated disk and served the double purpose of indicating the angle of lead or lag of the brushes with respect to the sectors and of clamping the brushes in any desired position. The brushes were connected in the galvanometer circuit of a Wheatstone bridge. The two galvanometers used at different parts of the study were of the d'Arsonval type, one having a figure of merit of 0.83×10^{-9} amperes per mm. for a distance of

212 cm. between the scale and mirror. The other was not as sensitive. A single storage cell was used as the battery. The selenium cell was placed in the light-tight box previously described and connected in the Wheatstone bridge circuit.

The selenium cell used in the preliminary experiment was one of the Bidwell type[1] bought from Max Kohl and will be designated as Cell No. 1 throughout this paper. Three selenium cells were used for the second part of the experiment. Cell No. 1 had a sensitiveness of 1.0 : 1.5 (ratio of change of resistance to dark resistance). Cells *22B* and *50C* were made by Giltay. The sensitiveness of *22B* was 1.0 : 1.4 and that of *50C* was 1.0 : 2.2. The dark resistances of Nos. 1, *22B* and *50C* were 35,000, 290,000 and 20,000 ohms respectively. The cells frequently became almost insensitive. By connecting a cell in series with two spark gaps, two Leyden jars in multiple and a 3 in. spark induction coil and sending oscillatory discharges through the cell for about five minutes the sensitiveness was generally restored. The cell would be in a very unsteady state of equilibrium for a day following such treatment.

For illumination a Nernst lamp, the terminals of which were kept at a constant voltage, was used for a part of the experiment and a Welsbach burner for the remainder.

Method of Taking Observations.

The apparatus was so arranged that when the indicator on the graduated circle pointed to zero the galvanometer was thrown in the circuit at the same instant as that at which light fell upon the cell. If the indicator pointed to 10 degrees, the galvanometer was thrown in the circuit at a time later than the exposure equal to the time required for the disk to rotate through 10 degrees. The speed of rotation was given by the speed counter and this time interval was easily calculated. A separate switch served the purpose of throwing the commutator out of the bridge circuit. The brush contacts were tested in this manner.

Results.

The relation between time and resistance for the preliminary experiment is shown in Fig. 1. It is seen that during the first hour the rate of change of resistance increased with the speed. For a speed of 215 R.P.M. the resistance change during the first hour was 210 ohms, for a speed of 690 R.P.M. the change was 260 ohms, and for a speed of 1,200 R.P.M. the change was 310 ohms. During the remainder of the time the cell approached the equilibrium resistance. This equilibrium

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[1] Shelford Bidwell, Proc. Phys. Soc., London, Vol. XI., p. 61.

resistance is the resistance of the cell at the time that the decrease on exposure is just equal to the increase in the dark. It is evident from the curve that the equilibrium resistances will not be equal although the total quantity of light falling upon the cell is the same.

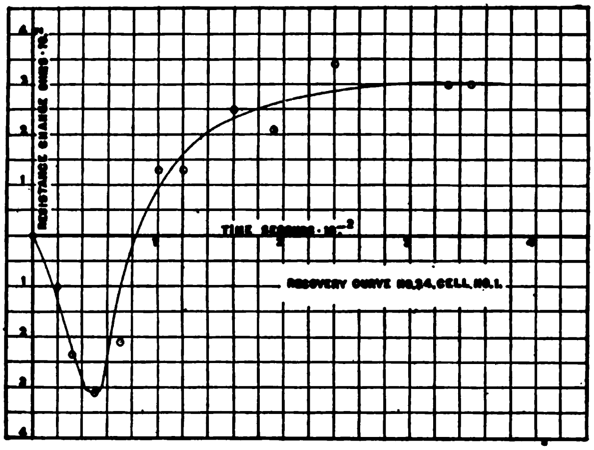

Fig. 2.

Exposures made to a Nernst lamp at a distance of 60 cm. Speed of disk, 490 R.P.M.

Figs. 2 to 5 show the results of the experiment proper. The loops in curves below the *X* axis apparently indicate that the resistance of the cell a short time after exposure was less than the resistance when the cell was actually exposed to the light. It was first thought that this

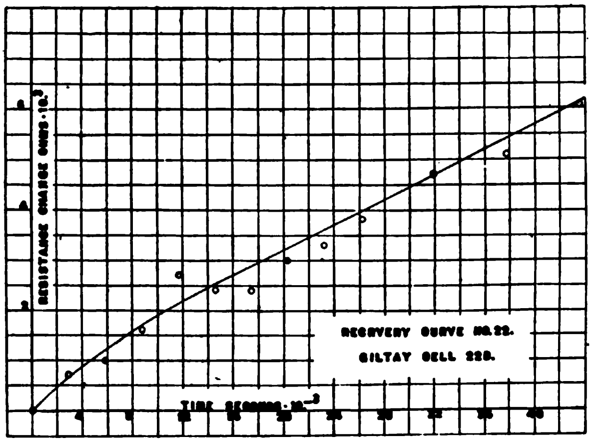

Fig. 3.

Exposures made to a Welsbach burner at a distance of 70 cm. Speed of disk 560 R.P.M.

negative loop might be caused by some contact electromotive force at the brushes and an auxiliary experiment was performed by substituting for the hard rubber disk a commutator made of two brass rods set at right angles to each other. Brushes made of the same material as the commutator were used. Then a resistance of the same magnitude as that of the selenium cell was introduced in the circuit, by reversing a switch which at the same time cut the selenium cell out of the circuit. No negative loop appeared and it was concluded that the loop was due to some change within the cell.

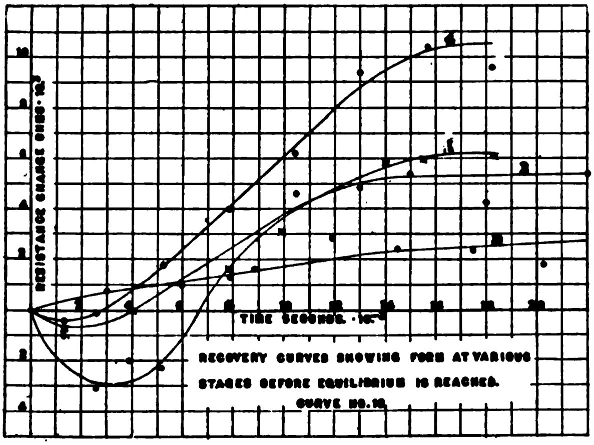

Fig. 4.

Exposures made to a Welsbach burner at a distance of 70 cm. Speed of disk for curves Nos. 1, 2, 3 and 4, 1200, 560, 1250 and 2400 R.P.M. respectively.

The negative loop did not appear if the readings were obtained within a very short time after the first exposure, as shown in Fig. 3. If the motor was run for an hour or longer before the readings were taken, a pronounced negative loop was obtained, as shown in Fig. 2. Fig. 4 shows some of the many various forms of curves obtained. Curve No. 1 was obtained when the cell had reached its equilibrium value.

DISCUSSION OF RESULTS.

The results of this experiment can be explained if it be assumed[1] that selenium consists of three components, A, B, and C, which under the action of light change according to the reaction; $A \rightleftharpoons B \rightleftharpoons C$. Assume that the resistivity of the component A is infinite, that of B approaches the resistivity of metals, and that of C has a value near that of A. Assume, further, that component A changes to B at the rate a_1, B changes

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[1] F. C. Brown, PHYS. REV., Vol. XXXIII, p. 1, 1911.

to C at the rate b_1, that the respective reverse rates are a_2 and b_2, and that the rate a_1 is much greater than the rate b_1. Then for a very short exposure some of A changes to B, but very little of B changes to C. During the recovery period between exposures, which is approximately thirty times as long as the time of exposure, some of C will change to B and some of B will change to A, but as the recovery rate is slow compared to the changing rate for one exposure, so for very short repeated exposures there will be a larger transformation to B than for longer repeated exposures. The resistance of the selenium cell depends upon the amount of B present and consequently for a time the rate of change should be greater for short exposures. The differences in the equilibrium values are due to the different amounts of B present.

In accordance with this hypothesis, the negative loop in Figs. 2 to 5 is easily explained. For a single short exposure, A changes to B, but very little of B changes to C. The change of C to B for a single short exposure can be neglected, but for repeated short exposures more and more of B changes to C, so that a longer time will be required for recovery. After the machine has been running for a time, a certain amount of B will change rapidly to C on exposure and then change slowly from C to B in the dark. Obviously there will be an apparent negative change due to the lag of the $B \leftarrow C$ change behind the $B \rightarrow C$ change. The curves of Figs. 2 to 5 then give a difference value between exposure and recovery values.

The equation of the resistance-time curves will have constants depending upon the rates of change a_1, a_2, b_1 and b_2. The conductivity will depend upon the amount of B present and F. C. Brown[1] has shown the relation to be given by the equation

$$i = K(L/N + C_1 e^{m_1 t} + C_2 e^{m_2 t}), \qquad (1)$$

in which L, N, C, m_1, C_2 and m_2 are constants which depend upon the rates of change of the components. By writing five equations using different values for t the constants may be eliminated. For convenience the substitutions $x = e^{m_1}$ and $y = e^{m_2}$ were made. The final equation then is:

$$\frac{y^2(i_2 - i_1) - (i_4 - i_3)}{y^2(i_1 - i_0) - (i_3 - i_2)} = \frac{y^2(i_1 - i_0) - (i_3 - i_2)}{y(i_1 - i_0) - (i_2 - i_1)} - y, \qquad (2)$$

in which i_0, i_1, i_2, i_3, and i_4 are the values of the conductivity at times t_0, t_1, t_2, t_3, and t_4 respectively. By choosing values of x and y the constants for a family of curves may be calculated, giving the various forms of the curves between the equilibrium value of the cell and the condition

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[1] Loc. cit.

existing after the first few exposures. The constants for a family of such curves are given in Table I. The equation which most nearly represents curve No. 3 of Fig. 4 was found by a cut and try process to be:

$$i = 1017 + 4005e^{-3.21t} + 4063e^{-2.91t}. \qquad (3)$$

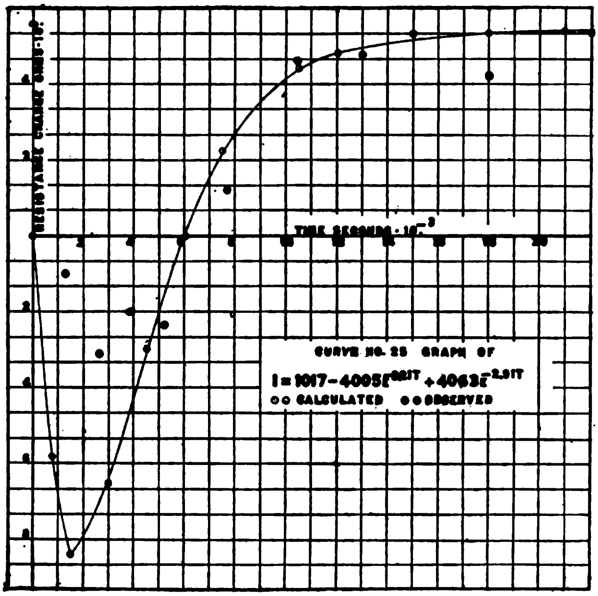

Fig. 5.

Comparison of theoretical and experimental data.

This equation is shown in graphical form in Fig. 5 together with the points experimentally determined. It is seen that beyond four one thousandths of a second the curves are almost identical. No values of

TABLE I.

Values of the Constants upon which the Rates a_1, a_2, b_1 and b_2 depend.

x	y	L/N	C_1	C_2	m_1	m_2
0.0343	0.06	1,017	− 2,121	2,179	−3.4	−2.82
0.0243	0.07	1,017	− 1,188	1,246	−3.74	−2.66
0.0373	0.057	1,017	− 2,788	2,846	−3.3	−2.87
0.0443	0.05	1,017	− 9,666	9,724	−3.12	−3.0
0.0463	0.048	1,017	−32,480	3,253.8	−3.08	−3.04
0.0423	0.052	1,017	− 5,668	5,726	−3.17	−2.96
0.0403	0.054	1,017	− 4,005	4,063	−3.21	−2.92[1]
0.0393	0.055	1,017	− 3,491	3,549	−3.24	−2.9
0.0143	0.08	1,017	− 812	870	−4.33	−2.52
0.0043	0.09	1,017	− 615	673	−7.68	−2.4

[1] Values chosen for equation (3).

x and y which give finite values for the different rates will make the curves compare favorably below four one-thousandths of a second. Then if the general equation for conductivity holds it is evident that the curve should reach a minimum a very short time after exposure. The theory offered explains this. For a very short exposure there will be a change $A \rightarrow B$. Since the rate a_2 is smaller than the rate a_1, if the galvanometer be thrown in the circuit in such a short time following exposure that but little of $A \leftarrow B$ change has taken place, the resistance must be lower than when measured during the exposure, for a_1 is finite and at a speed of 500 R.P.M. the time of exposure is less than one thousandth of a second. The effect of the C component will not enter for every short exposures. Because of the time of contact of the brushes, the time interval following exposures could not be measured to a greater degree of accuracy than two thousandths of a second. This explains why the curves plotted from experimental values do not have a minimum nearer the ordinate.

The results of these experiments are in accord with the three component theory of selenium.

It is intended, in a later paper, to show the variation of resistance of a light-negative cell under similar conditions.

This investigation was suggested by Professor Arthur L. Foley and I wish to thank him for his kindly interest and direction during the course of the experiment. I wish also to thank Dr. R. R. Ramsey for his many helpful suggestions.

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A STUDY OF THE MAGNETIC QUALITIES OF STRESSED IRON AND STEEL.[1]

By C. M. Smith and Geo. W. Sherman, Jr.

Introduction.

It was the purpose of this investigation to study the effect of stress on the magnetic qualities of iron and steel, using as test pieces short cylindrical bars. In the earlier studies[2] of magnetic properties as affected by stress, the test piece always took the form of a long slender wire; however the material of which the wire is composed is probably greatly modified as to its magnetic qualities by the process of drawing, and so the results obtained on samples of this form may not be at all representative of the material itself.

The chief desideratum in all magnetic testing is to secure results on a given test piece which will be as nearly as possible representative of the material from which the piece was formed, and this is achieved more nearly by those methods which use a short-circuited bar, thus doing away with the effects of polar fields.

The compensated permeameter as designed by Burrows[3] was used in these tests, one of the two test bars being clamped in an Olsen machine, thus making it possible to secure data for normal B–H curves, while the bar was under varying stress conditions.

It has been suggested that some definite relation should be found between the magnetic and elastic constants of steel, and to this end the present study has been directed.

The materials studied were rail steel, wrought iron, mild steel and silicon steel such as is used in transformer plate and for dynamo electric machines.

Apparatus.

The test pieces were clamped vertically in the testing machine, surrounded by the magnetizing coils of the permeameter as shown in Fig. 1. The general form of the permeameter is shown by Fig. 2.

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[1] The experimental work was performed by Mr. Geo. W. Sherman.

[2] Villari, Pogg. Ann., 1868; Kelvin, Phil. Trans., 1879, Vol. 170, p. 55.

[3] Bulletin of the Bureau of Standards, Vol. 6, No. 1, May, 1909.

The magnetizing winding M_t, with an H constant of 105, was wound on a tube of brass enclosing the test bar T. An exactly similar coil M_a surrounds the bar A while the four similar compensating coils M_c, when supplied with proper current, furnish a magneto-motive force sufficient to overcome the reluctance of the yokes and joints, when the circuit is said to be compensated. The coil S_t is the induction or test coil, used in connection with S_a and S_c and a Leeds and Northrup type H galvanometer G_2 to determine the induction density in the test specimen. A six pole mercury reversing switch is used to reverse the magnetizing and compensating currents, and to reverse the terminals of the ammeter in the magnetizing circuit. Compensation was effected by the zero method of Burrows, using a galvanometer G_1, of the Leeds and Northrup portable type. The standard of mutual inductance was wound on a marble spool and calibrated at the Bureau of Standards.

Fig. 1.

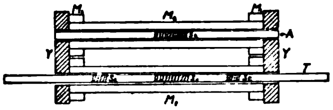

Fig. 2.

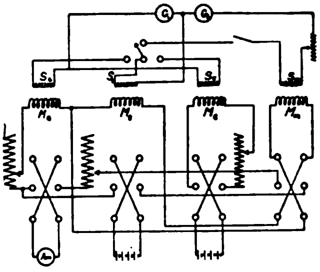

Fig. 3.

THEORETICAL.

The magnetizing field strength is calculated from the formula for the field at the center of a long solenoid

$$F = 0.4\pi ni. \tag{1}$$

The magnetic circuit is not uniform as in the case of a ring, so some modification has to be made. A winding of one hundred turns is placed about the center of the test bars to measure the induction density. A similar coil is placed at the center of the auxiliary bar A. The two coils are placed in opposition inductively, and in series with a ballistic galvanometer. When the two main magnetizing currents are adjusted so that the galvanometer shows no deflection on reversal, the flux at the center of one bar must equal the flux at the center of the other. Near the end of the test bar are placed two coils of fifty turns each which can be connected additively and in opposition to the test coil at the center.

Burrows[1] has found that if, with this arrangement, a balance again be attained, by sending current through the four end compensating coils and opposing the test coils against the two coils S_e, the magnetic circuit behaved essentially as a perfect magnetic circuit. That is, an exploring coil placed at any point about the circuit will receive the same charge, on reversal of the magnetizing current.

The induction density was determined in the ordinary ballistic manner, and is given by

$$B = \frac{10^8 MiD}{ASd}. \tag{2}$$

M is the mutual inductance,
i is the current through the mutual inductance,
d is the deflection due to the reversal of i,
A is the area of the test bar,
S is the number of turns in the test coil,
D is the deflection due to the reversal of the flux in the circuit.

The above applies only when M is kept constant and D and d are allowed to vary in the usual manner. The zero method worked out by Burrows requires a variable mutual inductance with the two secondary circuits opposed so that D and d neutralize each other and the equation becomes

$$B = \frac{10^8 Mi}{AS}. \tag{3}$$

The zero method is troublesome as there is a difference in the time constants of the two circuits and it may be advantageously replaced by the "Equal Deflection Method." This method was first suggested by Mr. Sherman and worked out by Mr. A. F. Wagner[2] of Purdue University. If a current be used through the primary of the mutual inductance sufficient to produce a deflection d equal to D, equation (3) becomes

$$B = \frac{10^8 Mi_m}{AS} = Ki_m. \tag{4}$$

This wipes out all inaccuracy due to calibration.

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[1] Loc. cit.

[2] To be published elsewhere.

PROCEDURE.

In order to investigate the possible influence of the large mass of metal of the testing machine on the bar when clamped in place, so far as compensation was concerned, specimen *A* was tested both in and out of the machine. The two curves were found to coincide exactly. This investigation is very good evidence of effective compensation, as otherwise the shunting of the test bar with the heavy mass of iron in the testing machine would alter the curve.

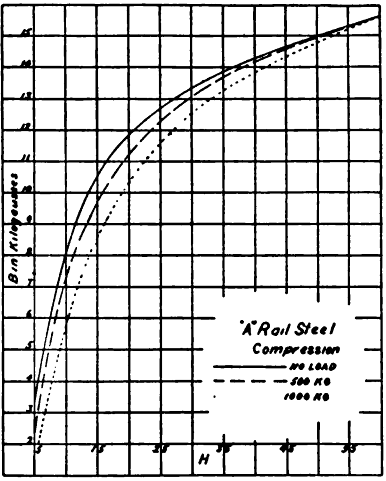

Fig. 4.

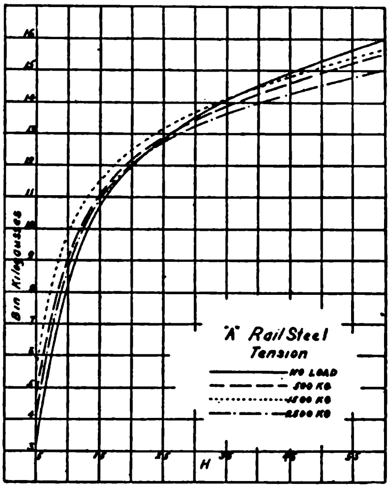

Fig. 5.

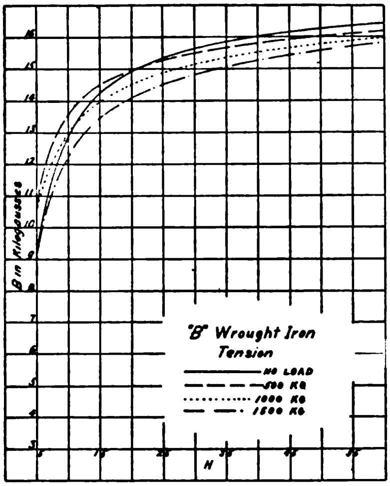

Fig. 6.

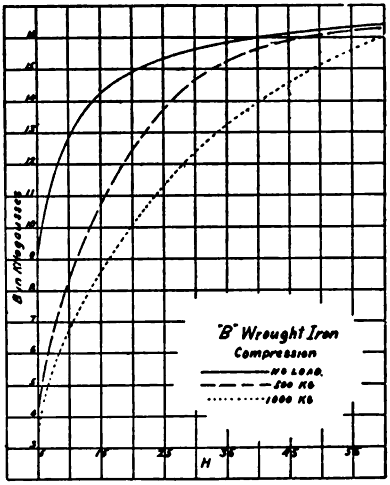

Fig. 7.

Each of the samples tested was about sixty cm. long and one cm. in diameter. The auxiliary bars were of the same cross section and material and were about forty cm. long. At first the samples were put through several cycles of loading to get rid of the effect of previous stress.

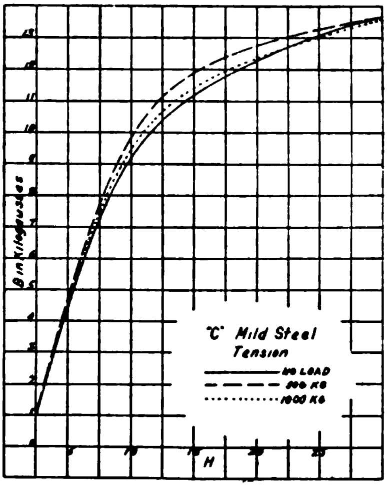

Fig. 8.

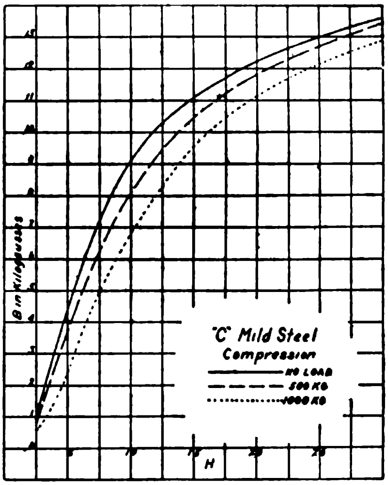

Fig. 9.

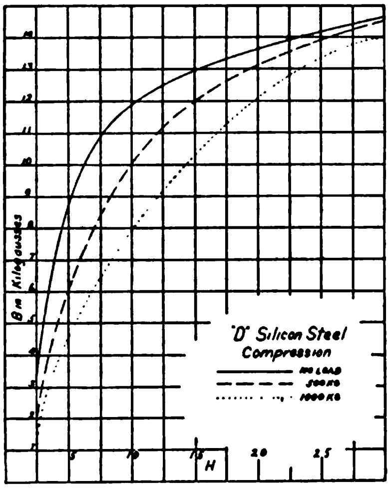

Fig. 10.

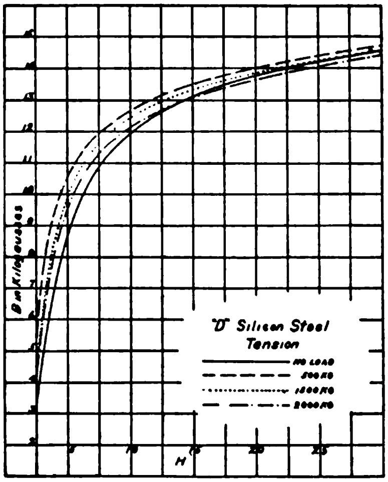

Fig. 11.

After carefully demagnetizing the sample by reversal of direct current, it was subjected to a magnetizing field and by use of the auxiliary and compensating windings, was brought to a uniform state of magnetization. With the sample and permeameter in the machine and no stress applied, values of B were determined for varying loads and for a fixed value of H, as shown in the previous section. The sample was then put under a stress of 500 kg. per sq. cm. As only the test bar was under stress, the circuit would not be uniform in flux, so the compensating and auxiliary currents had to be changed. When all was again balanced the new value of B was determined. This procedure was continued at steps of 500 kg. per sq. cm. until a load of 2,000 kg. or 2,500 kg. per sq. cm. was reached. The same process was repeated at other values of magnetizing force up to saturation.

After the tension tests were made, the test bars were cut down to forty-five cm. in length. After being placed in the permeameter, iron blocks with holes made to just fit over the bars, were slipped on. The permeameter was then placed in the Olsen machine and compressive force applied to the blocks. The iron blocks served to fix the ends of the rod and thereby allow double the load to be applied without causing measurable side deflection. Compression loads of 500 and 1,000 kg. per sq. cm. were applied and the corresponding B values found as before.

Sample of Data Taken by "Equal Deflection Method."

Bar A. Diameter of bar = 0.993 cm.
H constant = 105. B constant = 28,120.

I_h = .048	.095	.124	.209
H = 5	10	13	22
R = 5,000	5,000	10,000	10,000
No load.			
D = 3.35	8.73	5.81	7.19
I_m = .112	.290	.350	.434
B = 3,150	8,150	9,840	12,200
1,500 *Kg./Sq. Cm. (Tension).*			
D = 5.10	9.90	6.39	7.53
I_m = .172	.336	.387	.450
B = 4,840	9,460	10,730	12,650
2,000 *Kg./Sq. Cm. (Tension).*			
D = 4.72	9.96	6.30	7.45
I_m = .159	.338	.378	.447
B = 4,470	9,500	10,620	12,560

R = *resistance in series with bal. galv.*
I_m = *calibrating current.*

DISCUSSION.

In all the samples tested the Villari reversal effect was found for tension, but not in all cases for compression. From the form of the curves it was thought, however, that reversal would take place at higher values of magnetizing force. All of the samples were magnetically soft and follow the general form of the curves which, according to Ewing,[1] are typical for soft material.

As tension was applied to the specimen at low values of H, the permeability first increased and later, as more tension was applied, the permeability decreased and for heavy tension loads decreased below the original value. Above the Villari critical value this effect was reversed as might be expected. The effect of compression was to decrease the permeability at low values of H and to increase it at high values of H, but in much greater degree than for tension.

It will be seen in comparing the tension and compression curves that, when a bar is placed under compression and a low value of magnetizing force is applied with a successively decreasing load, the permeability gradually increases with a steady decrease in this rate of increase as zero load is approached. If new tension is applied, the permeability still increases at a diminishing rate until a certain value of load is reached at which the increase ceases and reverses, the permeability becoming smaller as more load is applied. The change in rate seems nearly constant, and in the same direction throughout.

Purdue University,
June 9, 1914.

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[1] Ewing, Magnetic Induction in Iron and Other Metals, 1903 edition.

DIOPTRIC FORMULÆ FOR CYLINDRICAL LENSES COMBINED AT OBLIQUE AXES.

BY CHARLES SHEARD.

THE solution of the problem of transposing double cylinders at oblique angles into their equivalents with axes at right angles (the so-called cross-cylinder combination) was first worked out, as far as the writer is aware, by C. F. Prentice and is given in his book on Ophthalmic Lenses, pages 53–83. His method is somewhat involved and lengthy and the final equations given for obtaining the values of the dioptric powers of the cylinders and their axial angles involve several terms and algebraic processes. The writer of this paper will present in outline a much simpler and clearer solution of the problem. The method of curvatures has doubtless been previously applied to cylinders, but no such solutions as are given in the succeeding paragraphs are known to me. This paper also calls attention to the superiority of the method of curvatures in the solution of a large number of problems in geometrical optics.

In Fig. 1, let *abmcd* represent a cylinder with *op*, its axis, making angles α and β respectively with the Y and X (90° and 0°) axes. The dioptric power in a direction at right angles to the axis *op* and the X, Y plane is determined by the curvature of the cylindrical surface; *i. e.*, by *amb*. Since the dioptric power is the reciprocal of the focal length, and

$$d = \text{dioptric power} = 1/f = (n - 1)1/r,$$

then $d \propto 1/r$. By geometry, $\overline{ap}^2 = 2r \cdot mp - \overline{mp}^2$ and neglecting the term $\overline{mp}^2$,

$$r = \frac{\overline{ap}^2}{2mp},$$

or

$$d \propto \frac{1}{r} \propto \frac{1}{\overline{ap}^2}, \tag{1}$$

since *mp*, the thickness of the cylinder, is a constant for any particular case.[1] Fig. 1 represents a cylinder of definite thickness, *mp*, and definite

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[1] The foregoing solution assumes a cylindrical element in which the thickness is very small in comparison to the radius of curvature of the curved surface, which is the case in actual practice. Therefore the arcs obtained by the intersections of the cylinder by planes while actually portions of ellipses may be taken as portions of circles.

semi-chord, *ap*. If the thickness of the cylinder is kept constant, it will be apparent to the reader that the value of the line *ap* will be altered when the curvature of the arc *amb* is changed. Since the width, *ab*, and the thickness, *mp*, are constants of a specified cylinder, the chords *ab*, *sb* and *ak* may be used in the measurement of the curvatures of the arcs *amb*, *sfb* and *afk* respectively in the manner indicated in equation (1).

The dioptric power of the cylinder in directions parallel to the *X* and *Y* axes may be obtained by passing planes through the cylinder intersecting its cylindrical surface in arcs *afk* and *sfb*.

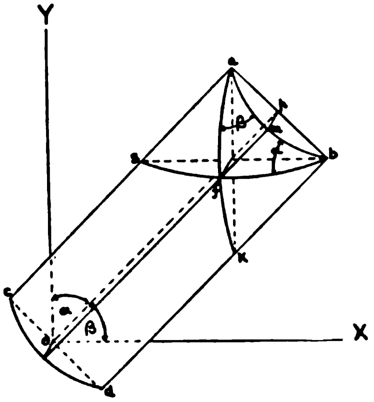

Fig. 1.

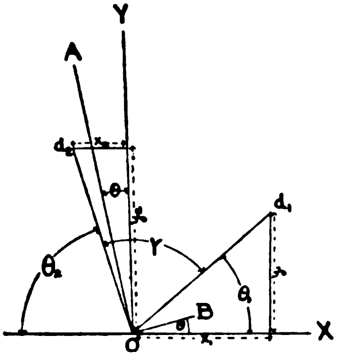

Fig. 2.

Since $ak = ab/\cos\beta$ and $sb = ab/\cos\alpha = ab/\sin\beta$, by substitution in (1) we obtain

$$\begin{aligned} d_y &= \text{vertical dioptric power} = d\cos^2\alpha = d\sin^2\beta, \\ d_x &= \text{horizontal dioptric power} = d\cos^2\beta. \end{aligned} \tag{2}$$

Hence the dioptric powers of any cylinder taken in right-angled directions are proportional respectively to the square of the cosine and sine of the angle which the axis of the cylinder makes with one of the coördinate axes.

In Fig. 2, Od_1 and Od_2 represent graphically the powers, d_1 and d_2, of two cylinders with axes at the oblique angle γ and making angles θ_1 and θ_2 with the *X* axis. Let *OA* and *OB* represent the right-angled equivalents of Od_1 and Od_2. The angle *BOA* is 90°; θ represents the angle which the axes of the equivalent cylinders make with the *X* and *Y* axes. Let the dioptric values of *OA* and *OB* be indicated by the symbols *A* and *B*.

The total dioptric power of the two cylinders Od_1 and Od_2 is (see Fig. 2)

(*a*) along *X* axis, $x_1 + x_2 = d_1\cos^2\theta_1 + d_2\cos^2\theta_2$,

(b) along Y axis, $y_1 + y_2 = d_1 \sin^2 \theta_1 + d_2 \sin^2 \theta_2$,

or

$$\Sigma x + \Sigma y = d_1 + d_2 = A + B. \tag{3}$$

$d_1 + d_2$ must be equal to $A + B$ since the sum of the dioptric powers before and after transposition must remain the same.

Treating the cross cylinders A and B, which are to be equivalent to the original oblique-angled combination, in a similar manner it will be seen that

$$A \sin^2 \theta + B \cos^2 \theta = \Sigma x_{A,B} = X, \tag{4}$$

$$A \cos^2 \theta + B \sin^2 \theta = \Sigma y_{A,B} = Y. \tag{5}$$

Combining by substraction (4) and (5) one obtains

$$(A - B) \sin^2 \theta - (B - A) \cos^2 \theta = \Sigma x_{A,B} - \Sigma y_{A,B}$$

or

$$(B - A) \cos 2\theta = \Sigma y_{A,B} - \Sigma x_{A,B} = Y - X \tag{6}$$

and by addition of equations (4) and (5) one obtains

$$A + B = Y + X = d_1 + d_2. \tag{7}$$

Therefore $\Sigma x + \Sigma y$ for cylinders of powers d_1 and d_2 is equal to a similar quantity for cylinders of powers A and B.

If the X, Y axes be rotated until the Y axis coincides in turn with Od_2 and OA it will be seen that

$$\begin{aligned} A^2 + B^2 &= d_1^2 + d_2^2 + 2d_1d_2 \cos^2 \gamma \\ &= (d_1 + d_2)^2 - 2d_1d_2 \sin^2 \gamma, \end{aligned} \tag{8}$$

from which it follows that

$$A \cdot B = d_1d_2 \sin^2 \gamma. \tag{9}$$

Equations (6), (7) and (9) give the complete solution of the problem.

Example.—To transpose 1.50 cyl. ax. 120° + 1.00 cyl. ax. 80° into the equivalent cross cylinder. We have

$$A + B = 2.50, \tag{7}$$

$$A \cdot B = 0.6195, \tag{9}$$

$$(B - A) \cos 2\theta = \Sigma y - \Sigma x = 1.6896. \tag{6}$$

Solving equations (7) and (9), $A = 2.21$ diopters and $B = 0.29$ diopter Equation (6) gives $\theta = 14° 12'$. Hence the solution,

2.21 cyl. ax. 104° 12′ + 0.29 cyl. ax. 14° 12′.

By neutralization there was obtained

2.25 cyl. ax. 104° + 0.25 cyl. ax. 14°

and by Prentice's formulæ

$$2.22 \text{ cyl. ax. } 104^\circ\, 47' + 0.28 \text{ cyl. ax. } 14^\circ\, 47'.$$

Equations (6), (7) and (9) are applicable to the solution of the cross cylinder equivalents of any combination of converging or diverging cylinders if due regard is had for the signs intrinsically associated with the dioptric powers of the cylinders involved. Furthermore, the algebraic sign of the quantity $(Y - X)$ determines the quadrant in which the axis of A or B lies; that is to say, whether A has its axis at an angle θ or $\theta + 90^\circ$.

Tables have been prepared by the writer to facilitate in the calculation of such transpositions.

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RATE OF DECAY OF PHOSPHORESCENCE AT LOW TEMPERATURES.

BY E. H. KENNARD.

THE first attempt to start out from a definite physical assumption and deduce a formula for the rate of decay of phosphorescent light was made in 1891 by H. Becquerel,[1] who obtained the expression

$$I = \frac{I_0}{(1 + \alpha t)^2}, \tag{1}$$

where I = intensity, t = time. Soon afterward Wiedemann and Schmidt developed their classical theory, according to which phosphorescence is due sometimes to the gradual restoration of a chemical equilibrium that has been disturbed, sometimes to the recombination of ions that have been formed by the exciting cause. If we adopt the latter hypothesis and add certain simple assumptions (cf. equations (2), (3) below) we are led[2] to the same simple expression that was obtained in another way by Becquerel.

This formula appears to hold[3] for the phosphorescence following the electrodeless ring discharge in gases. In solids it holds throughout in some cases,[4] but usually the values that would have to be assigned to I_0 and α undergo a rapid decrease at some point in the process of decay, remaining fairly constant before and after. Many possible causes for this deviation can be imagined: one of the most promising, according to Merritt,[5] is physical heterogeneity. This assumption leads to curves of the observed type, and it is supported by the nature and origin of those substances which show strong and lasting phosphorescence. All are crystalline; in many cases the presence of an impurity seems essential; and they are either crystallized from solution, which is likely to result in segregation of constituents, or are formed by mixing and heating without the occurrence of melting.

To test the explanation further, it seems desirable to obtain curves

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[1] Cf. Kayser, Handbuch der Spectroscopie, IV., p. 717.

[2] Nichols and Merritt, PHYS. REV., 22, p. 279, 1906.

[3] Trowbridge, PHYS. REV. 26, p. 515, 1908.

[4] Cf. C. A. Pierce, PHYS. REV. 26, p. 312 and p. 454, 1908.

[5] PHYS. REV., 27, p. 367, 1908.

for a number of amorphous solids which at some temperature have been liquid and therefore homogeneous. This condition is satisfied by many substances that phosphoresce at low temperatures, and accordingly the temperature of liquid air was chosen in the present investigation.

Apparatus.

The specimen, *S* (Fig. 1), was supported on the end of a brass tube, *B*, dipping into liquid air (*A*); the tube was screwed into a brass cup, *C*, which helped to keep the temperature of the specimen uniform. Over this was a tin hood (*H*) carrying two windows, one of quartz (*W*) to admit the ultra-violet exciting light from a mercury vapor lamp (*M*) and the other of glass to allow the phosphorescent light from the specimen to fall on a potassium amalgam photo-electric cell (furnished by Muller-Uri). Just above the windows were mounted three shutters (*T*) by which the exciting light could be cut off and the cell exposed to the phosphorescent light and then again screened. The platinum terminal of the cell was connected to an electrometer while the amalgam was subjected to a negative potential of a few volts.

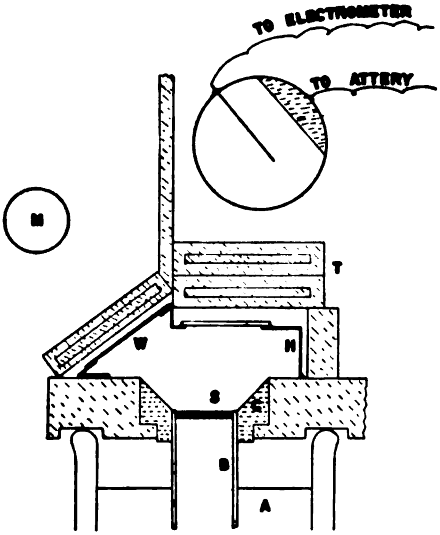

Fig. 1.

The shutters consisted of wooden slides drawn by phosphor-bronze springs; they were released by electromagnets which were operated by mercury keys tripped by a seconds pendulum swinging in an arc of about 24°. Often however, the second shutter was released by the first and followed it about 0.02 second behind, so that exposure began almost immediately after the exciting light was cut off.

The intervals of time at which the shutters followed each other, if longer than a second, were calculated from the position of the keys and the period of the pendulum. Otherwise all intervals were measured in subsidiary experiments by means of a ballistic galvanometer: one shutter was made to open a shunt and thus to cause a certain current to flow through the galvanometer, the next, to open another shunt and thereby to stop the current. The calculation of the interval involves only the period of the galvanometer and the ratios of resistances. The accuracy of these measurements is discussed below.

The slowest shutter required about 1/150 sec. to close the opening (about 23 mm. square) not counting the rather longer flying start and the time lag of the electromagnet.

The formation of frost inside the hood was slight and caused no trouble, but in order to stop the deposition of frost on the outside of the windows it was found necessary to blow a steady stream of dry air over the hood.

OPERATION.

In taking a run, the mercury lamp was first started and given at least half an hour to warm up. Meanwhile, the liquid air was applied and time was allowed for the specimen to become thoroughly cooled. Fresh liquid air was usually added every 20 min. For each observation, the electrometer was insulated and the zero point determined; after 30 sec. an auxiliary shutter was opened and excitation began; 30 sec. later another scale reading was taken and the pendulum was released, thus operating the shutters at the desired intervals; after 60 sec. (usually) another scale reading was taken and after another minute a fourth; the electrometer was then earthed. The extra readings allowed correction for the ever-present leak.

For instance, in a typical case the second shutter exposed the cell 0.02 sec. after the first had cut off the exciting light and 9.93 sec. later the third shutter ended the exposure; the four readings at intervals of 1 min. were:

303.2 302.3 372.4 371.3

The electrometer having been this time at its final potential during practically the whole of the minute between the second and the third readings, the correction for the leak is 372.4 − 371.3 = 1.1, and the corrected deflection is 372.4 − 302.3 + 1.1 = 71.2 mm.

Usually seven to ten points on the curve were determined and the series was then at once repeated in reverse order. Where possible the total emission from the start was measured in each observation, but in the case of paraffine it was necessary to measure separately successive portions of the emission and then add the deflections together.

SUBSIDIARY OBSERVATIONS.

A little uncertainty attaches to the intervals of time involved, but the error in the results due to this source is relatively small. The calculated times must be accurate within 1/50 sec., since the velocity of the pendulum past each key and the velocity of separation of wire and mercury were not less than 35 cm. per sec. Measurements of the 0.6 interval gave 0.602, 0.600, 0.598 sec. on three separate days, the ballistic galvanometer readings on each day agreeing within 1 per cent.; similarly 0.310, 0.309 sec. with the same agreement for another interval. When the first shutter was releasing the second the interval between them came out 0.022, 0.021 sec. with several per cent. variation. The degree of agreement of these observations is a test both of their own accuracy and of the uniformity with which the shutters operated. It may accordingly be asserted with confidence that each time interval occurring in the main experiment was known both within 10 per cent. (when small) and within 1/50 sec.

It has been shown by Richtmyer, Elster and Geitel, and others that a steady photo-electric current under a small potential difference is proportional to the intensity of the light. But there might still be a time lag in the action of the cell sufficient to introduce an error into measurements of short-time emission. So a tiny carbon lamp fed by a constant current from storage cells was substituted for the specimen and a double run with varying times of exposure was taken. The results, given below, show no sign of appreciable lag in the photo-electric cell down to exposures of 0.1 sec. t is the interval of exposure (corrected for measured differences between the shutters by subtracting 0.011 sec.), δ is the deflection of the electrometer in mm. corrected for leak.

	δ			
t	*Down*	*Up*	*Mean*	δ/t
0.097 sec.	28.4	28.0	28.2	291
0.193	55.2	54.9	55.1	285
0.297	87.6	86.7	87.2	293
0.394	114.8	114.1	114.5	291
0.493	143.2	144.3	143.8	291
0.587	172.7		172.7	294

PRINCIPAL OBSERVATIONS.

The results obtained thus far on paraffine are shown in the tables and in Fig. 2; some observations on cetyl alcohol and kerosene are exhibited in Fig. 3. In the tables, E_1 and E_2 denote the total emission of phosphorescent light from 0.02 sec. up to time t, where t is measured from the instant at which the exciting light was cut off; the observations for E_1 were taken in the order of increasing intervals, while those for E_2 followed immediately in reverse order. E is the mean of E_1 and E_2. As only relative values are desired, the emission of light is expressed in mm.

of deflection of the electrometer produced by the photo-electric emission caused by the light.

In order to test the theoretical formulas, the value of $R = (t - 0.02) \div E$ is given in the tables for each observation, and it is this quantity

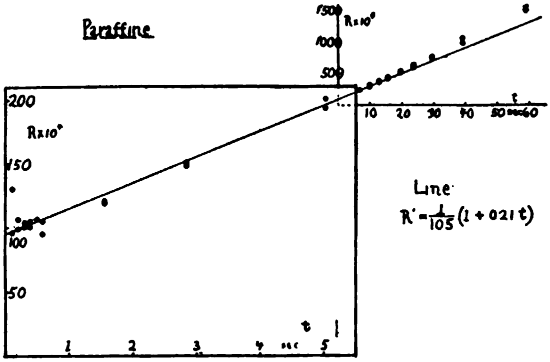

Fig. 2.

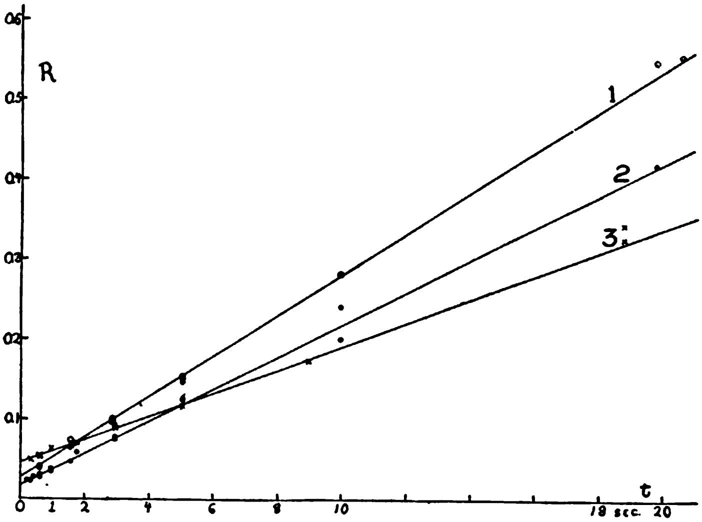

Fig. 3.

(1) Cetyl alcohol: $R' = \frac{1}{36.3}(1 + 0.92t)$. (2) Cetyl alcohol: $R' = \frac{1}{57.8}(1 + 1.16t)$.

(3) Kerosene: $R' = \frac{1}{22}(1 + 0.32t)$.

that is shown on the plots. Fig. 1 contains values of R corresponding to E_1 and E_2 in Table I.

TABLE I.

Paraffine.

t	E_1	E_2	E	$R\times10^4$	$R'\times10^4$	$R''\times10^4$	δ
0.02							
0.60	65.8	64.5	65.15	89	92	92	+ 5
1.56	139.8	135.4	137.6	112	111	112	0
2.85	207.4	202.7	205.05	138	136	136	− 2
5.03	278.8	277.9	278.35	180	179	178	− 2
9.95	361.5	358.3	359.9	276	275	276	0
19.6	409.8	403.6	406.7	482	465	486	+ 4
59.1	424.7	419.2	421.95	1,400	1,237	1,418	+18

$$R' = \frac{1 + 0.243t}{124}, \quad \delta = R'' - R$$

$$R'' = \frac{1}{119.7}\frac{.14(t - 0.02)}{1 - e^{-.14(t-00.2)}} + .147(t - 0.02).$$

TABLE II.

Paraffine.

t sec.	E_1 div.	E_2 div.	E div.	$R\times10^4$	$R'\times10^4$	$R''\times10^4$	δ
0.022							
0.105	8.9	6.4	7.65	108	97	98	−10
0.201	18.3	16.7	17.5	102	99	100	− 2
0.306	28.2	27.4	27.8	102	101	102	0
0.402	37.6	36.3	36.9	103	103	104	+ 1
0.501	44.8	44.8	44.8	107	105	106	− 1
0.595	55.2	60.5	57.8	99	107	108	+ 9
1.56	130.5	126.4	128.4	120	126	126	+ 6
2.85	191.6	186.7	189.1	150	152	151	+ 1
5.03	260.2	250.4	255.3	196	196	195	− 1
6.80	296.8	283.9	290.3	234	231	231	− 3
9.95	345.5	321.6	333.5	298	294	298	0
12.9	367.6	342.3	354.9	363	353	364	+ 1
15.7	378.3	355.7	367.0	427	409	428	+ 1
19.6	388.7	366.3	377.5	519	487	520	+ 1
23.6	397.8	372.2	385.0	612	567	616	+ 4
29.5	402.7	371.1	386.9	762	686	761	− 1
39.3	400.1	372.8	386.4	1,015	882	1,006	− 9
59.1	396.7	377.5	387.1	1,527	1,276	1,514	−13

$$R' = \frac{1}{105}(1 + 0.21t), \quad \delta = R'' - R$$

$$R'' = \frac{1}{103.5}\left\{\frac{0.14(t - 0.02)}{1 - e^{-0.14(t-0.02)}} + 0.125(t - 0.02)\right\}.$$

If formula (1) mentioned above holds, the curve for R should be a straight line. For we have

$$I = \frac{I_0}{(1 + at)^2}, \tag{1}$$

$$E = \int_{t_1}^{t} I dt = \frac{I_0(t - t_1)}{(1 + at_1)(1 + at)},$$

$$R = \frac{t - t_1}{E} = \frac{1 + at_1}{I_0}(1 + at), \tag{2}$$

and R is linear in t. In the present case $t_1 = 0.02$ sec. Arbitrary values were assigned to the constants in (2) and theoretical values of R were then computed. These are shown under the heading R' in the tables and are represented by the straight lines drawn on the plots. The numerical formula for R' is given in each case.

Each table and each curve in Fig. 3 represents a double series obtained on a separate day. The different series are in general not comparable as to magnitude of emission, and probably not entirely as to shape because the intensity of excitation varies.

But even the two parts of each double series do not agree very well. In the case of paraffine, the variation looks at first sight like weak "hysteresis." But in the other cases, the variations are erratic. As a further test five observations on kerosene, the interval being from 0.02 to 2.95 sec. after excitation, gave 31.3, 33.0, 33.3, 34.6, 30.8, the outgoing and returning readings in the double series just preceding (Curve 3) being 31.5, 34.3 respectively. The most probable cause of all these variations would seem to lie in fluctuations of the exciting light, which would result in variations of initial intensity.

The results seem however sufficient to warrant the conclusion that in the three substances examined R is linear in t and formula (1) holds at least during the first 10 seconds of decay, or until the phosphorescence has sunk to 1/10 or less of its initial value. Paraffine then shows a decided upward bend in the curve, which means that the decay is occurring more rapidly than is required by the formula. This cannot be due to the presence of several emission bands in the spectrum, each obeying (1), for it may easily be shown that such a combination would produce a downward bend in the curve. Paraffine thus differs radically from the ordinary phosphorescent substances, which exhibit an initial rapid decay. If any such effect exists here, it must at least be very weak or of extremely short duration.

It is interesting to consider what modification of the ionization theory would lead to a curve of the observed type. Nichols and Merritt[1] obtained from their assumptions the equations

$$\frac{dn}{dt} = -kn^2, \tag{3}$$

$$I = -A\frac{dn}{dt}, \tag{4}$$

where n = number of phosphorescent nuclei or of corresponding ejected electrons (or other ions) present, I = intensity of phosphorescence during decay. Hence

$$n = \frac{n_0}{1 + kn_0t}, \tag{1}$$

$$I = \frac{I_0}{(1 + at)^2}$$

where $a = kn_0$, $I_0 = Akn_0^2$.

The unusually rapid later decay suggests that something else besides the ejected electrons is destroying the phosphorescent nuclei. If we suppose that the ordinary free electrons may unite with a positive phosphorescent nucleus and cause luminescence (but that no luminescence arises from their normal recombination with an ordinary positive atom which has lost a free electron), we are led to the modified equation

$$\frac{dn}{dt} = -kn^2 - k'mn, \tag{5}$$

where m = number of free electrons. If m is practically uninfluenced by the presence of the electrons ejected from phosphorescent nuclei and is therefore practically constant, and if as before

$$I = -A\frac{dn}{dt},$$

we obtain after integrating

$$I = \frac{I_0e^{-\beta t}}{\left[1 + \frac{\alpha}{\beta}(1 - e^{-\beta t})\right]^2}, \tag{6}$$

where $\alpha = kn_0$, $\beta = k'm$, $I_0 = A(\alpha + \beta)n_0$, n_0 being the initial number of phosphorescent nuclei present.

It is evident from the deduction of (6), or by substituting in it $t = t_1 + (t - t_1)$, that the form of the right-hand member is unaltered if $t - t_1$ be taken as the variable instead of t, provided that I_0 be replaced by I_2, and α by α_1 to correspond to the displaced starting-point.

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[1] Nichols and Merritt, Phys. Rev., 22, p. 279, 1906.

Hence by integration

$$E(t_1, t) = \frac{I_1}{\beta} \frac{1 - e^{-\beta(t-t_1)}}{1 + \frac{\alpha_1}{\beta}(1 - e^{-\beta(t-t_1)})}, \tag{7}$$

and if $R = (t - t_1)/[E(t_1, t)]$

$$R = \frac{1}{I_1}\left[\frac{\beta t}{1 - e^{-\beta(t-t_1)}} + \alpha_1 t\right]. \tag{8}$$

Under the heading R'' in the tables are given values of R calculated from (8) in the numerical form given below the tables: the same value of β was designedly employed in both cases because theoretically it seems unlikely that it should vary from day to day. They agree rather well with the observed values given under R, which indicates that (6) represents the decay of the phosphorescence fairly well. But the accuracy of the observations is not sufficient definitely to establish any three-constant formula.

SUMMARY.

1. The rate of decay of phosphorescence at liquid air temperatures was studied in paraffine, cetyl alcohol and kerosene, employing a photoelectric cell and an electrometer.

2. From 0.1 sec. to 10 sec. paraffine obeys the law

$$I = \frac{I_0}{(1 + at)^2};$$

then the decay is more rapid (whereas in crystalline substances at ordinary temperatures the later decay is less rapid). Cetyl alcohol and kerosene appeared to obey this law as far as their phosphorescence could be followed (20 sec.).

3. Taking into account the free electrons, assumed constant in number, the formula becomes

$$I = \frac{I_0 e^{-\beta t}}{\left[1 + \frac{\alpha}{\beta}(1 - e^{-\beta t})\right]^2},$$

which represents the decay for paraffine within the fairly large errors of observation.

It is a pleasure to acknowledge my obligations to Professor E. Merritt for suggesting the problem and for advice in regard to the low temperature arrangements, and to Professor G. S. Moler for advice regarding the shutters.

CORNELL UNIVERSITY,
May, 1914.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

THE SPECIFIC HEATS OF MIXTURES OF ALCOHOL AND WATER AND OF SOLUTIONS OF NON-ELECTROLYTES IN THESE MIXTURES.[1]

BY WILLIAM FRANCIS MAGIE.

THE specific heats were determined of mixtures of one gram-molecule of ethyl alcohol with N gram-molecules of water. Between the limits $N = 5$ and $N = 50$ the heat capacity of such a mixture is very exactly obtained by adding to the heat capacity of the water, $18N$, the value ζ of the apparent heat capacity of the alcohol, given by the formula

$$\zeta = aN + \frac{b}{N} + c,$$

$$a = -0.0666, \quad b = -95.8, \quad c = 67.00.$$

For $N = 100$ or more, and for $N = 3$, this formula does not hold.

A gram-molecule of pyrocatechin dissolved in considerable water has an apparent heat capacity of 75.5, and in alcohol an apparent heat capacity of 57, independent of the concentration. When dissolved in mixtures of the sort described above the apparent heat capacity is apparently independent of the concentration of the pyrocatechin in a particular mixture, but differs with the proportion of alcohol in the mixture. The following table gives the results of the measurements. From 10 to 15 grms. of pyrocatechin were dissolved in about 700 grms. of the mixture. The N of the table represents the number of gram-molecules of water mixed with one gram-molecule of alcohol. The ζ represents the apparent heat capacity of the pyrocatechin.

N	50	25	12.5	8	5	3
ζ	70	61	49	35	34	53.5

The lowest values found are equal to the heat capacity of a gram-molecule of solid pyrocatechin.

Similar results were obtained for resorcin, except that the minimum value of ζ is considerably below that found for solid resorcin.

The only conceivable explanation of these results is that some association occurs in the solution, whereby the degrees of freedom of its constituents are diminished in number.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 24, 1914.

THE VILLARI CRITICAL POINT IN FERROMAGNETIC SUBSTANCES.[1]

BY S. R. WILLIAMS.

UNDER this subject a comparative study has been made of the change in length effect due to magnetization with the change in magnetization arising from a longitudinal pull. The first phenomenon is known as the Joule magnetostrictive effect and the second as the Villari reversal effect. The results indicate that for those substances in which a reversal of the Joule effect occurs, there also appears a Villari reversal. Specimens showing only an elongation for all field strengths exhibit only an increase of magnetization due to a longitudinal pull, while those rods, in which contraction for all field strengths appears, manifest a decrease of magnetization due to a longitudinal pull.

Nickel is the most interesting of all the specimens studied. It can be made to exhibit or not exhibit a Joule reversal and also a Villari reversal effect depending upon whether or not complete demagnetization has been previously secured. This is an important point and will doubtless clear up the controversy between the various investigators of this subject.

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[1] Abstract of a paper read by title at the New York meeting of the Physical Society, February 28, 1914.

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THE

PHYSICAL REVIEW.

MEASUREMENTS WITH A MOVING LAMP PHOTOMETER.

BY C. C. TROWBRIDGE AND W. B. TRUESDELL.

IN the following paper a modification of the ordinary photometer is described. The essential feature of the apparatus is a sliding carriage, supporting one of the lamps, which may be set in to and fro motion at uniform velocity in the photometric axis. Comparisons of equal field intensity are observed while the lamp is in motion, and recorded by a simple mechanism.

The purpose of the measurements which have been made with the apparatus was to ascertain the magnitude of errors occurring in photometric measurement in which one of the light sources is rapidly diminishing in intensity as in the photometry of phosphorescent gases and certain phosphorescent solids. The opinion was held that these errors might be of considerable magnitude, a view which has been verified by the series of measurements which are the subject of this paper. The use of the moving lamp device gives some informatión also on the rôle played by the fatigue of the retina of the eye in photometric measurements in general.

The paper is divided for convenience as follows:

1. The moving lamp mechanism and the method of measurement.
2. Judgment of screen equality and reaction time.
3. Retinal fatigue in the photometry of phosphorescence.
4. Use of the moving lamp apparatus in general photometry.
5. Retinal fatigue in different persons.

1. THE MOVING LAMP MECHANISM AND THE METHOD OF MEASUREMENT.

The apparatus used is shown in Fig. 1. By means of a belt carrying a metal *L*-shaped catch (*C*, Fig. 1) which engaged alternately projections *E* and *E′* on the lamp carriage, the lamp was made to approach and to recede from the photometer screen through a distance of 44 cm. respectively. The lamp carriage was equipped with a small electro-magnet with a pencil attachment controlled by a key. By this device the observer recorded the position of the moving lamp on a fixed strip of paper at the moment that the two sides of the comparison screen appeared uniformly illuminated. The lamp carriage was also equipped with a contact point, which entering a narrow trough of mercury, could be used to light a signal lamp for a moment in each advance and each recession of the light. The comparison screen employed was of the Conroy type (reflection at 60° incidence) and was viewed through an oval diaphragm and through a collimator about six feet in length. This type of screen and method of observation is not essential in the moving lamp mechanism, however. The left half of the oval collimator field was illuminated by the moving light, while the illumination of the right half was due to the standard lamp. In Fig. 2, *M* shows the field when the moving light is closest to the comparison screen; *N* shows the equality obtained as the moving light recedes; while *O* shows the contrast when the moving light is 44 cm. farther from the screen than in *M*. The investigation having been undertaken primarily to ascertain the errors in the photometry of phosphorescence; the conditions of the experiments were adjusted to correspond to the rate of the decay; that is the rapidity of fading of the light after stimulation of the substance. Accordingly the lamp was run at 13 cm. per second as this velocity produced a diminution of intensity which corresponded to the average rate of fading of the phosphorescent nitrogen gas found in previous experiments.[1] At this velocity the comparison screen changed from *M* (Fig. 2) through apparent equality, *N*,

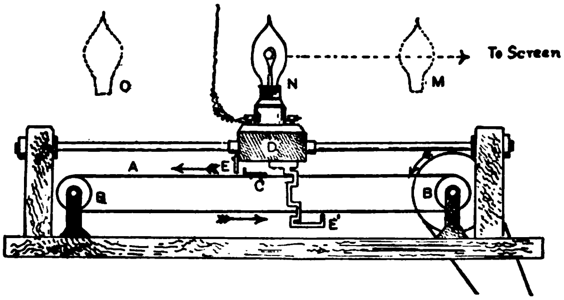

Fig. 1.

Essential Parts of Mechanism for Motion of the Lamp: A belt *A* moving on pulleys *B, B*, *C* a metal catch on belt, *D* lamp carriage, *EE'* projections attached to the lamp carriage to engage metal catch on the belt by which the uniform advancing and receding motion of the lamp is effected.

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[1] C. C. Trowbridge, Phys. Rev., XXVI., 6 June, 1908, and XXXII., 2 Feb., 1911.

to O so quickly as to produce the effect of a flash of reversal. An instant judgment was therefore possible. An explanation of the use of a uniform velocity to correspond to a phosphorescent decay is given in a later section.

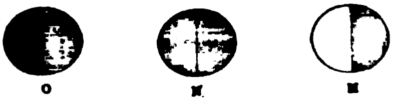

Fig. 2.

Photometric Fields of View. The Comparison Screen Corresponding to Three Positions of Lamp; *M*, moving light at least distance from comparison screen; *N*, moving light at position for apparent equality of fields; *O*, moving light at greatest distance from comparison screen.

With a velocity of 13 cm. per second, the moving lamp remained at each end of its run (shown by M and O in Fig. 1) for 1.2 sec. The distance from M to O was 44 cm.; hence $44 \div 13 = 3.4$ sec.; the time of the circuit of the lamp was therefore $(2 \times 3.4) + (2 \times 1.2) = 9.2$ sec. Thus, under the above conditions, five points of judgment of screen equality, *as the light receded*, were taken within 37 sec. while if readings in both directions were desired they could be taken within 42 seconds. On the other hand, ten settings of the screen by hand adjustment without an elaborate recording device could scarcely be accomplished in less than several minutes. The moving light thus eliminates the slowness of hand settings. It is possible, of course, that new errors enter, but the convenience is undoubted.

The method of using the instrument was as follows: The "movable lamp," set at the middle point of its run, was adjusted at a distance of 80.3 cm. from the screen, this being the calculated distance for .75 of an arbitrary standard intensity, I, used in the phosphorescent work referred to above. The comparison lamp on the opposite side was then adjusted for approximate screen equality. A mean position was obtained by using long cords attached to the "movable lamp" carriage so that the observer could make a series of adjustments free from the effects of muscular coördination. The muscular control of the instrument which one has with his eye at the eyepiece and his hands upon the carriage of a Lummer-Brodhun or Bunsen screen was lacking since the cords were somewhat elastic. The judgments were therefore eye decisions, but were made after various shiftings as is usual.

Each of a series of ten hand settings were made and recorded by means of the magnetic device. The record strip was then replaced by a blank, the light set in motion at $v = 13$ cm. per second and 5 observations of apparent equality were taken. The record blank was then replaced, since more than 5 observations tended to destroy the clearness of the record. After fifteen to fifty had been secured, the apparatus was adjusted for .50 *I* and later for .375 *I* and the procedure described was repeated for each position.

The first series of observations taken cover the period from January 25, 1912, to March 16, 1912, and were 1,200 to 1,500 in number. Those shown in the tables were taken March 15 and 16, and are given merely because the measurements of the three intensities were repeated on two successive days under similar conditions and with the apparatus unchanged by any modifications during that time.

Table I.

Distances from Moving Lamp to Screen in Centimeters.					
Hand Settings .75I.	**Readings of Position of Light Receding from Screen at $v=13$ Cm. per Sec.**				
77.5	77.9	78.6	76.1	77.0	
79.7	81.3	79.5	77.3	78.5	
80.6	81.4	80.1	80.1	78.8	
81.5	82.6	82.9	81.3	82.1	
81.6	85.5	83.4	84.4	83.6	
82.3	79.1	75.4	77.8	77.7	
83.3	79.4	78.0	78.0	81.0	
84.2	80.2	80.3	79.4	81.7	
84.6	82.3	80.7	80.5	84.2	
85.2	78.6	81.0	84.8	86.8	
82.0	80.8	81.3	80.0	81.0	Means
1.87	1.79	2.08	2.25	2.53	Av. dev.

Table II.

Distances from Moving Lamp to Screen in Centimeters.				
Hand Settings .50I.	**Readings of Position of Light Receding from Screen at $v=13$ Cm. per Sec.**			
92.8	93.8	91.5		
93.9	94.5	92.3		
94.5	95.5	93.5		
95.1	96.7	94.6		
95.3	97.8	97.5		
96.4	90.8	92.8		
96.8	93.0	93.1		
97.0	95.4	95.5		
99.1	95.7	98.7		
100.8	100.0	102.9		
96.2	95.3	95.2	Means	
1.85	1.84	2.72	Av. Dev.	

Table III.

Distances from Moving Lamp to Screen in Centimeters.					
Hand Settings. .375I.		Readings of Positions of Light Receding from Screen at $v=13$ Cm. per Sec.			
108.1	104.9	109.0	107.0	106.4	
109.0	105.7	109.3	110.2	106.5	
109.1	108.3	110.1	110.4	111.9	
109.5	108.4	111.0	110.6	112.6	
113.2	108.6	113.6	112.0	115.7	
113.4	109.1	108.7	104.3	107.1	
113.6	110.8	109.7	104.9	107.2	
115.7	111.5	111.3	108.2	108.0	
116.3	111.8	112.7	111.8	115.5	
117.1	113.1	113.5	114.3	116.8	
110.8		110.9	109.4	110.8	Means
2.46		1.52	2.61	3.74	Av. dev.

Tables I., II. and III. contain the data just referred to. It will be seen by inspection that the sets of observations on screen equality for the light receding at 13 cm. per second differ among themselves to about the same extent that the hand settings differ among themselves. The average deviations for hand settings are: — 1.87, 1.85, 2.46 cm., and for the motion readings, 2.1, 2.28, and 2.62 cm. It is to be especially noted that the averages of the receding sets are in very close agreement, namely: 80.8, 81.3, 80., 81.0; Table I.; 95.3, 95.2, Table II.; and 110.9, 109.4, 110.8 cm., Table III.

The figures in Table III. seem to indicate that low luminations give the best conditions for the use of the instrument. If we are to regard the hand adjustment as necessary to standardize, the mean position of the light receding at $v = 13$ cm. per second expressed in per cent., Table III. is found to be less than the mean hand adjustment by only .0036, while in Table II. the difference is .01+ and in Table I. it is .02.

2. JUDGMENT OF SCREEN EQUALITY AND REACTION TIME.

Before proceeding to the calculation of data relating to the errors in the photometry of phosphorescence, the sources of error affecting the readings of the moving lamp photometer must be examined and the necessary corrections determined and applied. In the uncorrected readings of the photometric comparisons, four sources of error seem to be important—(X_1) the judgment of screen equality; (X_2), the effect of the diminishing intensity of the receding light upon the eyes of the observer; (X_3), the reaction time of the observer; and (X_4) the reaction time of the electrical recording device.

By means of the momentary circuit closing device which could be instantly adjusted to throw a flash from the signal lamp into the illuminated collimator field, it was possible to ascertain the values of (X_3+X_4), namely, the combined reaction times of the observer and the recording instrument.

With a given velocity of the lamp carriage, the recording mechanism was used with the automatic signal lamp to determine the reaction time. Readings taken indicated a lag in the record behind the actual time of the signal in direct proportion to the velocity. For $v = 13$ cm. per second, which corresponds to the mean rate of phosphorescent decay of intensity at the values tested in this investigation, the lag (d) was 3.6 cm. Then $3.6 \div 13 = .28$ sec. which indicates that for the same observations upon the phosphorescent problems the decay at the values of intensity tested ($.75\ I$, $.50\ I$ and $.375\ I$), was .28 sec. later than the photometric judgment, assuming the observer to have this reaction time.

The data on this lag (denoted hereafter $X_3 + X_4$) are given in Table IV. Each number tabulated is the mean (taken by inspection from the nearly overlapping strokes of the recording pencil) of five successive observations. It will be noted that the figures for the three later dates are in close agreement. The values have been applied as a correction to the photometric observations during the corresponding periods of time.

TABLE IV.

Personal and Instrumental Reaction Time $(X_3 + X_4)$.

Feb. 13, 1912.	Feb. 23.	Feb. 24.	Mar. 15.
3.1 cm.	3.7	4.0	3.7
3.2	3.7	3.7	3.8
3.0	3.7	3.8	4.0
3.1	4.2	3.6	3.6
3.4	3.7	3.7	3.7
3.3			4.0
3.6			
3.5			
3.5			
Av. 3.3 cm.	3.8 cm.	3.8 cm.	3.8 cm.

3. RETINAL FATIGUE IN THE PHOTOMETRY OF PHOSPHORESCENCE.

A curve is shown in Fig. 3 (A and A'), which gives the relation between the luminous intensity of phosphorescent nitrogen gas and the time. This gas in the phosphorescent state has been shown to be chemically active by R. J. Strutt and has been called "active nitrogen" by him. The equation of the curve is

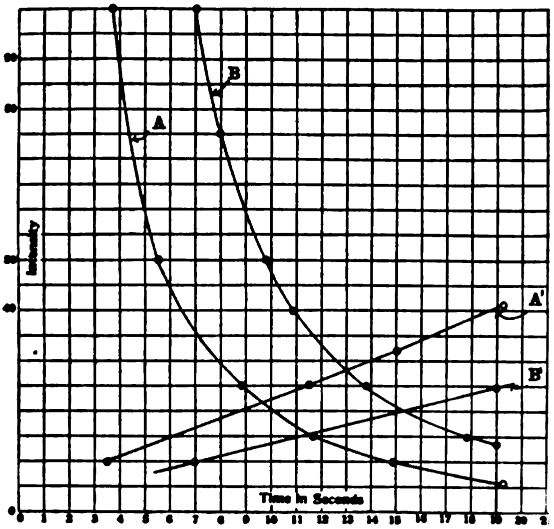

Fig. 3.

Phosphorescent Decay and the Photometric Law. *A*, curve of phosphorescent decay of nitrogen gas (active nitrogen); *A'*, same curve in which $1/\sqrt{I}$ is plotted instead of *I*; *B*, curve to show photometric decrease of intensity with distance, *or with time for uniform motion*; *B'*, same curve in which $1/\sqrt{I}$ is plotted instead of *I*.

$$I = \frac{1}{(a + bt)^2},$$

in which a and b are constants. This relation for a phosphorescent gas was first demonstrated by one of the authors of the present paper in 1907.[1] The other curve in Fig. 3, *B* and *B'* is the general photometric law,

$$I = \frac{1}{d^2}.$$

If the light I is made to recede from a screen with uniform velocity t^2 may be put in place of d^2 and under these circumstances the receding light acts as if it were a source of phosphorescence, diminishing in intensity, according to the determined law of phosphorescent decay. This circumstance affords opportunity for a study of the apparent and the real intensity in the photometry of phosphorescence. Employing the data given in the foregoing tables; first deducting the combined reaction times $(X_3 + X_4)$ from the mean positions of observed screen equality at the standard velocity of the receding light, and then calculating the intensity of the receding lamp at the corrected positions, the results shown in Table V. were secured. The figures in the last column may be

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[1] C. C. Trowbridge, Astrophysical Journal, V., 26, Sept., 1907, p. 102.

considered the corrected intensity values corresponding to three different points of a decay curve of gas phosphorescence for observer A. The intensity value in column 4 is the corrected intensity 1.

TABLE V.

Date.	No. Observations.	Standard.	Calc. Total Intensity (Moving Light).
Jan. 25 to Feb. 24	about 200	.75 I	.86 I
January 29	15	"	.85 I
March 9	15	"	.87 I
March 9	25	"	.854 I
March 15	45	"	.86– I
Jan. 25, Feb. 24	about 100	.50 I	.54 I
March 16	25	"	.55 I
January 25, February 24	85	.375 I	.41 I
March 16	30	"	.406 I

The agreement between the "corrected" intensities (last column) calculated from different experiments, is quite satisfactory, and indicates a consistent uniformity in the work done by one observer.

The time reaction of the observer[1] of the phosphorescent decays has been found to be in close agreement with the observer obtaining the results in Table V. and it seems legitimate to apply this correction for reaction time ($X_3 + X_4$) to one of the phosphorescent curves taken by the former, but the correction X_2 for the retinal fatigue used in Fig. 4 is that of the observer of the phosphorescent decay.

In Fig. 4, A shows the relation between the luminous intensity of a phosphorescent source and time (the rate of decay of gas phosphorescence, —see curve C on page 137),[1] and A' is the relation between the inverse square roots of intensity and time of the same set of observations. Curve B is obtained by correcting the abscissae of .75 I, .50 I, and .375 I on the phosphorescent decay curve, by the time value of $X_3 + X_4$, the combined reaction time, and by extrapolation of a few later points. Curve C has been corrected for both ($X_1 + X_2$) and ($X_3 + X_4$) and curve C' is the plot of the reciprocal square roots of intensity of curve C. These corrections shift the curves C and C' slightly, but do not materially affect the general relation of phophorescent intensity to time of decay.

The effect of the combined reaction times ($X_3 + X_4$) in phosphorescent measurements is to make the reading of the time nearly three tenths of a second later than it should be. The effect due to diminishing the intensity of the receding light or the decay of the phosphorescent source of

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[1] C. C. Trowbridge, PHYS. REV., XXXII., 2 Feb., 1911.

light, namely, the fatigue, is to judge the intensity equal when by the law of inverse squares it must be distinctly greater. But it so happens that for the rate of decay of phosphorescent gases, these two sources of error tend to counteract each other as is shown in Tables I., II., and III. (observer *A*), also curve *C*, Fig. 4 (observer *C*); that is, X_2 is opposite in character and in some cases is nearly equal to ($X_3 + X_4$).

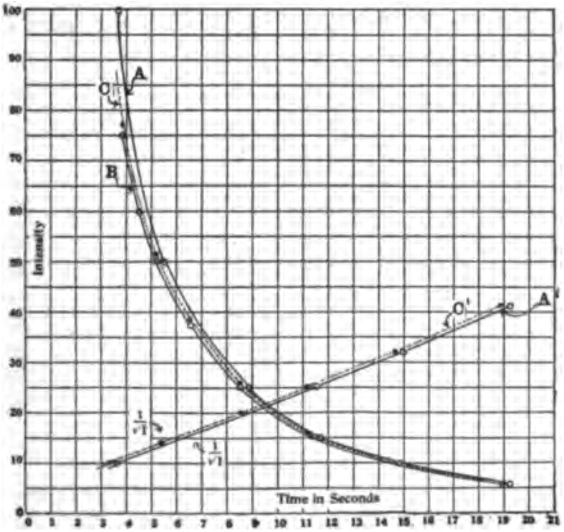

Fig. 4.

Corrections Applied to a Phosphorescent Decay Curve. *A*, curve of phosphorescent decay of nitrogen gas; *A'*, straight line form of Curve *A*, ($1/\sqrt{I}$); *B*, Curve *A*, corrected for reaction time when making observations; *C*, Curve *A* corrected for both reaction time and retinal fatigue; *C'*, Curve *C* (corrected Curve *A*) in straight line form showing little change from original Curve *A*.

In all experiments on photometry of phosphorescent sources the effect of the factor X_2 which has been assumed to be the temporary fatigue of the retina of the eye, should be ascertained. The errors due to X_2 might be of considerable importance, since failure to allow for the effect would for some persons result in photometric errors of fifteen per cent. especially with rapid phosphorescent decay.

Apparently none of the investigators who have made a study of phosphorescent decay have used this important correction; on the other hand it has required an investigation with special apparatus to determine it.

4. Use of Moving Lamp in General Photometry.

In order to determine what value the moving lamp apparatus might have for general photometry; a number of observations were taken for the equality of illumination when the moving light was *both approaching and receding from the screen* at the standard velocity, 13 centimeters per second. Here the reaction time correction cancels out and the mean position of the moving lamp is the arithmetical mean of the observed positions, subject to a correction. The three tables which follow exhibit the precision of the measurements. The observations were by three different observers, *A*, *B* and *C* at three distances of the moving lamp from the screen corresponding to .75 *I*, .50 *I* and .375 *I*, *I* being an arbitrary intensity standard.

TABLE VI.

1st Distance (80 26 Cm.), *I*=.75.

Observer.	Average Deviation Cm.	Percentage Deviation.	Percentage Error of Intensity.
A	.65	.008	.016 high
A	1.38	.016	.032 "
A[1]	.27	.003	.006 low
B	.72	.009	.018 high
B	.76	.009	.018 "
C	.60	.007	.014 low
Mean......			.016

TABLE VII.

2d Distance (97.9 Cm.), *I*=.50.

Observer.	Average Deviation.	Percentage Deviation.	Percentage Error of Intensity.
A	1.70	.017	.034 low
A[2]	.74	.007	.014 high
B	.95	.009	.018 "
C	.87	.009	.018 "
Mean...........			.021

TABLE VIII.

3d Distance (112.8 Cm.), *I*=.375.

Observer.	Average Deviation.	Percentage Deviation.	Percentage Error of Intensity.
A	.50	.004	.008 low
A	.25	.002	.004 "
A[3]	.14	.001	.002 high
B	.10	.001	.002 low
B	.36	.003	.006 high
C	.24	.002	.004 "
Mean...........			.005

[1] Lummer-Brodhun screen.
[2] Lummer-Brodhun screen.
[3] Lummer-Brodhun screen.

The Lummer-Brodhun sight box was used in one set at each intensity in order to relate the series of measurements to a well known and standard screen.

Photometrically, the third distance used (112.8 cm. from the screen) is distinctly the best since the deviation in intensity measured falls below ½ per cent. With the Lummer-Brodhun screen this could be reduced to ⅙ per cent. and might prove useful for rapid work within the limits found. The hand-made Conroy comparison screen (60° prism) was used in most of the observations because the experiments were primarily made to correspond with a screen used in the experiments in the photometry of phosphorescence where the Lummer-Brodhun type could not be used.

Table IX. gives some information about the effect (X_2) of the diminishing intensity of the receding and advancing light upon the observer's judgment, which effect has been ascribed to what is called retinal fatigue. It shows that the intensity change is different on the two slides of the theoretically correct photometric setting, necessitating a correction if the photometer is to be used to give correct candle power.

Table IX.

Rates of Intensity Change (v = 13 Cm. Per Second).

Intensity Change. Intensities *a*, *b*, *c*.	Rate of Intensity Change.	Difference.
a. 1.42 *I* to .75 *I* Rec.	.39 *I* per sec	
.46 *I* to .75 *I* Adv.	.17 *I* per sec	.22 *I*
b. .83 *I* to .50 *I* Rec.	.19 *I* per sec	
.33 *I* to .50 *I* Adv.	.10 *I* per sec	.09 *I*
c. .58 *I* to .375 *I* Rec.	.12 *I* per sec	
.27 *I* to .375 *I* Adv.	.06 *I* per sec	.06 *I*

In Table IX. the intensity of the moving light is diminished from 1.42 *I* to .46 *I* in 3.4 sec. In *b* this decrease is from .83 *I* to .33 *I*, or a change from .83 to .50 *I* of .33 *I* in 1.7 seconds. In *C* the fall is from .58 *I* to .27 *I* or a change from .58 to .375 of .205 in 1.7 seconds.

But in the last case the rise from .271 to 37.5 *I* is .105 *I*, with the light advancing. The rate of change of intensity is thus the lowest of the three cases and it is more nearly balanced by the rate of change in the opposite direction. It was also at this intensity at which the precision of the observation was found to be best.

Probably these are the conditions to be met if the instrument is to be used as a photometer.

The apparatus is to be rebuilt with various improvements and one of these is an adjustment which may be varied to secure these best conditions for any intensity to be measured and at any distance from the screen.

The error due to reaction time of the electro-magnetic recording mechanism (X_4) is very small and has so far been included with the personal reaction time. The error of judgment of screen equality (X_1) has not been separated from the effect of the diminishing intensity of the receding light. Perhaps it is not even separable.

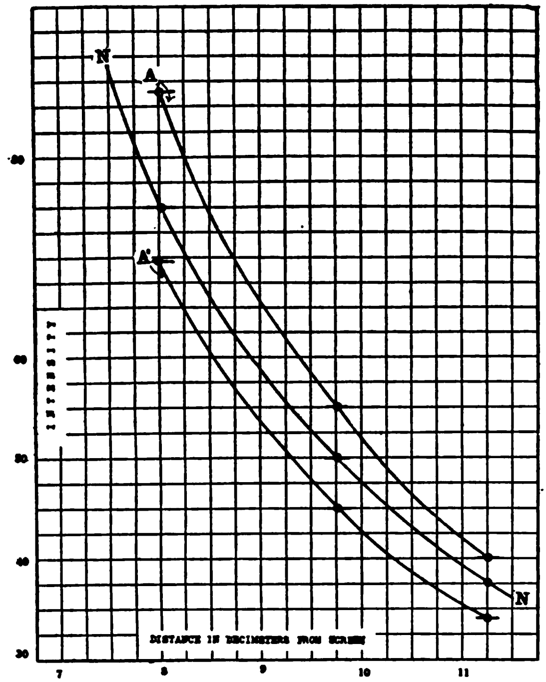

Fig. 5.

Deviations from True Intensity Curve for Both Advancing and Receding Light. *A*, three intensities receding light, *A′*, three intensities advancing light. Same observer. In obtaining these values the Lummer-Brodhun screen was used; *N*, the true intensity curve.

While these observations are only preliminary, the results nevertheless indicate that we may have there a method of photometry in which the settings are free from the trial and error method now employed. Furthermore in the latter case the movement of the screen, or of one of the lamps in the usual way when making a setting, must necessarily cause retinal fatigue effects that are confusing and which affect the photometric judgment to a certain extent at every setting, unless the movement is extremely slow.

In the moving lamp photometer the change in the retina is utilized to obtain the equality of illumination. With the method now in vogue in lamp testing laboratories, namely, that of substitution in which the lamp under test replaces the standard, and with the electric recording devices which are used, it is doubtful if the moving lamp will prove of advantage except for special purposes. On the other hand, using the substitution method with the moving lamp photometer might give the best results since the correction referred to in a previous paragraph, namely, that owing to the difference in retinal fatigue for receding and advancing light would not need to be applied.

5. RETINAL FATIGUE IN DIFFERENT PERSONS.

The moving light photometer affords a means of measuring the so-called fatigue of the retina of the eye by a new method.

Retinal fatigue is a difficult quantity to measure and has been chiefly studied by means of light from revolving discs. In the present method retinal fatigue is measured by a photometric comparison. The reaction time of the sensation wave from the retina to the brain appears to be included in the personal and instrumental reaction times ($X_3 + X_4$) and hence the quantity X_2 on which the measurement of retinal fatigue with the moving light depends, is chiefly the retina effect alone. With the small range of luminosities of the experiments described in this paper the effect of the change in intensity on the size of the pupil of the eye is probably but a small part of the value of X_2; there is, however, the possibility that the pupil change is greater than is suspected.

TABLE X.

	1912.	Observer.	No. of Obs.	Mean Distance.	Reaction Distance.	Corrected Distance.	Hand Setting.	Intensity.	
1	Jan. 25	*A*	25	79.7	3.7	76.0	80.8	.85 −	.86 −
2	Jan. 29	*A*	15	79.5	3.7	75.8	80.8	.85	
3	Feb. 24	*A*	50	78.1	3.7	74.4	80.8	.88 +	
4	Mar. 9	*A*	15	78.6	3.7	44.9	80.8	.87	
5	Mar. 9	*A*	25	79.5	3.8	75.7	80.8	.85	
6	Mar. 15	*A*	45	80.4	3.8	76.6	82.0	.86 −	
7	May 14	*A*	20	80.5	4.0	76.5	82.2	.86 +	
8	1913 Jan. 7	*A*	70	79.9	4.6	75.3	80.0	.85 −	
9	1912 May 7	*B*	25	82.1	3.7	78.4	80.7	.79 +	.80 +
10	May 14	*B*	20	81.7	3.6	78.1	81.6	.82 −	
11	May 28	*C*	20	84.1	4.1	80.0	80.9	.77 −	.77 +
12	May 28	*C*	10	83.5	4.1	79.4	80.9	.78 −	

The quantity X_2 may be measured under the following conditions; brightness of the light source, the rate at which this source is diminishing in intensity, and color. Thus far, only greenish white and yellowish white light have been used. Values of X_2 have been obtained for three persons for receding light, as set forth in Tables X., XI., XII., and XIII., and show at once that X_2 is widely different for different individuals. The effect must depend on the time of recovery after some organic change in the retinal cells when they are subjected to light and may be due in part to the rate of the replacement of the visual purple, as has been suggested.

Table XI.

	1912.	Observer.	No. of Obs.	Mean Distance.	Reaction Distance.	Corrected Distance.	Hand Setting.	Intensity.	
1	Jan. 25	*A*	100	98.5	3.5	95.0	97.8	.53	.55−
2	Feb. 23	*A*	20	99.6	3.8	95.8	99.9	.54	
3	Mar. 16	*A*	25	95.4	3.8	91.6	96.2	.55	
	1913								
4	Jan. 10	*A*	40	96.6	4.6	92.0	97.9	.57−	
	1912								
5	May 7	*B*	20	98.0	3.7	94.3	97.5	.53+	.53+
6	May 28	*C*	30	100.0	4.1	95.9	97.5	.52−	.52+
7	May 28	*C*	10	99.7	4.1	95.6	98.4	.53−	

Table XII.

	1912.	Observer.	No. of Obs.	Mean Distance.	Reaction Distance.	Corrected Distance.	Hand Setting.	Intensity.	
1	Jan. 31	*A*	55	110.4	3.5	106.9	111.6	.41	.40+
2	Feb. 23	*A*	60	111.6	3.8	107.8	112.8	.41	
3	Mar. 16	*A*	30	110.3	3.8	106.5	110.8	.40+	
4	May 14	*A*	20	114.3	4.1	110.2	113.6	.40−	
	1913								
5	Jan. 17	*A*	50	112.3	4.5	107.8	113.2	.41	
	1912								
6	May 7	*B*	25	111.9	3.7	108.2	111.3	.39	.39
7	May 7	*B*	25	113.0	3.7	109.3	111.3	.39−	
8	May 14	*B*	25	113.9	3.7	110.2	113.2	.39+	
9	May 28	*C*	30	114.5	4.1	110.4	111.5	.38	.38+
10	May 28	*C*	15	112.9	4.1	108.8	111.2	.39	

In Table X. the intensity of the moving light is that which has been called .75 *I*. The apparent intensities, due to retinal fatigue, X_2, are for three individuals, *A*, *B* and *C*, .86−, .80+ and .77 respectively.

In Table XI. the results for intensity .50 *I* are given. These are for

A, *B* and *C*, .55, .53 and .52 respectively, in Table XII. the results for intensity .375 *I* are .40+, .39 and .38. These results are strikingly represented in Fig. 6 by three curves showing the apparent intensities as modified by retinal fatigue, and the curve showing the true intensities.

In a series of measurements extending from January 1912 to January, 1913, observer *A*'s results recur to the intensity values 86, 55 and 40 instead of true values of 75, 50 and 37.5. The prejudgment of the light value thus varies in a consistent manner at different intensities in the case of the persons tested. The effect might be explained by saying that with the receding lamp the retina is fatigued by the brightness of the light while at the near point. When the lamp is advancing there is a similar prejudgment also; for example, with the light approaching the screen, observer *A* obtained 70, 45 and 35 instead of 75, 50 and 37.5. The full significance of this effect ascribed to retinal fatigue is not understood at present; but the instrument has obvious possibilities for the study of some optical and psychological questions.

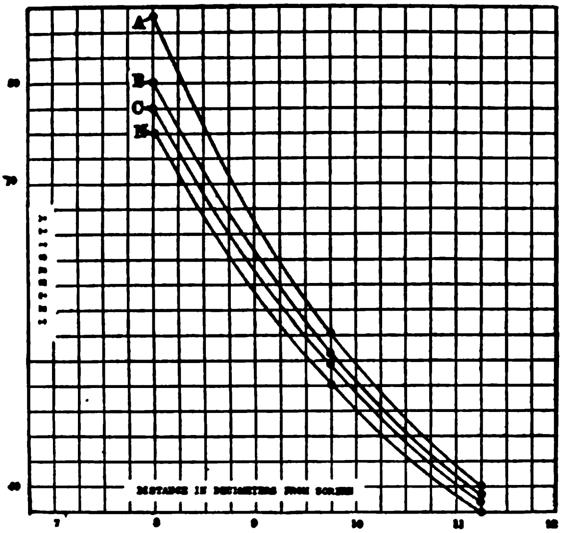

Fig. 6.

Deviations Shown by Three Observers *A*, *B* and *C*, from True Intensity Curve Due to Retinal Fatigue when Using a Light Receding with Uniform motion. *N*, true intensity curve (hand settings); *A*, *B*, and *C*, observed intensity curves (moving light).

In Table XIII. is given the percentage deviation from true intensity due to retinal fatigue for the three observers *A*, *B* and *C*. The actual rates of diminution of intensity of the stimuli producing a uniform field of light on the retina of the eye are also given. The intensity *I* taken as an arbitrary standard is equivalent to about 3.0 lux (meter candles).

The figures given are purely relative, however. If a study of retinal fatigue is undertaken by this method it would seem a simple matter to standardize the measurements; using the Lummer-Brodhun screen as the standard of comparison for all observations and expressing the rate of diminution in decrease in lux per second.

Another method would be to adopt some prism screen standard, diaphragmed to a definite size, and specify the decrease in intensity of the light in factions of a lumen per second. The field in this case might well be a meter's distance from the eye viewed through a collimator.

TABLE XIII.

Observer.	True Intensity.	Mean Rate of Change of Total Intensity.	Observed Intensity.	Difference of Observed from True Intensity.	Per Cent. Deviation from True Intensity.
A	.75 *I*	.63 *I* per sec.	.86 *I*	.11 *I*	14.6
B	.75	.63	.80 *I*	.05 *I*	6.6
C	.75	.63	.77 *I*	.02 *I*	2.6
A	.50 *I*	.39 *I* per sec.	.55 *I*	.05 *I*	10.0
B	.50	.39	.54 *I*	.04 *I*	8.0
C	.50	.39	.52 *I*	.02 *I*	4.0
A	.375 *I*	.28 *I* per sec.	.41 *I*	.035 *I*	9.3
B	.375	.28	.39 *I*	.015 *I*	4.0
C	.375	.28	.38 *I*	.005 *I*	1.3

SUMMARY.

In conclusion the paper may be summarized thus:

1. A simple mechanism for producing a uniform motion of a lamp in the photometric axis with the method of recording observations, while the lamp is in motion, is described.

2. Analysis of the following corrections that must be applied to the observations is made:

   (*a*) Personal equation of the photometric judgment, X_1.
   (*b*) Retinal fatigue, X_2.
   (*c*) Reaction time of the observer, X_3.
   (*d*) Reaction time of the recording device, X_4.

3. By means of the receding lamp moving at uniform velocity, it has been possible to investigate quantitatively the errors in photometric measurements of phosphorescent decays. These errors may run as high as fifteen per cent. or more for rapid decay, and are apparently chiefly due to fatigue of the retina of the eye.

It should be added that the corrections applied to the decay curves of phosphorescent gases and phosphorescent solids to allow for retinal fatigue would not alter the general laws of decay which have been formulated by those who have made the subject a study, but the corrections would shift the curves to some extent.

4. The moving lamp photometer has been studied with a view to its use in general photometry and it has been shown that at least in special investigations it would be advantageous since it has the following desirable features:

(*a*) It is more rapid than hand adjustments.

(*b*) There is no possibility of muscular coördination control as in hand movements, the settings being strictly eye judgments.

(*c*) The effect called retinal fatigue is used to obtain positions of screen equality, while by hand adjustments this fatigue is usually a source of error in each setting.

(*d*) The settings appear to be more accurate than hand adjustments; yet it is doubtful if the moving lamp would prove of advantage over the methods used in lamp testing laboratories where an electric recording device is used and the substitution method of comparison of lamps is employed.

(*e*) When this moving lamp device is used for general photometry the substitution method should be employed, if a correction is to be avoided.

5. The rôle played by the fatigue at the retina of the eye in photometry is shown and figures are given for this effect in the case of three persons showing a marked difference of retinal fatigue among the three individuals tested.

Phœnix Physical Laboratory,
Columbia University,
June, 1914.

CHARACTERISTICS OF CONTACT RECTIFICATION WITH A SILICON CARBON CONTACT.

BY RALPH C. HARTSOUGH.

INTRODUCTION.

During the last thirty-five years much experimental evidence has been brought to bear upon the phenomenom that a large current will pass in one direction and small current, if any at all, in the opposite direction, through contacts of certain dissimilar solids.[1] Though the greater part of these investigations have been valuable, there are many unsettled and disputed problems connected with the phenomenon of contact rectification. The question as to whether we are dealing with a film effect, the chemical or physical structure of certain materials, or the mere ease of giving up electrons, has received more or less attention. There has been some disagreement as to the area of contact in its effect upon the rectifying property. Some investigators have had much difficulty with the uncertainty of the rectifying properties of the contacts used. Among these uncertainties three are very much in evidence: (1) the direction of flow of rectified current was different with various voltages; (2) at various places on the materials used there would be lacking a rectifying property and often different places on the same specimen would rectify

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[1] Ferdinand Braun, Pogg. Ann., 153, p. 556, 1874; 157, p. 350, 1878. F. Streintz, Akad. Wiss. Wein. Sitz. Ber. III., 2a, p. 345, 1902. P. G. Nutting, PHYS. REV., 19, p. 1, 1904. L. W. Austin, Bull. Bur. Standards, 5, 1, p. 133, 1908. G. W. Pierce, PHYS. REV., 28, p. 153, 1909. 29, p. 478, 1909. A. E. Flowers, PHYS. REV., 3, p. 25, 1914. R. H. Goddard, PHYS. REV., 34, p. 423, 1912.

in opposite directions; (3) the rectifying property would be lost by increasing the voltage a very slight amount.

These difficulties and many more of a minor character have greatly hindered the bringing of conclusive evidence to bear upon this most interesting phenomenon.

EXPERIMENTAL WORK.

After a careful consideration of all the former experiments on the phenomenon of contact rectification, it seemed that to make any subsequent and real additions to previous investigations, a more perfect pair of contacts would have to be found. It seemed from the standpoint of convenience and accuracy that, if a rectifying contact could be found which would stand up against from five to ten volts and give 0.5 to 1.0 ampere rectified current, we should then be in a better position to study its characteristics. Also a very important feature for this proposed contact would be durability of its surfaces as well as permanency of the adjustment of contact of its surfaces. Durability here calls for absence of any change, physical or chemical, which would alter its rectifying action. By the durability or permanency of adjustment of the contact is meant that a certain pressure would remain the most efficient in the rectifying action under given conditions.

A systematic search was begun with the above conditions in mind. With all the better contacts used by former experimenters and a long list of metals and non-metals, a selective and sorting arrangement was devised.[1] About one hundred different metals, alloys and non-metals were included in this process, which was essentially as follows: All which apparently allowed more current to pass from than to the substance were put in class *A* and the opposite in class *B*. Those which were uncertain in their action were placed in another series. After these three series had been carefully worked over, a cross-testing process was carried on involving the use of both alternating and direct current in each of about one thousand pairs of contacts tested. It was found that a silicon carbon contact most nearly fulfilled the proposed ideal conditions for a contact as outlined above. However, after much careful investigation with various grades of silicon, furnished through the courtesy of The Carborundum Co., of Niagara Falls, New York and many grades of arc carbon furnished through the courtesy of the National Carbon Co., of Cleveland, Ohio, it was found that the purer material up to a certain point gave the best results. A grade of silicon and carbon was found which admirably filled all the hoped-for essentials.

[1] Some of this work was done in Nebraska Wesleyan University.

CONSTRUCTION OF CONTACT AND METHODS OF CONNECTION.

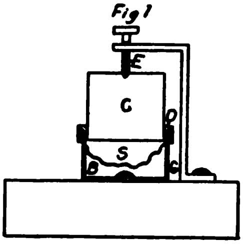

Fig. 1.

In Fig. 1 is given a sectional view of the arrangement of the contact. *S* is the silicon mounted in molten type metal and allowed to cool. *C* is arc carbon, insulated and held in place away from the brass cup container *g*, by a hard rubber ring *D*. *E* is a pressure adjusting screw.

To connect the rectifier so as to obtain a total effect of the a. c. current as d. c. Two methods of connection were experimented with. In Fig. 2 is essentially the aluminum valve rectifier connection. This arrangement requires no auxiliary devices and is not bothersome from that standpoint. However, to get the most efficient effect, all contacts should be adjusted to work the same under like conditions. In Fig. 3 is shown rectifier connected after the manner of a mercury arc rectifier. All together the writer had better satisfaction from this connection than from the previous one in Fig. 2.

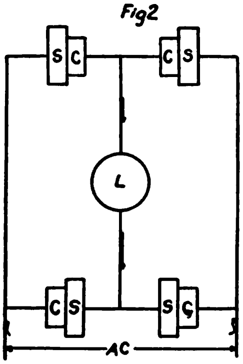

Fig. 2.

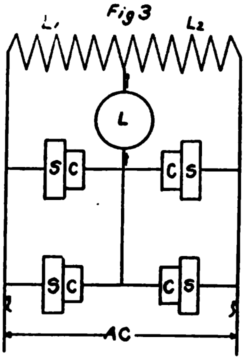

Fig. 3.

EFFECT OF TEMPERATURE AND LOW AIR PRESSURE.

A slight unsteadiness was noticed when the temperature was about 300° C., but aside from this, temperature seemed to have no effect whatsoever. There is reason to believe that the unsteadiness observed was due to expansion in the metals and thereby a change in pressure. No effect was noticed when the contact was placed in a vacuum of one mm. pressure. It apparently worked as well as when in the normal air pressure.

DISCUSSION OF SURFACES.

The surfacing and finishing as well as a suitable mounting of the silicon received not a little attention. The grinding was done on a carborundum wheel, but the finishing, a most important part, was accomplished on a thick plate glass, using alcohol and carborundum powder to bring it to as perfect a plane surface as could be obtained. It was found best not to put a high polish on this silicon surface. The surfacing of the carbon was essentially the same as that of the silicon. All surfaces were made chemically clean and dried thoroughly.

It is very hard to make even a good estimate of the amount of surface in contact after using the best of care in preparation and manipulation. However, two points applied gave approximately twice the magnitude of rectified current that one point gave. Also, two pairs of surface areas, surfaced exactly the same way, one area being only a little over twice that of the other, gave very approximately twice the rectified current. We are led to conclude that the magnitude of the rectified current is approximately proportional to the area of contact.

Ratio of Contact Areas.

Observer: R. C. H. March 30, 1914.

Conditions:

1. All carbons surfaced by same process.
2. Same voltage used throughout test (5.0 volts, a. c.).
3. Same surface of silicon used throughout test.

Carbon, Area $\overline{\text{Cm.}}^2$	Force in Grams.	Rectified Current, Amperes
One point[1]	30	0.050
Two points[2]	30	0.095
1.34	50	0.115
3.44	100	0.050

Discussion of Photographs of Surfaces.

The character of the surface of both the silicon and the carbon is so closely allied with efficient rectification that a very thorough study of these surfaces was carried on. Many degrees of fineness of grinding compounds was experimented with to ascertain if possible to what extent the smoothness of surface affected the rectifying property. In photograph No. 1, is shown the best rectifying surface of silicon magnified to 400 diameters. No. 3 is the best rectifying surface of carbon (same magnification). No. 2 is a highly polished surface of silicon and No. 4 is a polished surface of carbon. Both No. 2 and No. 4 are magnified 400 diameters. Below is given a summary of the action of these photographed surfaces in experiment.

---

[1] Point of same order as knitting needle.

[2] Thirty grams force on each point.

Observer: R. C. H. April 21, 1914.

Conditions:

1. Same voltage throughout test (5.0 volts a.c.).
2. Same method used for surfacing throughout test.

Surfaces.	Contact Area $\overline{\text{Cm.}}^2$	Degree of Rectification.	Pressure per $\overline{\text{Cm.}}^2$
No. 1 and No. 4..........	1.87	slight	300 grams
No. 1 and No. 3..........	3.44	complete[1]	40 grams
No. 2 and No. 3..........	3.44	fair[2]	60 grams
No. 2 and No. 4..........	1.87	poor	500 grams

To Show Forming Process.

Observer: R. C. H. April 14, 1914.

Conditions:

1. Started freshly surfaced silicon and carbon.
2. Same surfaces kept in contact throughout test.
3. Used 3.44 $\overline{\text{cm.}}^2$ area carbon and silicon surface with 30 grams per sq. cm.

Time.	P.D. Contact.	Approximate Rectification Ratio.[3]	Rectified Current Amperes.
9.15 A.M................	3.8	35 : 1	0.23
9.17 A.M................	5.8	50 : 1	0.64
9.20 A.M................	6.1	40 : 1	0.86
9.22 A.M................	6.3	100 : 1	1.10
9.23 A.M................	3.6	300 : 1	.56

After looking into the results of the different grades of surfacing and after carefully studying the photographs of surfaces, we are led to think that the contact rectification takes place through the action of small points. There is no doubt in the writer's mind that the "forming effect," mentioned above, is a bringing of more points on the surface into activity. In the photographs No. 1, a good rectifying surface of silicon shows a large number of points as compared to No. 2, a poor rectifying surface of the same silicon. Surface No. 2 was afterward made a good rectifying surface by finishing as No. 1 was surfaced. The size and number of these projections are vitally tied up with the rectifying property. Pressure would bring more points into contact, theoretically, but we find that increased pressure does not increase the rectifying properties of a surface. Pressure evidently has a crushing effect on the points of the surface, thereby destroying the property of rectification. The writer's theory is that in the "forming processes" there are points which

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[1] These surfaces could be formed so as still to give complete rectification with an applied voltage of 10–12 volts and rectified current of 1.0 to 1.5 amperes.

[2] These surfaces would not form. By "Forming" we mean that a better rectifying surface resulted from certain manipulation as given below.

[3] Rectification ratio is the area of the current curve above compared to that below the zero line. The current curves were studied by the use of the oscillograph.

are very close yet not active as rectifiers and an increase in voltage causes an arc across these points bringing them into contact. This theory is compatable with the fact that two surfaces which have been "formed," when separated have to be "re-formed" again the same as the first time. This shows that there is no chemical formation and that this forming is evidently a point phenomenon. A surface once "formed" will hold constant as a rectifier as long as it is undisturbed. With the carbon surfaces the same conclusions are drawn. No. 3, a good rectifying surface, shows more point area than No. 4, a poor rectifying surface. All surfaces were illuminated from the side in photographing them.

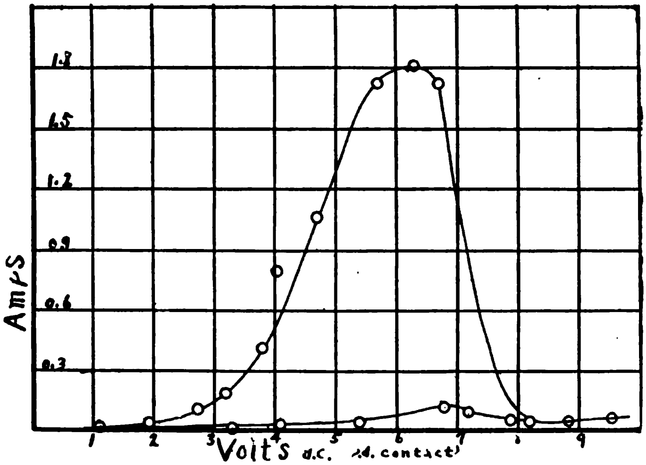

Fig. 4.

Current E.M.F. Data.

It was in this part of the experimental work that an effect was noticed which started a long search for a time element in the rectifying action. This effect was the fact that a rectifier which would give complete rectification with an alternating E.M.F., would allow, with equivalent value of direct E.M.F., as much as fifteen per cent. to twenty per cent. of the current to pass in the high resisting direction. Fig. 4 shows current E.M.F. curved with the direct current. The contact used

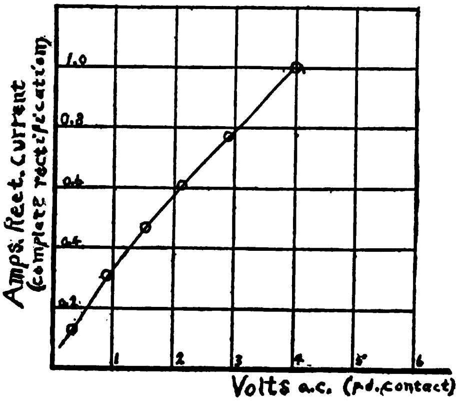

Fig. 5.

gave complete rectification with alternating E.M.F. at 5.0 volts. The upper curve gives the current when the voltage was applied from silicon to carbon, the lower curve, the corresponding current when the voltage was applied in the reverse direction. These results could be repeated time after time. Here is given a set of readings very much like the readings from which the curve in Fig. 4 was plotted.

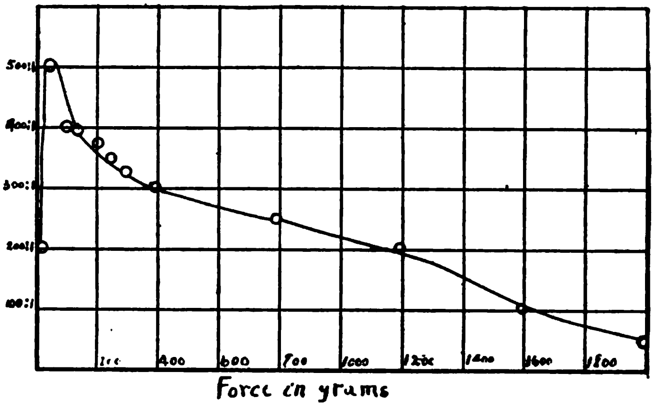

Fig. 6.

Direct Current Applied to Contact.

Observer: R. C. H. March 27, 1914.

Silicon to Carbon.		Carbon to Silicon.	
Volts (P. D. Contact.)	**Amperes.**	**Volts (P. D. Contact.)**	**Amperes.**
1.14	0.013	1.15	0.004
2.03	0.067	2.04	0.02
2.87	0.114	2.93	0.018
3.1	0.162	3.2	0.018
3.9	0.30	4.0	0.01
4.7	0.455	4.85	0.017
5.0	0.50	5.4	0.017
6.0	0.70	6.1	0.017
6.8	0.2	6.9	0.018
7.2	0.007	7.2	0.007
8.1	0.024	8.2	0.011
9.0	0.018	9.0	0.013
9.3	0.018	9.3	0.013
10.4	0.018	10.4	0.018

(Storage Battery used)

Fig. 5 is a current-voltage using alternating E.M.F.

Current Pressure Data.

That pressure is closely related to the rectifying property has been pointed out by almost every experimenter, but I believe not one has

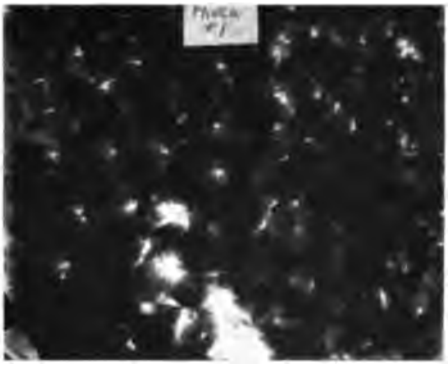

Fig. 1.

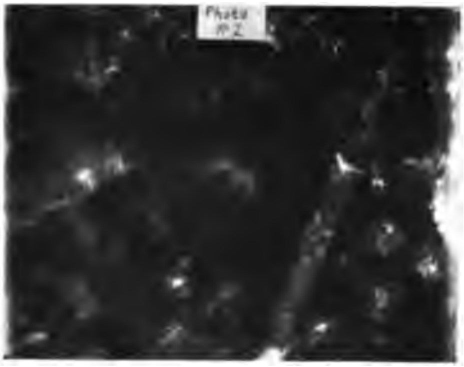

Fig. 2.

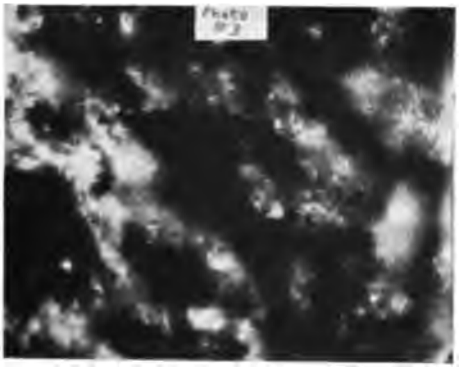

Fig. 3.

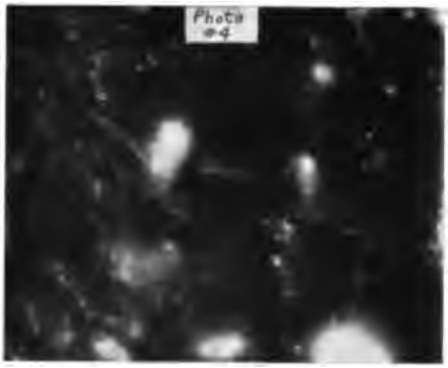

Fig. 4.

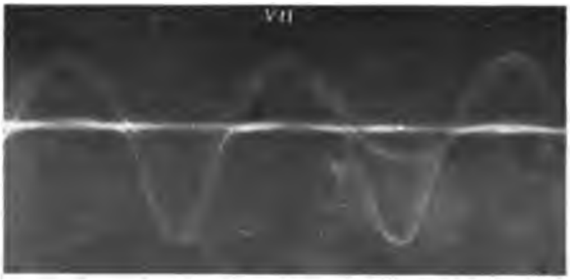

Fig. 7.

RALPH C. HARTSOUGH.

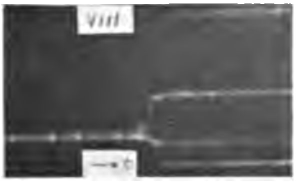

Fig. 8.

Fig. 9.

Fig. 10.

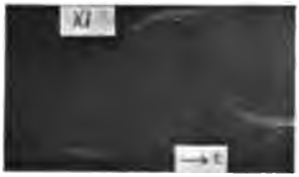

Fig. 11.

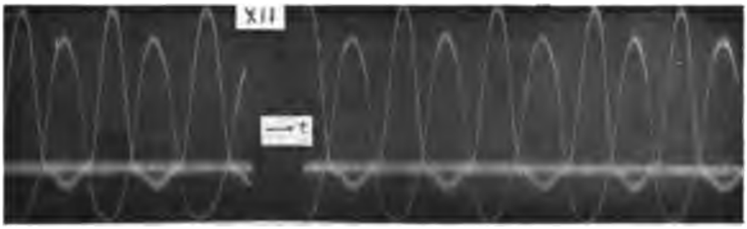

Fig. 12.

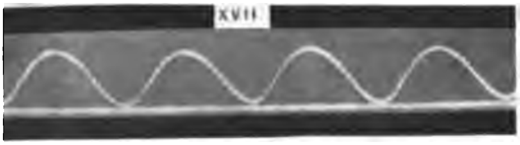

Fig. 17.

RALPH C. HARTSOUGH.

shown the exact relation. The following readings are characteristic of many taken. They show the effect of pressure, not only upon rectification but also upon the potential drop.

Observer: R. C. H. April 22, 1914.

Conditions:

1. Same voltage throughout experiment (5.0 volts a.c. applied).
2. Same contact area (3.44 $\bar{\text{cm}}.^2$).

Volts P.D. Contact.	Estimated Rectification Ratio.	Forces in Grams.
4.25	200 : 1	25
3.85	500 : 1	50
3.70	400 : 1	100
3.65	400 : 1	150
3.60	375 : 1	200
3.55	350 : 1	250
3.50	325 : 1	300
3.40	300 : 1	400
3.15	250 : 1	800
3.00	200 : 1	1,200
2.80	100 : 1	1,600
2.25	50 : 1	2,000

Fig. 6 is a curve plotted from these results.

Discussion of Oscillograph Curves.

Of the many investigators who have used the oscillograph in studying the peculiarities of contact rectification we believe none have reported the fact before mentioned—that a rectifier giving complete rectification, as in Fig. 7, will allow, with direct E.M.F., some current to pass in the high resisting direction. It was thought that by the sudden application of direct E.M.F. in the high resisting direction a building up process, if present, would be detected by a lag in the current curve. Fig. 8 is a result of such manipulation, while Fig. 9 is the result of an equivalent non-inductive resistance replacing the rectifier. No difference in slope of the two curves can be detected. Then the apparatus was arranged to get the current curve from zero to low current value, reversed to high current value, finally to zero, which is shown in Fig. 10. The heavy line is the current curve, the lighter one the E.M.F. This total change on reversal of current occurred in about 1/100 of a second. Fig. 11 represents the same manipulation having the rectifier replaced by an equivalent non-inductive resistance. No difference was detected in the two curves.

Alternating current was now resorted to as offering an explanation of the time element in the rectifying action. In Fig. 12 is given a curve of

60-cycle current beginning with the instant of application to the rectifier. Within the twelve inches of film which revolved on the drum at a speed of 600 R.P.M., a reducing of the current curve in the high resisting direction is noticable. There are 1½ cycles missing on the print as the twelve-inch film was too long for the paper. Time goes from the break toward the right of the page, then from the left up to the break again. Fig. 7 is an oscillograph record taken on the rectifier after operating a very short time, of the order of one and two seconds. These curves clearly show a building-up effect which almost completely cuts off current in one direction. To get a complete record of this process as well as the time element would require a record six to ten feet long, perhaps more, traveling at a speed of 600 feet per minute. This was not available. Lower frequencies were investigated as to the effect they might have on this action. A thirty-cycle also a fifteen-cycle current did not show any marked difference in contrast with the 60-cycle current in Fig. 12. Fig. 17 is a record of the current taken with the apparatus connected as in Fig. 3. This is complete rectification utilizing both halves of the a.c. wave.

CONCLUSIONS.

The most popular explanation of contact rectification is that electrons are given up more easily by some solids than others, and that the rectification ratio is of the same order as the ratio of emission of electrons at the contact of dissimilar solids. If the "forming theory," given in this paper is correct, the theory of the passage of electrons is still intact. It seems from Fig. 12 that, if the frequency should be increased many times and a similar record taken, we might get a larger amount of current in the high-resisting direction, for the first few cycles. The writer intends to try this out at a later date.

Also the "forming action" will require not a little study in its details to understand more accurately what happens.

The writer wishes here to express his appreciation and thanks to Dr. F. E. Kester, of the University of Kansas, for his untiring help and inspiration to me in this work. To those in the department of physics who so kindly helped in many ways my sincere thanks are given.

Much of this investigation was made possible through the courtesy of Professor C. A. Johnson, of the school of electrical engineering of the University of Kansas, who gave of his time to furnish needed apparatus. The writer wishes to express his indebtedness to him.

BLAKE PHYSICS LABORATORY,
UNIVERSITY OF KANSAS,
May 21, 1914.

MAGNETIC RESISTANCE CHANGE OF PURE IRON.

By R. A. Heising.

About the year 1856, Kelvin noticed that the resistance of metals changed when placed in a magnetic field. Since then a score of men have investigated these changes for all the principal metals. Among them may be mentioned Beattie,[1] for iron, cobalt and nickel, Patterson,[2] for non-magnetic metals, Williams,[3] and Knott,[4] for magnetic metals, Heaps,[5] for iron and other metals, and Grunmach,[6] on various metals. It was early noted that the behavior of the magnetic metals was entirely different from that of the non-magnetic metals. In the case of the non-magnetic metals, the resistance is increased by transversal magnetization while in the magnetic metals, a decrease is predominant. In the case of iron, the most important magnetic metal, the results of the different investigators varied, there being about as many different results as there were investigators. The primary cause was a difference in the iron used. Since the invention of the method of producing pure electrolytic iron by Burgess and Hambuechen[7] made it possible to secure pure iron in quantities sufficient for investigation of its properties, it was thought worth while to find the resistance change of pure iron and see if it threw any light on the subject.

The iron used in this work was electrolytic iron whose greatest impurity was hydrogen, which was about .072 per cent. and which was over twice the combined weights of all other impurities. The wire was made by Dr. Roebuck in 1911. It was drawn to size 36 through diamond dies and annealed between drawings. The wire was annealed at about 700° in order to soften and to drive off the hydrogen.

The longitudinal measurements were carried up to $H = 1{,}700$ only, at which point the longitudinal change almost reaches a maximum. A solenoid was used for the purpose. The solenoid was made of a water jacketed brass spool with an inside diameter of 2.2 cm., 1 meter long,

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[1] Phil. Mag., Mar., 1898.

[2] P. M., Vol. 8, 1902.

[3] P. M., Vol. 6, 1903; Vol. 9, 1905.

[4] Proc. R. S. E., Vol. 33, p. 200, 1913.

[5] P. M., Vol. 22, 1911.

[6] Ann. d. Phys., Vol. 22, 1907.

[7] Iron and Steel Mag., Vol. 8, 1904.

wound with 3,096 turns of No. 14 D.C.C. wire in six layers with a mean diameter of 4.2 cm. The specimen consisted of two pieces of wire each 40 cm. long stretched on opposite sides of a thin strip of wood and soldered at the upper ends to manganin leads and at the bottom to a piece of copper tubing which slipped over the end of the wood strip. The wires were of such length that the demagnetizing factor was practically zero. The whole specimen was placed inside a long thin Dewar flask filled with kerosene. The Dewar flask was used to prevent rapid changes in temperature due to the solenoid current or to changes in temperature of the water through the jacket.

The field was computed from the ammeter readings. The ammeter was calibrated before and after using and no change found. The computed values of field checked within a small fraction of a per cent. with the measured values found by placing a coil of known dimensions inside the solenoid and comparing the mutual inductance with that of a standard mutual inductance.

The resistance change was measured with a slide wire bridge as shown in Fig. 1.

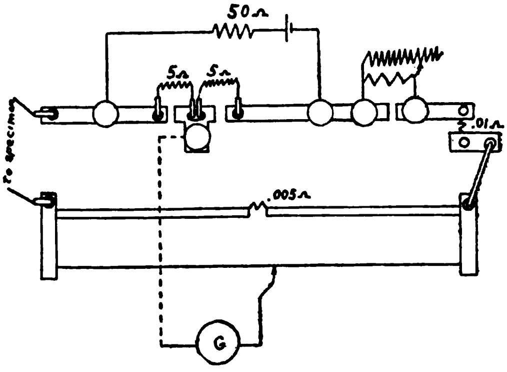

Fig. 1.

The specimen was demagnetized by sending an alternating current through the solenoid and gradually decreasing the current. Care was taken to see that the mutual inductance between the solenoid and bridge was eliminated so that any change in current caused by the slow heating of the solenoid would not give spurious points of balance. That was secured by throwing the solenoid current on and off when the battery was off the bridge and moving the leads to the specimen until there was no throw of the galvanometer.

Readings were taken in the following manner: bridge balanced with the specimen demagnetized, solenoid current thrown on, bridge balanced again, all readings recorded, current thrown off, specimen demagnetized, and bridge balanced again to see if the balance point was the same as at first. No correction was necessary for the leads as they showed no resistance change.

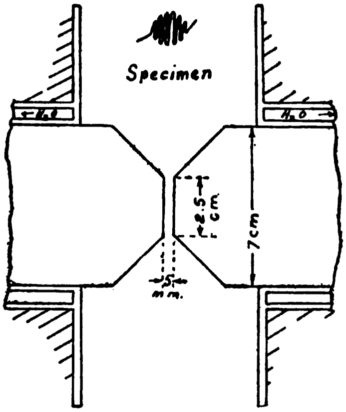

Fig. 2.

For the transverse effect, the first specimen of electrolytic iron used was single silk-covered wire wound in the form of a flat coil, non-inductively, and held between two discs of hard rubber. The whole was immersed in melted rosin. The thickness of the specimen and discs was such as to just fit the gap between the pole tips. It was thought that this would hold the wire perpendicular to the field. The curve secured showed an increase in resistance at low fields similar to the curves for iron by others. Using nickel, Jones and Malam[1] showed that the curves of similar shape secured by various men for that metal were wrong in that if the wire was within a fraction of a degree of being perpendicular to the field, no resistance increase would be found at low fields. In order to see if this was true also for iron and if the increase found was due to kinks or incorrect setting in the field, a second attempt was made using greater care in preparing and mounting the specimen.

The wire in this case was bent in the form of a grid as shown in Fig. 2, and clamped with pieces of mica between brass plates in order to hold it flat. The annealing was done in a vacuum after bending in order to remove all strains introduced in bending. Leads of No. 36 copper wire were soldered to the iron wire and extended to the edge of the plates where they were soldered to No. 25 wire which led to mercury cups on the outside of the plates for purposes of connection. Beside the iron wire between the plates was placed a coil of fine copper wire between mica plates and placed in the other arm of the bridge to prevent troublesome temperature variations. The copper wire coils had similar mercury cup contacts on the outside. Both coils were made non-inductive by bringing the leads back across the outside directly over the coils. A correction in $\Delta R/R$ was made for the copper wire in the field by determining its resistance change separately, it being comparatively easy as its values fall on a smooth curve. The brass plates with specimen were mounted

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[1] P. M., Vol. 27, Apr., 1914, p. 649.

in a gimbal like support to allow of adjustment until perpendicular to the field. The leads to the bridge were of twisted No. 12 copper wire. Asbestos was fastened to the outer sides of the plates to aid in preventing rapid changes in temperature.

The coils of the magnet were water cooled. The space between the coils and surrounding the specimen was filled with cotton-batting. The temperature regulation was solely by the cooling water. To prevent sudden changes, it was passed into a tank before coming to the magnet so that small changes in temperature would be averaged over a long period of time. A thermometer was placed near the specimen at the edge of the pole tips to determine the temperature. Trouble from conduction from the magnet coils was encountered only at the very highest fields when the current was exceedingly large. However, there was sufficient time for observation before such heat reached the specimen.

A calibration curve of the field strength in terms of the exciting current was made before and after using, without the specimen in. The readings were made using a flip coil and ballistic galvanometer and compared with a standard drop magnet and a standard mutual inductance. 3, 15, and 75 amp. shunts were used on the ammeter to get the readings on the upper part of the scale.

The same bridge was used in this work as in that on the longitudinal effect.

Readings of resistance change were taken from the point of retentivity of the magnet. As the resistance change for the transverse magnetization at the field strength in the gap due to the retentivity of the magnet was practically zero (being too small to determine with the apparatus used) it was not necessary to demagnetize the specimen each time. The field in the gap with no exciting current flowing was about 200 gausses. The demagnetizing effect of the specimen even slightly magnetized, reduces the internal field to a neglible quantity. Approximately the same manipulation was gone through in taking these readings as was done with the solenoid.

On energizing the magnet, the magnetism did not reach its final value for about 5 seconds. This was allowed for in taking readings.

LONGITUDINAL EFFECT.

A preliminary run was made for the longitudinal effect using ordinary soft iron wire. The first curve secured did not coincide with those that followed. The later ones were above at the start and fell below at larger fields. There seemed to be a continual shift in that direction for four or five runs after which there was no further change. The same thing was noted in the electrolytic iron. This was evidently due to magnetic hardness. It disappeared after repeated magnetizations. The longitudinal curve for the electrolytic iron, Fig. 3, was plotted from a set of about 250 readings. Of these, 90 per cent. fell within .8 per cent. of the values represented by the curve and 50 per cent. came within .3 per cent. of the values shown by the curves. This last is about as close as the individual readings could be trusted, so that considering the number of readings we can estimate the values as within .1 per cent. of being correct.

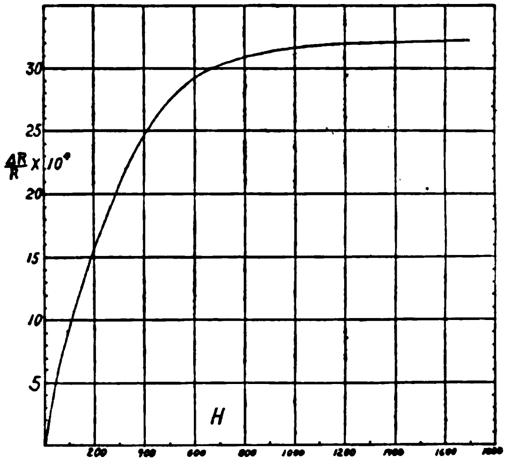

Fig. 3.
Longitudinal Effect.
$\Delta R/R$—external field.

In the longitudinal effect, a small "hysteresis" was noted such as is mentioned by Barlow[1] for nickel and iron and Jenkins[2] for nickel.

The longitudinal effect was carried up to only 1,700 C.G.S. units field strength. The effect reaches a maximum a short distance further and is but little above the value at 1,700. That the longitudinal effect reaches a definite maximum seems evident from the work of Heaps.[3] The slight drop in his curve at high fields is ascribed to the transverse effect where the wire is bent around the mica. Williams[4] with iron at high temperatures finds no decrease for fields several times the value giving complete saturation. Jones and Malam[5] using nickel show that it is reasonable to expect a definite maximum in the longitudinal effect with

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[1] Proc. R. S. L., Vol. 72, Apr., 1902.

[2] P. M., Vol. 27, Apr., 1914, p. 731.

[3] P. M., Vol. 22, 1911,

[4] P. M., Vol. 6, 1903; Vol. 9, 1905.

[5] P. M., Vol. 27, Apr., 1914, p. 649.

no increase or decrease afterward at higher fields. The most interesting part of the curve however, is the part up to $H = 1,700$.

The longitudinal effect when plotted as $\Delta R/R$ against H show no direct relation to the magnetization, but rises steadily long after reaching that field strength which causes the bend or knee in the magnetization curve, and slowly approaches a maximum shortly beyond $H = 1,700$. However, this maximum occurs at practically the same field strength that produces saturation. If the effect is plotted as a function of the intensity of magnetization, another relation is apparent. In Fig. 4, Curve A is plotted in this way and on the same sheet is plotted the magnetization curve for electrolytic iron (from Terry, PHYS. REV., Vol. 30, 1910) with $4\pi I$ horizontal and H vertical to the values of H to which he carried it. The whole line parts of the curves are known, while the dotted parts are estimated values considering the saturation point of the iron as $4\pi I = 20,400$. It is seen from the curves that the magnetization curve and the $\Delta R/R = 4\pi I$ curve turn at the same values of I. Over 90 per cent. of the resistance change occurs during the final rotation of the elementary magnets which produces the last quarter of the saturation value. It is to secure this last orientation that 98 per cent. of the field is applied.

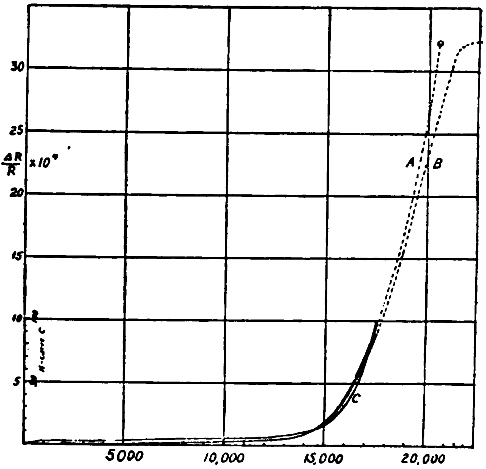

Fig. 4.
Longitudinal Effect.
Curve A, $\Delta R/R$—$4\pi I$
Curve B, $\Delta R/R$—flux density.
Curve C, H—$4\pi I$ (magnetization curve).

Williams[1] has secured longitudinal $\Delta R/R - H$ curves for nickel up to the critical temperature and for iron almost there. His work shows the

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[1] P. M., Vol. 6, 1903; Vol. 9, 1905.

dependence of the change of resistance on the magnetic properties of the metals since the change for nickel disappears with the magnetic properties, and the $\Delta R/R - H$ curves for iron show a shifting of the maximum point to the smaller fields at high temperatures, a fact which is true also of the saturation point. All this tends to confirm the statement that when saturation is complete the change in resistance is a maximum.

The chemical composition of the metal has much to do with the result as is shown by the comparison of maximum values. Heaps gets a maximum of 28×10^{-4} for soft Norway iron and 13×10^{-4} for piano wire at 23° while the maximum for electrolytic iron is about 32×10^{-4} at 6°. The difference in temperature could hardly make the difference.

TRANSVERSE EFFECT.

In the transverse magnetizations, a hardness similar to that noted in the longitudinal was found. Continued magnetizations for some time after putting in the specimen caused the resistance to increase about .01 per cent. after which it remained constant.

In a preliminary run with unannealed wire, a curious behavior was noted. At high fields above saturation, the total resistance change did not occur simultaneously with the field, but jumped to a certain value and then approached a final value slowly, the latter change being about 17 per cent. of the total at $H = 25,000$, and requiring one minute to attain. Then on throwing off the field, the $\Delta R/R$ did not come back along the original curve nor along any curve that could be plotted, but stayed entirely below it and fell to a point considerably below the zero, then slowly decreasing to zero. It required several minutes for this decrease to occur. This behavior could not have been caused by inductance between the magnet and bridge nor to flux through the specimen changing as these were guarded against by reversing terminals, and also the field, and the results were the same. However, with the annealed wire, these lags were absent. The field was left on for several minutes on several occasions to see if they were present, but only a small lagging change was noted which on determining the correction curve for the copper coil was found to be due to a decrease in the resistance of the copper after its first increase in the field. It was of about the same relative magnitude to the $\Delta R/R$ of the copper as was that of the iron to its $\Delta R/R$ and may be somewhat analogous in character. With the sample of unannealed iron, no copper compensating coil was used so the lag could not have been due to that.

The results for the transverse magnetization, about 100 points, are plotted to three sets of abscissas in Fig. 5, Curve A for $\Delta R/R$ against original field between the pole tips before placing specimen in, Curve *B* for $\Delta R/R$ against flux density inside the wire, and Curve *C* for $\Delta R/R$ against field inside the wire. Curves *B* and *C* were computed and are approximate only. The wires in the grid were on an average about three diameters apart and an average external field was computed, that is the field due to the magnet plus the demagnetizing field from the neighboring magnetized wires. Considering this as the applied field, the wire was treated as a cylindrical body in a magnetic field and the ordinary formula applied. The form of the grid is shown in Fig. 2. The intensity of magnetization at saturation was taken as $4\pi I = 20{,}400$ which is about that of the best Swedish wrought iron. Curve *B* is practically the same as a $\Delta R/R - 4\pi I$ curve would be up to values of $4\pi I = 18{,}000$.

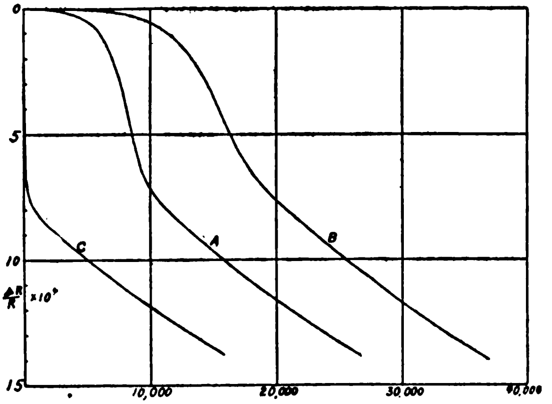

Fig. 5.
Transverse Effect.
Curve *A*, $\Delta R/R$—external field.
Curve *B*, $\Delta R/R$—flux density.
Curve *C*, $\Delta R/R$—internal field.

Curve *A*, plotted between $\Delta R/R$ and external *H*, which is the way others have plotted their results, confirms for iron what Jones and Malam showed for nickel, that the initial increase in resistance of magnetic metals when transversely magnetized is due to longitudinal components of the field where there are kinks in the wire, or the wire is not exactly perpendicular to the magnetic field. The true transverse resistance change is entirely negative from the start, as one of Grunmach's specimens showed. The decrease in resistance for the transverse case is approximately the same as the increase in the longitudinal up to $4\pi I = 16{,}000$, but here they diverge rapidly and at saturation the longitudinal is about 4 times the transverse effect in magnitude.

Curve C, plotted between $\Delta R/R$ and internal H shows that the decrease consists of two distinct parts, that due to the orientation of the elementary magnets, and that due to the added field after saturation. The first apparently reaches a minimum at saturation, but the second, which has scarcely begun then, continues to decrease as long as the field is increased with no sign of stopping at $H = 16{,}000$. This second decrease is not directly proportional to H but varies about as $H^{.85}$. Since the true saturation value of the iron is not known and the two curves overlap, it is hard to say where the first ends and the second begins.

To compare the magnitude of the transverse effect with that secured by other observers, the internal field strength should be used as the form of the sample has much to do with the value of $\Delta R/R$ at any external field. As others have their results plotted as function of external field only, the comparison cannot be made in this case.

Mathematical theories accounting for resistance change in magnetic fields have been worked out by J. J. Thomson[1] and E. P. Adams.[2] Jenkins has shown that these theories come far from fitting the observed facts. S. R. Williams[3] suggests an explanation by giving the elementary magnets shapes. This theory explains part of the phenomena observed quite simply but does not fit all the facts. A satisfactory explanation of the effect of a magnetic field upon resistance is yet to be found.

SUMMARY.

1. Electrolytic iron has slightly greater $\Delta R/R$ for the longitudinal effect than ordinary iron.

2. Iron, when transversely magnetized, does not increase in resistance at small fields, but continually decreases.

3. Continual decrease of resistance in transverse case after saturation is proportional to internal H raised to the .85 power.

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[1] Rap. Pres. a Cong. Int. de Phy., Vol. 3, 1900.

[2] Phys. Rev., Vol. 24, 1907.

[3] Phys. Rev., Sept., 1913.

TABLE OF VALUES.

Longitudinal.		Transverse.			
$R = 5.075$ Ohms, 6°.		$R = 2.14$ Ohms at 12°.		H(internal).	$\Delta R/R \times 10^4$.
H	$\Delta R/R \times 10^4$.	H	$\Delta R/R \times 10^4$.		
5	.22	2,000	...	1,000	8.11
10	.82	4,000	.27	2,000	8.68
20	2.09	6,000	.95	3,000	9.11
30	3.22	7,000	1.91	4,000	9.54
50	5.37	8,000	3.66	6,000	10.33
100	9.33	9,000	5.71	8,000	11.11
150	12.65	10,000	7.06	10,000	11.84
200	15.61	11,000	7.81	12,000	12.50
250	18.26	12,000	8.30	14,000	13.13
300	20.61	14,000	9.22	16,000	13.73
350	22.84	16,000	10.		
400	24.66	18,000	10.80		
450	26.18	20,000	11.56		
500	27.38	22,000	12.26		
550	28.35	24,000	12.92		
600	29.18	26,000	13.55		
650	29.76	27,000	13.85		
700	30.20				
750	30.57				
800	30.86				
850	31.12				
900	31.33				
950	31.50				
1,000	31.60				
1,100	31.85				
1,200	31.97				
1,400	32.08				
1,700	32.24				

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THE THERMOELECTRICITY AND MAGNETOSTRICTION OF HEUSLER ALLOYS.

By L. O. Grondahl.

IN an earlier paper it has been shown that the thermoelectromotive force of at least some Heusler alloys is affected by a magnetic field,[1] and that the effect varies quite considerably from specimen to specimen; in some even passing from a negative value in one field to a positive value in another, while in others it is always positive. This decided difference between the curves for different specimens made it seem possible to determine whether or not, as has been suggested from time to time, there is any connection between this effect on the thermoelectromotive force, and magnetostriction.

The specimens used were the same as in the work already referred to, and were again kindly loaned for the purpose by Dr. A. A. Knowlton. The general plan followed in the study of magnetostriction consisted in determining the change in length in different fields at about 0° C., room temperature, and 100° C. If the relation exists, it was thought that the difference between the first and the last of these quantities might bear a definite relation to the change in the electromotive force caused by the field when the junctions were at those two temperatures.

The method employed for the measurement of the magnetostriction was the same as that employed by Guthe and Austin.[2] The coil used was made of No. 8 double cotton-covered wire, is about 40 cm. long, 30 cm. external diameter, and 4 cm. internal diameter, and has a calculated constant of 122.5 gauss per ampere. The multiplying device was arranged as follows: Two pieces of ground glass, one stationary and the other movable, were strapped together with rubber bands. Between them was placed two thin glass rods as rollers. One of the rollers, which was .0903 cm. in diameter, had attached to one of its ends a glass arm 21.8 cm. in length, the other end of which carried one thread of a bifilar suspension. The bifilar supported a concave mirror which reflected the image of a carbon filament on a ground-glass scale at a distance of about 3 m. The scale distance and the distance between the threads was measured for every set of readings. The latter distance usually varied

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[1] Phys. Rev., Vol. XXXIII., p. 531, 1911.

[2] Bul. B. of S., Vol. II., No. 2, p. 315, 1906.

between 1 and 2 mm. and was measured with a micrometer telescope. The rubber bands held the movable piece of glass firmly against a brass rod which bore against the end of the alloy and was therefore moved when the alloy changed in length. In this work an error was present all the time in the high fields. It was rather small and the cause was not discovered until it was too late to repeat. The carbon filament used as a source of light was located about 1 m. away from the end of the coil. At the high fields the force on the filament was enough to bend it and thus cause a motion of its image on the screen. By the use of blank observations with a piece of brass in place of the alloy, the condition was found when this effect was a minimum. Only under this condition were observations taken. This was undoubtedly the case when the end of the filament happened to be the part reflected to the scale. The magnetostriction curves were taken to fields as high as 2,600 to 2,800 gauss, but are reported here only to 1,600. Below this value the error was always negligible and above this value the curves showed no new characteristics.

The alloys were placed in the inner space of a double brass tube which was made to fit into the coil. The temperatures were obtained by allowing ice water, water at room temperature, and steam, respectively, to flow through the space between the two tubes. The water was made to circulate through this space by means of a small propeller fitted into a brass tube as part of the circuit and used as a pump. The part of the water circuit outside of the coil was composed of a coil of brass tubing in a bath kept at the required temperature.

The change in length due to magnetization at approximately 0° C.,

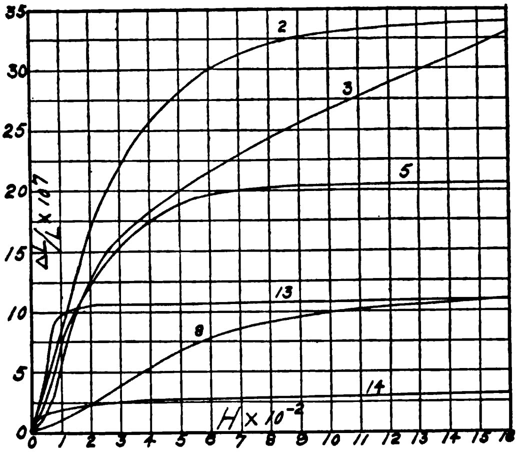

Fig. 1.

Magnetostriction at Zero.

20° C., and 100° C. are shown in Figs. 1, 2, and 3, respectively. The ordinates represent the fractional changes in length and the abscissæ represent the field in gauss. The curves are very similar to those obtained by Guthe and Austin. When it is measurable, $\Delta L/L$ is always positive and the three curves for the same alloy at the three different temperatures do not cross each other; hence there is at least no direct relation between the change of E.M.F. curves and the difference between the magnetostriction curves.

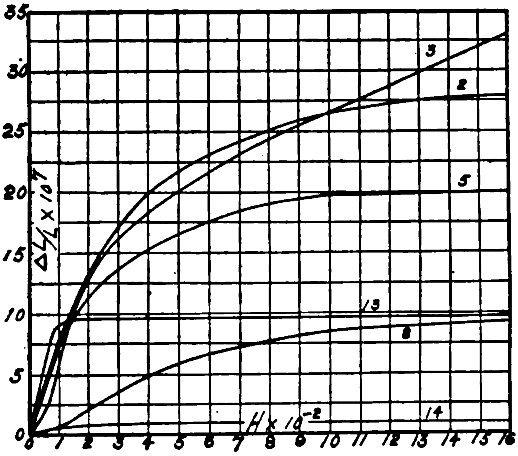

Fig. 2.

Room Temperature.

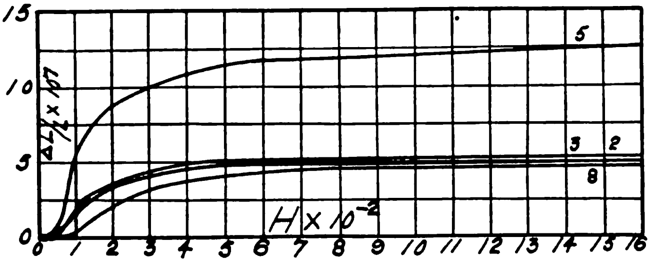

Fig. 3.

Steam.

The curves do show the interesting fact that the magnetostriction decreases very early with rise in temperature, even at temperatures considerably below the transformation range, and that above the transformation range it entirely disappears. The sensibility of the apparatus was such that a value for $\Delta L/L$ of 10^{-8} could have been easily detected. The transformation ranges as determined by Dr. Knowlton, are given in

Table I. It will be seen that in the case of specimens Nos. 2, 3, 5 and 8, although the transformation range is given as between 225° and 275° C., the magnetostriction has dropped to a very small value even at 100° C. Specimens No. 13 and No. 14 showed no magnetostriction at 100° C.

TABLE I.

Specimen.	Transformation Range.
2	225°–275° C.
3	225°–275° C.
5	225°–275° C.
8	225°–275° C.
13	40°–120° C.
14	10°– 30° C.

Measurements of the change of thermoelectromotive force in a magnetic field were then undertaken with two objects in view: First, to check the earlier work; and second, to determine the effect of temperature change, and especially to see whether or not this effect also disappears above the transformation range. The arrangement of apparatus was similar to that used in the previous work,[1] the only difference being the use of a clip to grip the end of the alloy in place of the solder. The results for four of the alloys are shown in Fig. 4, where positive ordinates correspond to an increase in the electromotive force against copper. The agreement with the earlier measurements is fairly satisfactory in that the nature of the curves is always the same. In the case of No. 14 where the effect shown is 10 times as great as in the previous work, the explanation was found in a mistake of a factor 10 in the former determination. The other

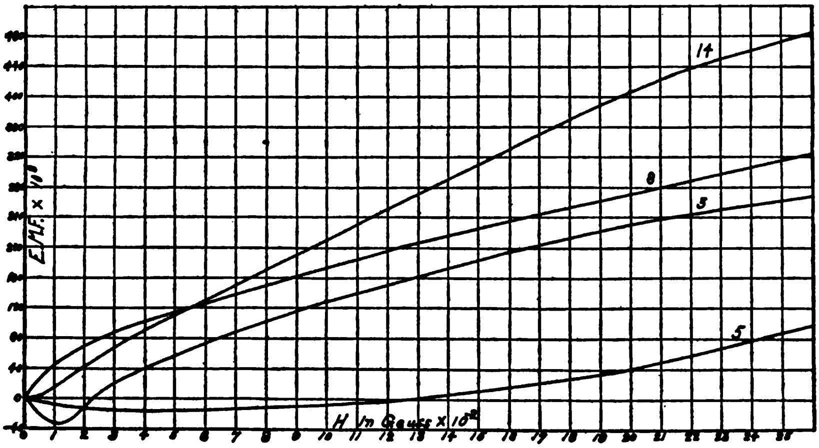

Fig. 4.

Change of Thermo E.M.F. with Magnetic Field.

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[1] Loc. cit.

smaller discrepancies may be explained by the inhomogeneity of the samples, and the possible difference in the positions of the temperature gradients in the two cases. In a still unpublished work, S. Karrer has shown that at least in the case of iron, the effect of a magnetic field on the thermoelectromotive force is practically all located in the temperature gradient.

To study this effect at higher temperatures, a non-inductively wound heating coil was placed around one junction and insulated from it by means of layers of asbestos and mica. The temperature of the hot junction was read by means of a mercury thermometer, the bulb of which was placed in contact with it. A constant current was conducted through the heating coil until the temperature became constant, when a set of readings was taken. The current in the heating coil was then slightly increased and the operation repeated. The results for Alloy No. 14 are given in Fig. 5. Since the temperature difference between the junctions was different in the different cases, it became necessary to compare the percentage change, rather than the change itself. Hence in Fig. 5 the ordinates represent the percentage change in electromotive force and the abscissæ the field strength. In this specimen all the curves were found to approximate a straight line. The accompanying Table II. gives the

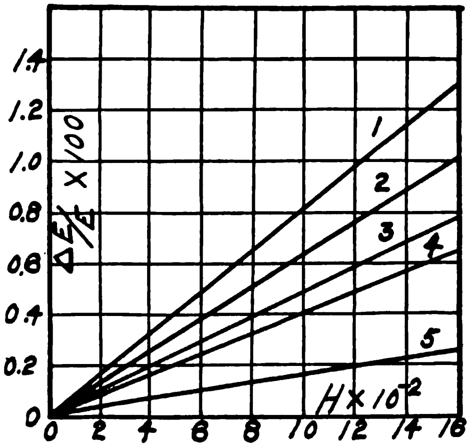

Fig. 5.

TABLE II.

I.	II.	III.	IV.
1	100	0	50
2	100	16	58
3	100	44	72
4	100	90	95
5	150	101	125

conditions under which these curves were taken. Column I. gives the number of the curve, Columns II. and III. the temperatures of the two junctions, and Column IV. the mean temperature in degrees Centigrade.

In another specimen the effect was found to increase with rise in temperature. However, in both the specimens tested, the effect was found to exist at a temperature for which the magnetostriction was not noticable. This is especially striking in the case of specimen No. 14. No. 11, the other specimen tried, is not reported in detail because the data were not complete. The effect of the magnetic field on thermoelectromotive force therefore seems to be entirely independent of magnetostriction. It seems that it might be interesting to determine the effect of the magnetic field on the inversion point, and that will be undertaken as soon as the writer can return to it.

The fact that these are the only Heusler alloys that have been found to show the effect of the magnetic field on the thermoelectromotive force, made it interesting to look for the Kerr effect. Professor Ingersoll of the University of Wisconsin who had an apparatus for Kerr effect determinations already in operation kindly consented to make this investigation. The result of his investigation was entirely negative.

Professor Ingersoll states that he could have detected a rotation as small as 1′ of arc and in most cases as small as 0.3′, but no effect was found. The portion of the spectrum investigated was between 1 μ and 1.5 μ.

RESULTS AND CONCLUSIONS.

For the alloys described above, the following statements are true:

1. The magnetostriction curves up to a field of 1,600 gauss, show no maximum and are always positive.

2. Some of the curves showing the effect of the magnetic field on the thermoelectromotive force pass through a maximum negative value and reverse.

3. The magnetostriction decreases as the temperature rises and disappears as the substance passes through the transformation range.

4. The effect of the magnetic field on the thermoelectromotive force changes but does not disappear even at temperatures considerably above the transformation range.

5. Hence, the two effects seem to be entirely independent of one another.

6. Although the effect of the magnetic field on the thermoelectromotive force in these alloys is very pronounced, Professor Ingersoll has found that they show no Kerr effect.

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April, 1914.

NOTES ON QUANTUM THEORY.

THE DISTRIBUTION OF THERMAL ENERGY.

By Gilbert N. Lewis and Elliot Q. Adams.

IN a previous paper[1] we have made use of a hypothesis called "the theory of ultimate rational units," to show that the interesting quantity, "*h*," which has come to be known as the "Wirkungsquantum," is, when expressed in suitable units, merely the square of the electron charge, multiplied by a simple numerical factor. This relation, in view of the important role of the constant "*h*" in the development of the quantum theory, leads to the inquiry whether the facts on which that theory rests cannot be explained by less radical assumptions.

The discontinuity of the radiation field postulated in the more extreme forms of the quantum theory leads inevitably to the assumption of a similar discontinuity in the field of a static charge. Such a conclusion would mean that the classical equations for the electromagnetic field possess only statistical validity. It is extremely doubtful, moreover, whether any of the proposed forms of the quantum theory can avoid these consequences, when carried to a full logical development. A theory which requires that an electric oscillator radiate energy at one amplitude and not at a slightly lower amplitude can hardly be brought into harmony with the accepted theories of the continuous field. According to the view now advocated by Planck, an oscillator in thermal equilibrium with its surroundings continues to have a finite energy of oscillation, even at the absolute zero of temperature. This view is equally at variance with the accepted views of electromagnetics and with those of thermodynamics.

Now, if there were no other way of explaining the very important facts to which attention has been called by the quantum theory, it would be proper to make such assumptions and to modify the body of physical theory in so far as might be necessary to render it consistent with them. But we believe that no such necessity exists.

The phenomena which must be considered fall into three classes: first, the distribution of energy in the spectrum of a black body; second, the heat capacity of material systems, and third, the photoelectric and photochemical effects.

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[1] Phys. Rev., N. S., *3*, 92 (1914).

Several phenomena of the first two classes prove beyond question the failure of the classical theorem of equipartition, according to which the total kinetic energy of any system in thermal equilibrium is equal to a universal constant multiplied by the absolute temperature and by the number of independent modes of motion possible to the system. This number of modes of motion may be also expressed as the number of data which suffice to determine the instantaneous momentum of every part of the system.

The third class of phenomena bears a less direct connection with the principle of equipartition, and we shall discuss these phenomena in another place. The present paper will deal therefore only with those portions of the quantum theory which are immediately related to the problem of the partition of energy.

THE DISTRIBUTION OF ENERGY IN THE SPECTRUM OF THE BLACK BODY.

The so-called Rayleigh formula for the distribution with respect to frequency of the energy of a hohlraum, is the one which corresponds to the simple equipartition theory. It may be derived most simply from the equation relating the energy, U_r, of a one-dimensional electric oscillator to the energy of the same frequency, u_ν, in a hohlraum in equilibrium with it, namely,

$$U_r = \frac{c^3 u_\nu}{8\pi\nu^2}, \tag{1}$$

u_ν being the derivative of the energy density of the hohlraum with respect to frequency. This equation is derived from classical electromagnetic theory, and while its complete validity for high frequencies may be doubted, there seem to be good grounds for believing that it cannot be held responsible for the flagrant departure of the Rayleigh equation from observed fact.

If the energy of the oscillator at the temperature, T, is $U_r = kT$, then the Rayleigh equation follows at once

$$u_\nu = \frac{8\pi}{c^3} k\nu^2 T. \tag{2}$$

This equation can be put in the form

$$u_\nu d\nu = \frac{8\pi}{c^3 h^3} k^4 T^4 x^2 dx, \tag{3}$$

where $x = h\nu/kT$ and h is a constant which does not appear directly in the Rayleigh formula, but which has been inserted for the sake of comparison with other radiation formulae. In this form, the Rayleigh formula obviously satisfies the Wien displacement law, which must be regarded as an inevitable consequence of thermodynamics. It is, however, evidently untrue, in that it requires that u_ν increase without limit with increasing values of ν, and gives for the total energy of a hohlraum, at any finite temperature, an infinite value. It has been suggested by Jeans that the Rayleigh equation may after all be valid for complete thermal equilibrium, but that such equilibrium never obtains in practice on account of the slowness with which it is established, and that the conditions which actually prevail represent a sort of false equilibrium. In discussing certain phenomena relating to specific heat we shall show the inadequacy of such an explanation.

To represent the actual distribution in the spectrum, Wien proposed the formula

$$u_\nu d\nu = \frac{8\pi}{c^3h^3}k^4T^4x^3e^{-x}dx. \tag{4}$$

This equation agrees qualitatively and to a large extent quantitatively with the existing data concerning radiation. In the region of low frequency and high temperature it gives, however, lower values of u_ν than have been obtained experimentally. For this reason Planck proposed the equation

$$u_\nu d\nu = \frac{8\pi}{c^3h^3}k^4T^4\frac{x^3}{e^x - 1}dx. \tag{5}$$

This not only is in better agreement with observation, but has the distinct theoretical advantage that it becomes identical with Wien's for large values of x and with Rayleigh's for small values of x. Indeed we have at present many reasons for believing that these two equations possess complete validity as limiting laws, true respectively for $x = \infty$ and $x = 0$. Aside from this it is difficult to judge whether the Planck equation can be regarded as having a theoretical basis. The various derivations of this formula offered by Planck are founded upon different forms of quantum theory which we regard as unsound in principle. But the simplicity of the equation recommends it, and although the experiments of Paschen[1] and of Rubens and Kurlbaum,[2] were not in complete accord therewith, we must nevertheless conclude that if Planck's is not the true radiation formula it represents an extraordinarily close approximation to the truth.

It must be noted, however, than an infinite number of equations may be set up, which would agree as well with the experimental facts as the

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[1] Ann. d. Phys. (4), 4, 277 (1901).

[2] Ann. d. Phys. (4), 4, 649 (1901).

Planck formula. Expressing the distribution of energy in accordance with the Wien displacement law as

$$u_\nu d\nu = \frac{8\pi}{c^3h^3}k^4T^4f(x)dx, \tag{6}$$

$f(x)$ is x^2 in the Rayleigh formula, x^3e^{-x} in the Wien, and $x^3/(e^x - 1)$ in the Planck.

The total energy density $\int_0^\infty u_\nu d\nu$, is therefore dependent at a given temperature upon $\int_0^\infty f(x)dx$, which has the value ∞ in the Rayleigh, 6 in the Wien, and $\pi^4/15 = 6.49+$ in the Planck formula.

Boissoudy[1] has suggested $f(x) = x^2e^{-x}(1 + x)$ for which $\int_0^\infty f(x)dx = 8$. This equation is not in as good agreement with the facts as that of Planck, but as examples of different types of functions which would represent the available data within the limits of experimental error, we may mention:

$$f(x) = x^2e^{-x}(1 + x^3)^{\frac{1}{3}}, \quad \int_0^\infty f(x)dx = 6.16+;$$

$$f(x) = x^3e^{-x}\coth x, \quad \int_0^\infty f(x)dx = (\pi^4/8) - 6 = 6.17+;$$

$$f(x) = x^3e^{-x} + x^2e^{-2x}, \quad \int_0^\infty f(x)dx = 6.25;$$

$$f(x) = x^2e^{-x}(1 + x^2)^{\frac{1}{2}}, \quad \int_0^\infty f(x)dx = 6.42+.$$

All of these equations become identical at the two limits with the equations of Rayleigh and of Wien, and it is evident that an indefinite number of such equations might be devised which would fit the true distribution curve with any desired degree of accuracy. Whether the Planck formula is the true one we must regard as an open question.

Whatever the true form of the distribution equation may be, it is unquestionably not the one obtained from the equipartition of energy, according to which the heat capacity not only of the hohlraum, but of any continuum, is infinite. In other words, according to that principle a material continuum is regarded as equivalent to a system composed of an infinite number of particles of infinitesimal mass, each capable of independent motion of the type which Boltzmann called "molekular ungeordnet." But such a system of particles cannot legitimately be

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[1] Comptes Rendus, *156*, 1364 (1913).

regarded as a continuum. A (physical) continuum must be defined as a region in which all properties vary continuously from point to point, and in which the properties at any point may be predicted with a probability approaching complete certainty when the corresponding properties at other points taken nearer and nearer to the given point are ascertained. Any system which we are justified in calling a continuum is one in which the motions of neighboring elements cannot be regarded as completely independent or unordered.

Let us consider a hohlraum permeated by electric and magnetic fields. If these fields are produced according to equations of accepted electromagnetic theory the hohlraum is a continuum, and the magnitude of the electric and magnetic vectors at any point will differ in magnitude and direction from those at any neighboring point by an amount which becomes indefinitely less as the two points are taken nearer and nearer together. Without knowing quantitatively how near to a point the vectors must be found in order to predict their values at that point within a certain probable error, we may nevertheless assert that starting from a given point each of these vectors must maintain a component in the original direction for a finite distance in every direction. If then any hohlraum containing radiant energy should have within it a charged body, this body would be acted upon through finite intervals of time by forces of the same general direction and would acquire a finite kinetic energy. If the hohlraum is in thermal equilibrium the average kinetic energy of the charged body will be that which belongs to it at that temperature. In other words, since the charged body acquires thermal energy from the hohlraum the latter cannot have an infinite heat capacity.

We must assume, not that a continuum must have an infinite heat capacity, but that the very definition of a continuum precludes this possibility. We may state this principle in another way, in the case of the hohlraum. The electric and magnetic vectors at a point determine the general magnitude and direction of the vectors in neighboring points. The existence of radiant energy of infinite frequency is precluded, for in such radiation it would be impossible to determine the electromagnetic vectors at a point from those at any other point, however near together these points might be. The failure of the Rayleigh radiation formula may therefore be ascribed not to discontinuity of the hohlraum, as in the quantum theory, but to its very continuity.

The hohlraum thus interpreted as a continuum presents a very interesting problem which we may state, somewhat inadequately perhaps, as follows:

In any continuum containing vector fields the knowledge of the magnitude and direction of a vector at a single point determines a most probable value of the vector at any other point. In the simplest case this probable value would be a vector in the same direction, and with a magnitude diminishing with increasing distance from the given point, to zero at infinite distance. The probable value will in general be a function only of the magnitude of the original vector and of the distance.

In a given hohlraum therefore the electric or magnetic vector at each point determines a probable value of the vector at every other point, and if the average value of the magnitude of the vector were known throughout the whole region considered, and if the probability function which we have mentioned above were also known, the determination of the macroscopic condition of highest probability in the system would present no difficulties except those of an analytical character. In other words, a knowledge of this probability function[1] is mathematically sufficient to give the true equation for the distribution of energy in the hohlraum.

We may call attention at this point to an interesting consequence of the theorem that the electric (or magnetic) vector in a hohlraum continues, for a finite distance from a given point, to have a component in the same direction as the vector at that point. Consider a hohlraum in thermal equilibrium at a given temperature, and some direction, say north, arbitrarily chosen. Choosing some point where the electric vector has a northerly component, the locus of the termini of lines drawn from this point and terminating at points where the north component disappears is a closed surface, and the whole volume of the hohlraum may thus be divided into a finite number of domains, in each of which either all the electric vectors possess a northerly component, or all possess a southerly component.

The average volume of these domains is obviously independent of the fixed direction chosen, and in fact depends on the temperature alone. It follows readily from the Wien displacement law that this average volume is inversely proportional to the cube of the absolute temperature. In other words, the *average entropy* of one of these domains is the same under all conditions of thermal equilibrium.

It is extremely interesting to note that from our theory, which may be considered in many respects the opposite of a quantum theory, we are thus led to a consideration of domains which are closely analogous to the " Elementargebiete " which Planck employs so frequently.

Another parenthetical observation may not be out of place here. A single molecule (at a given instant) cannot be regarded as a macroscopic

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[1] The possibility must be considered that this probability function may depend upon the average strength of the field, as it would, for example, in case the continuum were an elastic solid, subject to severe strains.

system, as Planck uses this term. Only when enough molecules are taken to enable us to deal satisfactorily with averages can the system be so regarded. Now the interesting question arises: what is the minimum volume of a hohlraum in which the instantaneous conditions may be treated statistically without any great danger of error? The answer to this question is that a hohlraum, in order to be regarded as a macroscopic system at any instant must contain a considerable number of these elementary domains.[1]

Returning now to the consideration of the probability function, discussed above, we may, if it is desirable, consider the energy of the hohlraum resolved by a Fourier analysis with respect to frequencies. This convenient, although by no means necessary, resolution enables us to divide the radiation into a number of approximately monochromatic constituents. Now it is evident that the probability function due to the character of the hohlraum as a continuum is without effect upon the very long wave-lengths, but that it makes the existence of any given amount of energy at smaller and smaller wave-lengths less and less probable. This new factor of probability, which could be derived from the other probability function, if that were known, by mere mathematical transformation, may be regarded as the factor which multiplied by the u_ν of the Rayleigh formula gives the true radiation formula. Thus, if Planck's formula is correct, this factor is $x/(e^x - 1)$, where x is, as before, $h\nu/kT$. In any case this factor, according to the Wien displacement law, must be a function of ν/T only.

Since free space maintains undamped any electromagnetic disturbances which may be imposed upon it, it may seem strange to regard the properties of a hohlraum in thermal equilibrium as in any way intrinsic, but that it may be so regarded is shown by the fact that the properties of a hohlraum are entirely independent of the nature of the walls or of the mechanism of radiation. Nevertheless we may hope to obtain important information concerning the hohlraum from the laws of the distribution of energy in material systems.

THE DISTRIBUTION OF THERMAL ENERGY IN MATERIAL SYSTEMS.

There are two important cases in which the equipartition law gives a quantitative explanation of observed specific heats. The heat capacity

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[1] The volume of one of these elementary domains can be roughly estimated by assuming that *in ultimate rational units* the average volume of the elementary domain is unity at unit temperature. Except for the factor 4/3, this is equivalent to assuming that the average entropy of any elementary domain is unity (or k in ordinary units). This would make the volume of the elementary domain of the order of 10^{-3} c.c. at one centigrade degree above the absolute zero. A value of the same order of magnitude is obtained by assuming that the volume of the elementary domain at any temperature is the cube of λ_m, that is, the wave-length of maximum energy at that temperature.

of a monatomic gas is precisely that which is calculated from the assumption that the atoms possess no energy except kinetic energy of translation; and the heat capacity of most solid elements at ordinary temperatures is that calculated from the assumption that each atom possesses this same translational energy and, on the average, an equal amount of potential energy. There is, however, nothing apparent in the derivation of the equipartition law which would lead us to expect the heat capacity of solids to diminish at lower temperatures, approaching zero at the absolute zero of temperature, nor which would account for the failure of the atom to acquire energy of rotation as well as energy of translation.

Such departures from the principle of equipartition Jeans formerly attempted to interpret as due to the non-attainment of true thermal equilibrium. According to this theory a monatomic gas at a given temperature would rapidly attain the appropriate translational energy, but would acquire rotational energy, and other forms of energy within the atom, only with extreme slowness. The following argument seems to us sufficient to show the inadequacy of this explanation. A monatomic gas, like argon, has a dielectric constant higher than unity. This means that in an electric field certain modes of motion, of rotation or displacement, manifest themselves in the atoms. An atom of argon therefore, subjected to the electromagnetic field of radiation at a given temperature, must respond rapidly in these modes of motion, which apparently, however, make no appreciable contribution to the heat capacity. Moreover, that theory could in no possible way account for the diminution in the heat capacity of solids at low temperatures, for it would have to be admitted in any case that in the region of validity of Dulong and Petit's law the establishment of true temperature equilibrium is rapid. Now as the temperature is lowered the theory would require that at some temperature the establishment of thermal equilibrium would take place with a very small but measurable speed; such a phenomenon, if existent, would certainly have been observed.

The situation seems to be this: a particle subject to no constraints acquires at each temperature the kinetic energy demanded by the equipartition theorem, but a constraint in any mode of motion diminishes the kinetic energy associated with that mode of motion and this diminution is greater the more powerful the constraint.[1] This is in brief the phenomenon for the explanation of which the quantum theory has been invoked. Before proposing a different explanation we may put the problem in another form.

It is customary to correlate with the energy of a hohlraum the energy

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[1] We use the word constraint in its most general sense, to mean any set of forces which operate against the free acquisition of rectilinear motion.

of a system of electrical oscillators by means of equation (1), connecting the energy of an oscillator of frequency, ν, with the energy density of a hohlraum associated with the same frequency, and this is the method by which Einstein was led to the first formula for the heat capacity of solids at low temperature. The approximate validity of this and other similar formulæ has been regarded as a substantiation of the quantum theory, but the authors of these formulæ have recognized that their derivation depended only upon the assumption that the Planck formula is a satisfactory empirical equation for the distribution of energy in the hohlraum,

By combining equation (1) and the Wien displacement law, equation (6), we obtain for the energy of any electrical oscillator

$$U_\nu = kTF(x), \tag{7}$$

where $F(x)$, as we know from a study of the spectrum, approaches unity for small values of x, that is, at high temperatures or low frequencies, but approaches zero at low temperatures and high frequencies. It has been universally assumed, and we think with perfect propriety, that there is nothing unique about an oscillator which happens to be charged, and that this same equation would hold for any mechanical oscillator. Hence, any oscillator of given frequency will have at high temperatures the energy predicted by the equipartition principle, but zero energy at the absolute zero, where $F(x) = 0$.

Now while it is permissible to regard a material system as composed of a large number of simple harmonic oscillators of definite frequencies, this resolution is not necessary nor is it perhaps always advantageous. Thus Debye employs this method of representing the conditions prevailing in a solid body as a mere mathematical expedient, but regards his system as a quasi-continuum, pervaded by chaotic thermal vibrations.

In general a process which is associated with a definite frequency cannot be regarded as *elementary*, for in periodic motion a system may pass through very different conditions of velocity and acceleration in a single period. If a system be divided into elementary parts, each undergoing motions which are in general periodic it is entirely justifiable to make a Fourier analysis and consider the distribution of energy among frequencies, but it is perhaps more fundamental to arrange the elementary parts according to the instantaneous *accelerations*, and to determine the distribution of energy among the accelerations.

Such a resolution may be made in a number of ways but in general it will be true that just as the energy belonging to any frequency is less than that required by the equipartition law, and by a factor which is a function of ν/T only, so also the energy of any portion of the system is on the average less than equipartition demands and by an amount which is greater, the greater the acceleration of that portion of the system, and the lower the temperature.

In so far as this conclusion is based upon the Wien displacement law it may be regarded as thermodynamic in character and independent of the specific constitution and properties of any particular system, but at most thermodynamics can only tell us that the factor in question must be of this form if it exist. What change, therefore, must be made in the derivation of the partition law to permit the entrance of this factor involving acceleration?

We believe that the true answer to this question is suggested by our previous remarks concerning the continuum. The deviations from the equipartition law occur in systems subject to powerful constraints, and in such a system the motion of its parts can not be regarded as independent. In a system of mathematical particles such an assumption might be justifiable, but in a system of actual particles each particle determines a field of force and to these fields must be attributed some part of the mass of the system. We are dealing therefore not with a mere assemblage of discrete particles nor with a pure continuum, but with a quasi-continuum which approximates the one or the other according as the character of the constraints and the nature of the problem emphasize the singular points which are the atomic centers or the field in which these centers are, as it were, imbedded.

It has frequently been assumed that the failure of the equipartition law will necessitate at the very least some striking amendment of the accepted principles of mechanics. It seems to us, on the contrary, that the fault is not in these principles but in their application.

In the derivation of the equipartition theorem it is assumed that the idea of a *collision* is unambiguous. In the ideal collision identifiable objects of definite mass[1] exchange momenta according to simple laws, but any actual collision presents a far more difficult problem. The particle concerning which we possess the most specific information is the electron, and yet it has hitherto been impossible to solve the simple problem of the motion of two electrons which at a given distance are started toward one another in a straight line with a known velocity. In certain limiting cases we are justified in regarding the mass of the system as localized at the electron centers, but in general it must be regarded as distributed throughout the infinite electron fields. Likewise the localization of the mass of any particle at the center of the particle leads in some

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[1] We are in no way concerned here with the change in mass with velocity demanded by the principle of relativity. The deviations from the principle of equipartition are especially pronounced at the lowest temperatures where the smallest velocities are concerned.

cases to no confusion, but this is not the case when the fields of force of the particles appreciably overlap. In the case of a quasi-continuum, where mutual constraints are in evidence, some part of the mass of the system must be assigned to the fields of these constraints.

The fact that these considerations have been ignored is, in our opinion, alone responsible for the erroneous theorem concerning the partition of energy, and while we are still unable to offer any quantitative solution of this difficult problem we believe that all the phenomena of the distribution of thermal energy may be qualitatively and semi-quantitatively explained if we assume that, in a system subject to constraints, the acceleration of any portion of the system under given conditions is less than it would be in the absence of the constraints, and that this difference increases with the acceleration. This does not mean that we must give up the law that acceleration is equal to force divided by mass, but only that in a quasi-continuum it is hard to localize the forces and masses, and that formerly we have assumed a given mass to be acted upon by a larger force than the actual one.

Without attempting to analyze the situation further let us state once more our proposition, namely, that every portion of a system in thermal equilibrium possesses on the average a kinetic energy which is less than that given by the equipartition law by an amount which is larger, the greater the constraints, the higher the instantaneous acceleration, and the lower the temperature. We may now proceed to apply this general principle to specific cases.

The Heat Capacity of Solids.—According to the principle just enunciated, the deviation from the law of Dulong and Petit should be greater the lower the temperature, and at a given temperature should depend upon two factors, namely, the rigidity of the constraints acting upon the atoms and the mass of the atom. For the smaller the mass of the atom, the greater its average acceleration must be, in movements of a vibratory character, to give it a certain amount of energy.

In fact all of the elements which depart appreciably from the law of Dulong and Petit at ordinary temperatures are those of small atomic weight, and the element which shows the greatest deviation of all is carbon in the form of diamond, in which the internal constraints must be enormously greater than in the case of any other element. Boron is another element of high internal rigidity, and its heat capacity is about the same as that calculated for hydrogen in the solid state (at room temperature). What hydrogen lacks in rigidity it makes up in the smallness of its atomic weight.

The element which above all others is characterized by small atomic mass is electricity, and we believe that the failure of the electrons to contribute materially to the heat capacity of a metal is merely a deviation from the law of Dulong and Petit which differs in magnitude but not in kind from those which are already familiar. We are thus able to understand why the electrons in metals which from some points of view appear to be free, still contribute nothing to the heat capacity. Their mass is so extremely small that very weak constraints prevent their acquiring an appreciable amount of thermal energy at ordinary temperatures, at high temperatures we must expect them to increase the heat capacity of the metals.

The Heat Capacity of a Monatomic Gas.—Assuming that the moment of inertia of an atom is extremely small, any appreciable amount of rotational energy must be associated with very great acceleration, and therefore at any ordinary temperature the energy of rotation will be no considerable part of the thermal energy of a monatomic gas. In fact, Pier[1] has shown that the heat capacity of argon is constant within the limits of experimental error up to 2,350° C. If the moment of inertia of the atom were known, it would be possible to estimate the temperature range in which the heat capacity of a monatomic gas would pass from the value at lower temperatures to double that value. It is probable however that before the atoms acquire much rotational energy they will acquire an inner energy due to the vibration of their parts. Certain metallic vapors radiate thermally below 1,000° C. While the internal oscillations necessary to produce radiation may be associated with comparatively little energy, we should expect the heat capacity of a monatomic gas to increase soon after it begins to produce appreciable thermal radiation.

The Heat Capacity of a Diatomic Gas.—According to the principle of equipartition the molal heat capacity at constant volume of a diatomic gas should be $^3/_2R$ if the molecule can possess only translational energy, $^5/_2R$ if it may also rotate about the two axes perpendicular to the line of centers of the atoms, and $^7/_2R$ if it further possess kinetic and potential energy of vibration in the line of centers. According to the experiments of Eucken[2] the heat capacity of hydrogen gas is approximately $^3/_2R$ at all temperatures below 50° absolute; above that temperature it rises rapidly and then more slowly, approaching at about room temperature the value $^5/_2R$. From this point on the heat capacity increases linearly with the temperature up to the highest temperature where it has been investigated, 2,000° C. It has ordinarily been assumed, we believe correctly, that the heat capacity at ordinary temperatures is

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[1] Pier, Z. Elektrochem., *15*, 536 (1909).

[2] Eucken, Ber. Berl. Akad., *1912*, 141.

due to translational and rotational energy and not, to any large extent, to vibrational energy. The rotational energy apparently obeys the equipartition theorem above room temperature, but falls rapidly to zero with diminishing temperature. The attempts which have been made to explain this phenomenon by means of the quantum theory have been extremely unconvincing. The association of a certain amount of energy with a certain frequency has been an essential part of the quantum theory, but a rotating molecule possesses no natural period, and we could hardly expect the application of the quantum theory in this case to prove successful.

The principle which we have stated above affords a simple explanation of these phenomena. As in the case of the monatomic gas, we should expect the moment of inertia of the rotating molecule to determine the degree of departure from the equipartition law. If the various diatomic gas molecules have approximately the same dimensions, then a given energy of rotation of the hydrogen molecule corresponds to a higher acceleration than in the case of other gases, and we should expect hydrogen to be the first gas to approach in heat capacity $^3/_2R$ as the temperature is lowered. This appears to be the case.

Before the diatomic gases have attained their full energy of rotation, they apparently begin to have an appreciable energy of vibration, and the part of the heat capacity due to this cause appears to be a linear function of the temperature. Now in other cases which have been considered, the attainment of a new type of thermal energy with increasing temperature has initially a rate which increases rapidly with the temperature; thus Debye states the law that the heat capacity of a monatomic solid is proportional to the cube of the absolute temperature at low temperatures. We believe that this law is independent of the more specific assumptions from which he derived it, and that for all substances, crystalline or non-crystalline, when the inner constraints are large and the temperature low, the heat capacity is proportional to the cube of the temperature. In other words, this is the general property of a continuum, which quasi-continua approach as a limit. To this conclusion that the heat capacity of any continuum would follow the same law as that of the hohlraum we were first led by considerations analogous to those in our paper on the ultimate rational units. A very interesting derivation of this law has just been obtained by Professor Tolman from his principle of similitude.[1]

The heat capacity of a three-dimensional continuum is proportional to the cube of the temperature; that of an n-dimensional to the nth power

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[1] Phys. Rev., N. S., 4, 145, 1914.

of the temperature. A diatomic molecule capable of vibration only in the line of atomic centers approaches with increasing strength of constraint and decreasing temperature an ideal one-dimensional continuum. The part of the heat capacity of a diatomic gas which is due to vibration should therefore be a linear function of the temperature over a considerable range.[1]

The departure from the value predicted from equipartition of the heat capacity due to vibration should be the least in those cases where the largest amount of energy can be reached with the smallest average acceleration, that is, where the constraints are least and the masses greatest. Now iodine has not only the highest atomic weight of all the diatomic gases which have been investigated, but also the lowest internal constraints, as shown by the fact that it dissociates at a lower temperature than any other diatomic gas. In fact, iodine has the highest heat capacity of all the diatomic gases.

CHEMICAL LABORATORY OF THE UNIVERSITY OF CALIFORNIA,
BERKELEY, May 22, 1914.

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[1] Le Chatelier found that the molal heat capacity of diatomic gases at constant volume could be adequately represented over a wide range of temperature by the equation $c_v = 4.5 + aT$, and later measurements have been in pretty good agreement with this formula. According to the theory stated above the first constant of this formula should be $5R/2$ or nearly 5.0. The difference is probably due to the fact that owing to the scarcity of experimental data at higher temperatures the values at room temperatures have been given great weight and some of the diatomic gases probably do not acquire their full rotational energy until somewhat higher temperatures are reached.

ON PHYSICALLY SIMILAR SYSTEMS; ILLUSTRATIONS OF THE USE OF DIMENSIONAL EQUATIONS.

By E. Buckingham.

1. *The Most General Form of Physical Equations.*—Let it be required to describe by an equation, a relation which subsists among a number of physical quantities of n different kinds. If several quantities of any one kind are involved in the relation, let them be specified by the value of any one and the ratios of the others to this one. The equation will then contain n symbols $Q_1 \cdots Q_n$, one for each kind of quantity, and also, in general, a number of ratios r', r'', etc., so that it may be written

$$f(Q_1, Q_2, \cdots Q_n, r', r'', \cdots) = 0. \qquad (1)$$

Let us suppose, for the present only, that the ratios r do not vary during the phenomenon described by the equation: for example, if the equation describes a property of a material system and involves lengths, the system shall remain geometrically similar to itself during any changes of size which may occur. Under this condition equation (1) reduces to

$$F(Q_1, Q_2, \cdots Q_n) = 0. \qquad (2)$$

If none of the quantities involved in the relation has been overlooked, the equation will give a complete description of the relation subsisting among the quantities represented in it, and will be a complete equation. The coefficients of a complete equation are dimensionless numbers, *i. e.*, if the quantities Q are measured by an absolute system of units, the coefficients of the equation do not depend on the sizes of the fundamental units but only on the fixed interrelations of the units which characterize the system and differentiate it from any other absolute system.

To illustrate what is meant by a " complete " equation, we may consider the familiar equation

$$\frac{pv}{\theta} = \text{constant},$$

in which p is the pressure, v the specific volume, and θ the absolute temperature of a mass of gas. The constant is not dimensionless but depends, even for a given gas, on the units adopted for measuring p, v, and θ; the equation is not complete. Further investigation shows that the equation may be written

$$\frac{pv}{R\theta} = N,$$

in which the symbol R stands for a quantity characteristic of each gas and differing from one to another, but fixed for any given gas when the units of p, v, and θ are fixed. We thus recognize that R is a quantity that can be measured by a unit derived from those of p, v, and θ. If we do express the value of R in terms of a unit thus derived, N is a dimensionless constant and does not depend on the sizes of the units of p, v, and θ but only on the fixed relation which the unit of R bears to them. The equation is now a " complete " equation.

Every complete physical equation (2) has the more specific form

$$\Sigma M Q_1^{b_1} Q_2^{b_2} \cdots Q_n^{b_n} = 0. \qquad (3)$$

Such expressions as log Q or sin Q do not occur in physical equations; for no purely arithmetical operator, except a simple numerical multiplier, can be applied to an operand which is not itself a dimensionless number, because we can not assign any definite meaning to the result of such an operation. The reason why such an expression as Q^2 can appear, is that Q^2 may be regarded as a symbol for the result of operating on Q by Q. For example, when we write $A = l^2$, l^2 is a symbol for the result of operating on a length l by itself. We are directed to take the length l as operand and " operate on it with the length l " by constructing on it as a base, a rectangle of altitude l; and the result of this operation, which fixes an area A, is represented by l^2. Whenever functions that do not have the form of the terms in equation (3) appear to occur in physical equations, it is invariably found upon examination that the arguments of these functions are dimensionless numbers.

2. *Introduction of Dimensional Conditions.*—We have now to make use of the familiar principle, which seems to have been first stated by Fourier, that all the terms of a physical equation must have the same dimensions, or that every correct physical equation is dimensionally homogeneous. Let equation (3) be divided through by any one term and it takes the form

$$\Sigma N Q_1^{a_1} Q_2^{a_2} \cdots Q_n^{a_n} + 1 = 0, \qquad (4)$$

in which the N's are dimensionless numbers. In virtue of the principle of dimensional homogeneity the exponents a_1, a_2, $\cdots$ a_n of each term of equation (4) must be such that that term has no dimensions or that a dimensional equation

$$[Q_1^{a_1} Q_2^{a_2} \cdots Q_n^{a_n}] = [1] \qquad (5)$$

is satisfied.

Let Π represent a dimensionless product of the form

$$\Pi = Q_1^{a_1} Q_2^{a_2} \cdots Q_n^{a_n}. \tag{6}$$

so that equation (4) may be written more shortly

$$\Sigma N\Pi + 1 = 0. \tag{7}$$

Since Π is dimensionless, Π^x is dimensionless; and furthermore, any product of the form $\Pi_1^{x_1}\Pi_2^{x_2} \cdots \Pi_i^{x_i}$ is also dimensionless. Hence if Π_1, Π_2, $\cdots$ Π_i represent all the separate independent dimensionless products of the form (6) which can be made up in accordance with equation (5) from the quantities Q, equation (7) may be written in the form

$$\Sigma N\Pi_1^{x_1}\Pi_2^{x_2} \cdots \Pi_i^{x_i} + 1 = 0 \tag{8}$$

and still satisfy the requirement of dimensional homogeneity.

Now there are, so far as this requirement is concerned, no restrictions on the number of terms, the values of the coefficients, or the values of the exponents. Hence the Σ merely represents some unknown function of the independent arguments Π_1, $\cdots$ Π_i and equation (8) may more simply be written

$$\psi(\Pi_1, \Pi_2, \cdots \Pi_i) = 0. \tag{9}$$

By reason of the principle of dimensional homogeneity, every complete physical equation of the form (2) is reducible to the form (9) in which

$$[\Pi_1] = [\Pi_2] = \cdots = [\Pi_i] = [1] \tag{10}$$

and the number i, of separate independent arguments of ψ, is the maximum number of independent dimensionless products of the form (6) which can be made by combining the n quantities Q_1, Q_2 $\cdots$ Q_n in different ways.

We have next to find the value of i. Let k be the number of arbitrary fundamental units needed as a basis for the absolute system $[Q_1]$, $\cdots$ $[Q_n]$ by which the Q's are measured. Then in principle and if we disregard the practical considerations connected with the preservation of standards, etc., there is always, among the n units $[Q]$, at least one set of k which may be used as fundamental units, the remaining $(n - k)$ being derived from them.

Now each equation of the form (5) with a particular set of exponents a (corresponding to a particular dimensionless product Π) is an equation to which the dimensions of the units $[Q]$ are subject. But since $(n-k)$ of the units are derivable from the other k and the units are otherwise arbitrary, it is evident that each equation of the form (5) is in reality equivalent to one of these equations of derivation. There are therefore $(n - k)$ equations of the form (5) and the number of products Π which appear as independent variables in equation (9) is

$$i = n - k.$$

Furthermore, if $[Q_1]$, $[Q_2]$ $\cdots$ $[Q_k]$ are k of the n units which might be used as fundamental, the i equations (5) may be written

$$\left.\begin{aligned} [\Pi_1] &= [Q_1^{\alpha_1}Q_2^{\beta_1}\cdots Q_k^{\kappa_1}P_1] = [1] \\ [\Pi_2] &= [Q_1^{\alpha_2}Q_2^{\beta_2}\cdots Q_k^{\kappa_2}P_2] = [1] \\ &\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ [\Pi_i] &= [Q_1^{\alpha_i}Q_2^{\beta_i}\cdots Q_k^{\kappa_i}P_i] = [1] \end{aligned}\right\} \qquad (11)$$

in which the P's represent $Q_{k+1} \cdots Q_n$, *i. e.*, the quantities that are, temporarily, regarded as derived.

To make use of any one of equations (11) for finding the specific form of the corresponding Π, we replace each of the $[Q]$'s and the $[P]$ by the known dimensional equivalent for it in terms of whatever set of k fundamental units (such as mass, length, time, etc.) we may happen to find convenient. The resulting equation contains the k independent fundamental units, and since both members are of zero dimensions, the exponent of each unit must vanish. We therefore obtain k independent linear equations which suffice to determine the k exponents and so to fix the form of the Π in question. We have still, however, one arbitrary choice left which it is sometimes convenient to make use of. Since the Π's occur in equation (9) as arguments of an indeterminate function ψ and are subject only to the condition of being dimensionless, when we have found the specific form of any one of the Π's, we are at liberty to replace this by any function of it; for this function will also be dimensionless and will be independent of the remaining Π's. This remark enables us to dispense with fractional exponents, when they happen to result from equations of the form (11), and so to simplify the writing down of our results.

3. *Illustration.*—To make the meaning of the foregoing developments more evident we may treat an example. Let us suppose that we have to deal with a relation which involves one quantity of each of the following $n = 7$ kinds:

Name.	Symbol.	Dimensions.	
1. Force	F	$[mlt^{-2}]$	
2. Density	ρ	$[ml^{-3}]$	
3. Length	D	$[l]$	
4. Linear speed	S	$[lt^{-1}]$	(12)
5. Revolutions per unit time	n	$[t^{-1}]$	
6. Viscosity	μ	$[ml^{-1}t^{-1}]$	
7. Acceleration	g	$[lt^{-2}]$	

Three fundamental units are needed, *i. e.*, $k = 3$, but they need not be $[m, l, t]$ for we could also use $[F, \rho, S]$ or $[\rho, n, \mu]$ or several other combinations. On the other hand, such combinations as $[l, S, n]$ or $[S, n, g]$ could not be used.

We know by section (2) that any relation whatever which involves all the above seven quantities and no others, must be expressible by an equation which can be reduced to the form

$$\psi(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0 \qquad (9, a)$$

because $n - k = 7 - 3 = 4$.

To find a specific form of this equation, we select 3 of the quantities as fundamental and proceed to use equations (11).

Let us, to start with, set

$$F = Q_1, \quad \rho = Q_2, \quad D = Q_3$$

these being a possible set of fundamental units sufficient for deriving the others. Then S, n, μ, g, act as P_1, P_2, P_3, P_4 and we have, corresponding to equations (11),

$$\left.\begin{aligned} [F^{\alpha_1}\rho^{\beta_1}D^{\gamma_1}S] &= [1], \\ [F^{\alpha_2}\rho^{\beta_2}D^{\gamma_2}n] &= [1], \\ [F^{\alpha_3}\rho^{\beta_3}D^{\gamma_3}\mu] &= [1], \\ [F^{\alpha_4}\rho^{\beta_4}D^{\gamma_4}g] &= [1], \end{aligned}\right\} \qquad (11, a)$$

from which to determine the α's, β's, and γ's.

Taking the first of these equations and substituting the dimensions of $[F, \rho, D, S]$ in it we have

$$[m^{\alpha_1}l^{\alpha_1}t^{-2\alpha_1} \times m^{\beta_1}l^{-3\beta_1} \times l^{\gamma_1} \times lt^{-1}] = [1]$$

and since m, l, and t are independent, this can be satisfied only if α_1, β_1, and γ_1 are related as shown by the equations

$$\left.\begin{aligned} \alpha_1 + \beta_1 &= 0, \\ \alpha_1 - 3\beta_1 + \gamma_1 + 1 &= 0, \\ 2\alpha_1 + 1 &= 0, \end{aligned}\right\} \text{ or } \left\{\begin{aligned} \alpha_1 &= -\tfrac{1}{2}, \\ \beta_1 &= \tfrac{1}{2}, \\ \gamma_1 &= 1. \end{aligned}\right.$$

We therefore have $\Pi_1 = F^{-\frac{1}{2}}\rho^{\frac{1}{2}}DS$, which will be more convenient to write and satisfy the condition of being dimensionless equally well if we square it and write $\Pi_1 = \rho D^2S^2/F$.

If we follow a similar method with the remaining three equations of the set (11, a) we have

$$\Pi_2 = \frac{\rho D^4 n^2}{F}; \quad \Pi_3 = \frac{\mu^2}{F\rho}; \quad \Pi_4 = \frac{\rho D^3 g}{F}$$

and equation (9, a) takes the form

$$\psi\left(\frac{\rho D^2 S^2}{F},\ \frac{\rho D^4 n^2}{F},\ \frac{\mu^2}{F\rho},\ \frac{\rho D^3 g}{F}\right) = 0. \tag{9, b}$$

Our conclusion is that any equation which is the correct and complete expression of a physical relation subsisting among seven quantities of the kinds mentioned is reducible to the form (9, b).

If $[F, \rho, D]$ were the only triad that could be used as fundamental units for the seven kinds of quantity, equation (9, b) would be the only general form of the equation; but in reality several other triads can be used, so that other equations may be found which, while essentially equivalent to (9, b), present a different appearance. If, for instance, we select the triad $[\rho, D, S]$, a process like that which led to equation (9, b) gives us the equation

$$\psi\left(\frac{\rho D^2 S^2}{F},\ \frac{Dn}{S},\ \frac{\rho DS}{\mu},\ \frac{Dg}{S^2}\right) = 0, \tag{9, c}$$

to which we shall have occasion to refer later.

4. *The General Form to Which Any Physical Equation is Reducible.*—Equation (9), subject to equations (11), gives the necessary form of any relation which subsists among n quantities of different kinds: it is the final form to which the dimensional conditions reduce equation (2). Now equation (2) describes a particular form of the more general relation described by equation (1), in which several quantities of each of the n kinds may be involved,—all but one of each kind being specified by their ratios to that one. Dimensional reasoning can not furnish any information regarding the influence of these dimensionless ratios on the phenomenon which is characterized by the relation in question, nor can it tell us how they are involved in the equation which describes the relation. But we can not assume that they are without influence, and the possibility of their entering into the relation must be indicated in the final equation which corresponds to (1) as equation (9) does to (2). Since equation (9) follows from equation (2), it is correct for any fixed values of the r's, and it may therefore be generalized so as to be applicable to any and all values of the ratios r by introducing the r's as independent arguments of the unknown function ψ, which is then a function of *all* the independent dimensionless combinations of powers of *all* the quantities of all the n kinds which are involved in the relation to be described.

The general conclusion from the principle of dimensional homogeneity may therefore be stated as follows: If a relation subsists among any number of physical quantities of n different kinds, and if the symbols $Q_1, Q_2, \cdots Q_n$ represent one quantity of each kind, while the remaining quantities of each kind are specified by their ratios r', r'', $\cdots$, etc., to the particular quantity of that kind selected, then: any equation which describes this relation completely is reducible to the form

$$\psi(\Pi_1, \Pi_2, \cdots \Pi_i, r', r'' \cdots) = 0. \tag{13}$$

If k is the number of fundamental units required in an absolute system for measuring the n kinds of quantity, the number of the dimensionless products Π is

$$i = n - k.$$

If $[Q_1]$, $[Q_2]$ $\cdots$ $[Q_k]$ are any k of the units for measuring the Q's, which are independent and so might be used as fundamental units; and if the remaining units needed are denoted by $[P_1]$, $[P_2]$ $\cdots$ $[P_i]$, each of the Π's may be determined from a dimensional equation

$$[\Pi] = [Q^{\alpha}{}_1 Q^{\beta}{}_2 \cdots Q^{\kappa}{}_k P] = [1] \tag{14}$$

after substituting in this equation the known dimensions of the $[Q]$'s and of $[P]$ in terms of any suitable set of k fundamental units.

5. *Remarks of the Utilization of the Foregoing Results.*—Equation (13), representing a single relation connecting a number of variables, can, in principle at least, be solved for any one of them and put into the form

$$\Pi_1 = \varphi(\Pi_2, \Pi_3, \cdots, \Pi_i, r', r'', r''', \cdots) \tag{15}$$

in which Π_1 is any one of the Π's; or into the form

$$r' = \varphi_1(\Pi_1, \Pi_2, \cdots \Pi_i, r'', r''', \cdots) \tag{16}$$

in which r' is any one of the r's.

Although the form of φ is unknown so that neither of these equations gives any definite general information, they may nevertheless be useful in particular circumstances. Equation (15), for example, tells us that if Π_2, Π_3, $\cdots$, Π_i, r', r'', etc., are all kept constant, Π_1 is also constant, regardless of the form of the unknown function φ. And since Π_1 is a product of known powers of Q_1, Q_2, $\cdots$ Q_k, P_1, we know how any one of these $(k + 1)$ quantities varies with the others under the given conditions. To illustrate; equation (9, b) may be put into the form

$$\frac{\rho D^2 S^2}{F} = \varphi\left(\frac{\rho D^4 n^2}{F}, \ \frac{\mu^2}{F\rho}, \ \frac{\rho D^3 g}{F}\right) \tag{17}$$

and if $\rho D^4 n^2/F$, $\mu^2/F\rho$ and $\rho D^3 g/F$ are kept constant we have

$$S^2 = \text{const.} \times \frac{F}{\rho D^2}. \tag{18}$$

If we wish to treat some one quantity X as the unknown and get an equation of the form (15) with this quantity absent from the second member, X must be a factor of only one of the Π's. This means that in selecting the variables which are to act as Q's and P's in equations (11) or (14), X must be one of the P's.

To illustrate the application of the above remarks, let us consider the screw-propeller problem. Let F be the thrust exerted by a screw propeller of a particular shape specified by a number of ratios of lengths r', r'', etc. and of a size specified by the diameter D. The thrust must be supposed to depend on the number of revolutions per unit time n, the speed of advance S, and the density ρ and viscosity μ of the liquid. It may safely be assumed that the very slight compressibility of the liquid has no sensible effect on the thrust, but unless the propeller is very deeply immersed, there will be surface disturbances and we must expect the thrust to be affected by the weight of the liquid, *i. e.*, by the intensity of gravity g. It does not appear that any other circumstance, except the depth of immersion which may be specified by its ratio to D and represented by an extra r, can influence the thrust, and if we are right in this assumption there must be an equation

$$f(F,\ \rho,\ D,\ S,\ n,\ \mu,\ g,\ r',\ r'',\ \cdots) = 0 \tag{19}$$

corresponding to equation (1). This equation must be reducible to the form (13), and we have already given in equations (9, *b*) and (9, *c*), two of the forms which it might have in the case of constant r's, *i. e.*, for a propeller of fixed shape and immersion.

Now suppose that we wish to find out how the thrust depends on the density of the liquid, the diameter of the propeller, and the speed of advance. Then in using equations (11) we must use F as one of the P's and take $[\rho, D, S]$ as $[Q_1, Q_2, Q_3]$. We then get the Π's which appear in equation (9, *v*) and the equation corresponding to (13) is

$$\psi\left(\frac{\rho D^2 S^2}{F},\ \frac{Dn}{S},\ \frac{\rho DS}{\mu},\ \frac{Dg}{S^2},\ r',\ r'',\ \cdots\right) = 0.$$

By solving for $\rho D^2 S^2/F$, this may be put into the form

$$F = \rho D^2 S^2 \varphi\left(\frac{Dn}{S},\ \frac{\rho DS}{\mu},\ \frac{Dg}{S^2},\ r',\ r'',\ \cdots\right) \tag{20}$$

which tells us without any experimentation at all, that if we can keep Dn/S, $\rho DS/\mu$, and Dg/S^2 constant, the thrust of a propeller of any given shape (r', r'', etc., constant) is proportional to the density of the liquid, the square of the diameter, and the square of the speed of advance. The meaning of this result and the value of the information will be discussed later: at present we may return to generalities.

6. *Physically Similar Systems.*—Equation (13) is a convenient expression of the conclusions to be drawn directly from the principle of dimensional homogeneity. It is useful in various ways, as will be illustrated later, but at present we may develop from it the notion of similar systems.

Let S be a physical system, and let a relation subsist among a number of quantities Q which pertain to S. Let us imagine S to be transformed into another system S' so that S' " corresponds " to S as regards the essential quantities. There is no point of the transformation at which we can suppose that the quantities cease to be dependent on one another; hence we must suppose that some relation will subsist among the quantities Q' in S' which correspond to the quantities Q in S. If this relation in S' is of the same form as the relation in S and is describable by the same equation, the two systems are " physically similar " as regards this relation. We have to enquire what sort of transformation would lead to this result, *i. e.*, what are the conditions which determine that two systems shall be similar as regards a given physical relation.

The original relation subsisting in S is reducible to the form (13), or

$$\psi(\Pi_1, \cdots, \Pi_i, r) = 0, \tag{13}$$

r representing *all* the independent ratios of quantities of the same kind which enter into the relation. The changes of the Q's during the transformation will, in general, result in a change of the numerical value of each Π or r. But these expressions remain dimensionless, so that to each of the arguments of ψ there corresponds, after the transformation, an expression Π' or r' of the same form in terms of the transformed quantities Q'; and these are all the independent dimensionless products of powers that can be made up of the quantities Q'. Hence the equation which describes the relation subsisting in S' among the quantities Q' is reducible to the form

$$\psi'(\Pi'_1, \cdots \Pi'_i, r') = 0. \tag{13'}$$

The requirement that S and S' shall be similar as regards this relation, means that the operators ψ and ψ' must be identical, and this will occur if the transformation leaves the numerical values of all the Π's and r's unchanged. For ψ and ψ' will then be applied to identical operands: and while two different functions of the same variables may vanish simultaneously for discrete sets of values of the variables, they can not do so for a continuous infinity of sets, and yet equations (13) and (13′) are satisfied without restriction. It follows that ψ and ψ' can not be different, or in other words; the systems S and S' are similar as regards this relation if corresponding Π's and r's are equal in the two systems. The nature of the transformation which leaves a system similar to itself may therefore be specified as follows:

(*a*) Any k quantities of independent kinds may be changed in completely arbitrary ratios, after which we must

(*b*) Change one quantity of each of the $(n - k) = i$ remaining kinds in such a ratio as to keep the numerical value of its Π unchanged; and finally we must

(*c*) Change the remaining quantities of each of the n kinds in the same ratio as the one quantity of that kind already mentioned, thereby keeping the ratios r unchanged.

The last and simplest of these conditions means that the system must remain similar to itself as regards each separate kind of quantity. If, for example, the sizes and shapes of some of its parts are essentially involved in the relation, the transformation must leave the system geometrically similar to itself as regards these parts, although other and unessential parts may change in any way. We have, in all, k arbitrary choices of ratios of change and since each of these may be made in an infinite number of ways there is a k-fold infinity of systems S' which are similar to any given system S as regards any particular physical relation. If the above conditions are fulfilled for all possible physical quantities which can pertain to a physical system, the transformed system will be similar to the original one as regards any possible relation between physical quantities and the two will be physically similar in *all* respects.

When absolute units are used, the validity of a complete physical equation is unaffected by changes in the fundamental units. Hence in changing from a system S to a similar system S' it is immaterial to the validity of the equation in question whether we do or do not retain our original fundamental units. If we alter the sizes of the fundamental units $[Q_1] \cdots [Q_k]$ in the same ratios as the kinds of quantity $Q_1 \cdots Q_k$ which they measure, the numerical value of any quantity of one of these kinds will be the same in both systems. And if we do not change the relations of the derived and fundamental units of our absolute system, every derived unit $[P]$ will change in the same ratio as every quantity P of that kind, so that the numerical value of every quantity in the system S will be equal to the numerical value of the corresponding quantity in the similar system S'.

This change of units will occur if the concrete primary standards which preserve the units partake of the transformation. To an observer whose quantitative information was all obtained by measurements based on such standards, not only all physical relations, but the numerical values of individual quantities, would appear the same in two similar systems: he could not distinguish the two systems nor detect a transformation of one into the other.

The foregoing theorem may, if we choose, be applied to an imaginary transformation of the whole physical universe, but in this grandiose general form it is of only metaphysical interest; for it is merely a statement about what would happen if we were to bring about certain changes which it is obviously quite beyond our powers to effect. Nevertheless in particular elementary instances the notion of physical similarity is useful and it is convenient to have the conditions of physical similarity formulated in a general way. One of these is always that of similarity with respect to each separate kind of quantity, such as length, speed, density, etc., which may enter into the physical phenomenon with which we happen to be concerned.

Let us suppose that a relation subsists among certain physical quantities. Dimensional reasoning suffices to tell us that if the relation is complete, the equation which describes it is reducible to the form (13); and if $Q_1 \cdots Q_k$ are the quantities which are used as independent in finding the Π's of equation (13), the equation can always be solved in the form

$$P_1 = Q_1^a Q_2^b \cdots Q_k^k \varphi(\Pi_2, \Pi_3, \cdots \Pi_i, r)$$

the form of the operator φ remaining to be found by other means. Without going into any abstruse consideration of conceivable modifications of the universe to which the various quantities may be regarded as pertaining, it is obvious that so long as we can, experimentally, control enough of these quantities to keep all the Π's and r's constant, any function of these arguments must also remain constant, no matter what its form may be. The practical application of the notion of similarity is based on this remark. The conditions necessary for this simplification are given by setting each Π and r equal to a constant: and when this is done the nature of the possible simultaneous variations which fulfil the requirements at once becomes evident.

It usually happens that some of the quantities concerned in the relation are obviously attached to or are properties of some body or some material system of limited extent and can be changed in value only by changing to a different body or system. If this second system is similar to the first as regards each separate kind of quantity all the r's which pertain to the system are the same. If we can so arrange the circumstances in which the system is placed that upon substituting one body or system for the other, $\Pi_2, \Pi_3, \cdots \Pi_i$ as well as any remaining r's, retain their values unchanged, the equation

$$P_1 = Q_1^a Q_2^b \cdots Q_k^k \times \text{const.}$$

is satisfied for both systems with the same value of the constant. The phenomenon characterized by the relation then occurs in a similar manner for both systems, and we say that the bodies or systems are similar with respect to this phenomenon. If the relation is a dynamical one, all essential parts of the two systems must be geometrically similar and have similar distributions of density, elasticity, etc., so far as these properties affect their behavior. If, in addition, the Π's of the relation are kept the same for one system as for the other, the systems are said to be "dynamically similar," though they might, of course, not be similar as regards some other dynamical relation nor behave similarly in some different sort of experiment.

The notion of physical similarity does not appear to have been developed and used to any extent except in this most obvious form of dynamical similarity. But the more general conception of a similarity which extends to other than merely dynamical relations, evidently follows directly from the dimensional reasoning, based on the principle of homogeneity, which culminated in equations (13) and (14).

7. *Remarks.*—In his article entitled "The Principle of Similitude," appearing in the April, 1914, number of the PHYSICAL REVIEW, Mr. Richard C. Tolman announces the discovery of a new principle and illustrates its value in reasoning about the forms of physical equations, by treating several examples. The statement of the principle is couched in such general terms that I have difficulty in understanding just what the postulate is, but it seems to me to be merely a particular case of the general theorem given in the foregoing section. Mr. Tolman selects length, speed, quantity of electricity, and electrostatic force as the four independent kinds of quantity which suffice for his purposes, and after subjecting them to four arbitrary conditions, he proceeds to find the conditions to which several other kinds of quantity are subject in passing from the actual universe to a miniature universe that is physically similar to it. Now I do not know whether the developments set forth above have ever been published in just this form, but it is certain that they are merely consequences of the principle of dimensional homogeneity, which is far from being either new or unfamiliar. The unnecessary introduction of new postulates into physics is of doubtful advantage, and it seems to me decidedly better, from the physicist's standpoint, not to drag in either electrons or relativity when we can get on just as well without them. Accordingly, my object in publishing the foregoing sections, which are a

fragment of a longer paper that I have had in hand for some time, is to call attention to the fact that in the present instance no new postulate seems to be needed. My feeling that Mr. Tolman's "Principle of Similitude" is not really new may, of course, be mistaken. But for the purposes to which he puts it, it is, at all events, superfluous, and this I shall proceed to prove by treating some of the problems he has used as illustrations.

The relations that involve temperature may be passed over, because Mr. Tolman's reasoning is based on the assumption that absolute temperature has the dimensions of energy, and this assumption is not permissible. If by "absolute temperature" is meant temperature measured by what is commonly called the thermodynamic scale, then the ratio of two temperatures is, by definition, the ratio of two quantities of heat. In a similar way, two intervals of time, measured by our ordinary time scale, have the same ratio as two angles through which the earth has rotated about its axis during these intervals; or two forces have the same ratio as the lengths by which they can stretch a given spiral spring. We do not, however, conclude that time has the dimensions of angle and force the dimensions of length; nor can we say that temperature has the dimensions of energy. The units needed for measuring thermal quantities can not all be derived from mass, length, and time or from any other set of three fundamental units which suffice for mechanics, and a fourth unit is indispensable. In practice, this special thermal unit is nearly always temperature; it is fixed by the arbitrary selection of the interval between the freezing and boiling points of water and the arbitrary assignment of a particular numerical value to this interval. We do not at present know of any method by which this or any other interval of temperature can be fixed by, *i. e.*, derived from, purely mechanical quantities without some further act of arbitrary choice than the selection of three mechanical units. We may therefore turn to Mr. Tolman's electromagnetic problems.

For the measurement of electric and magnetic quantities, one new fundamental unit is needed, beyond the three of mechanics, so that there must, in general, be four in all. In the electromagnetic system the new unit is permeability $[\mu]$ and the four are $[m, l, t, \mu]$. In the electrostatic system it is dielectric inductivity $[\epsilon]$, and the four are $[m, l, t, \epsilon]$. Other sets are sometimes more convenient, for example $[l, t, C, R]$, $[C]$ being current and $[R]$ resistance: this system corresponds to the "international" units, a system in which the ampere and the ohm are, by definition, fundamental units. Various systems might be used in dimensional reasoning without altering anything but a little algebra; the only important thing is the number of fundamental units required as the basis of the system.

8. *Illustrations of the Treatment of Electromagnetic Problems by the Method of Dimensions*: (*A*) *Energy Density of an Electromagnetic Field.*—We assume that the energy density u is completely determined by the field strengths E and H, and by the permeability μ and dielectric inductivity ϵ of the medium. If, or when, this assumption is valid we have

$$f(u, E, H, \mu, \epsilon) = 0. \tag{21}$$

The dimensions of these quantities on the $[m, l, t, \mu]$ system are

$$\left.\begin{aligned} [E] &= [m^{\frac{1}{2}}l^{\frac{1}{2}}t^{-2}\mu^{\frac{1}{2}}], & [u] &= [ml^{-1}t^{-2}], \\ [H] &= [m^{\frac{1}{2}}l^{-\frac{1}{2}}t^{-1-}\mu^{\frac{1}{2}}], & [\epsilon] &= [l^{-2}t^{2}\mu^{-1}]. \end{aligned}\right\} \tag{22}$$

We wish to get a relation that can be solved for u; hence u must not be one of the Q's of equation (14). Although in general, electrogmagnetic units require four fundamental units, three are enough in this instance: for example we may take $[E]$, $[\mu]$, and $[\epsilon]$ as fundamental units and from them derive the remaining two units

$$\left.\begin{aligned} [H] &= [E\epsilon^{\frac{1}{2}}\mu^{-\frac{1}{2}}], \\ [\mu] &= [E^{2}\epsilon]. \end{aligned}\right\} \tag{23}$$

With $n = 5$ and $k = 3$, $n - k = i = 2$ and there are only two of the Π's. To determine them we have by equation (14)

$$\left.\begin{aligned} [\Pi_1] &= [E^{a_1}\epsilon^{\beta_1}\mu^{\gamma_1}H] = [1], \\ [\Pi_2] &= [E^{a_2}\epsilon^{\beta_2}\mu^{\gamma_2}u] = [1]. \end{aligned}\right\} \tag{24}$$

To determine the exponents, we might substitute the dimensions given in equations (22); but since we already have the dimensions of $[H]$ and $[u]$ in terms of $[E, \epsilon, \mu]$ by equations (23), it is easier not to refer back to the complicated $[m, l, t, \mu]$ equations but use the $[E, \epsilon, \mu]$ system at once, Equations (24) then give us

$$\left.\begin{aligned} [\Pi_1] &= [E^{a_1}\epsilon^{\beta_1}\mu^{\gamma_1}E\epsilon^{\frac{1}{2}}\mu^{-\frac{1}{2}}] = [1] \\ [\Pi_2] &= [E^{a_2}\epsilon^{\beta_2}\mu^{\gamma_2}E^{2}\epsilon] = [1]. \end{aligned}\right\} \tag{25}$$

From the first of these we obtain the values

$$\alpha_1 = -1, \qquad \beta_1 = -\tfrac{1}{2}, \qquad \gamma_1 = \tfrac{1}{2},$$

so that we may write

$$\Pi_1 = \frac{\mu^{\frac{1}{2}}H}{\epsilon^{\frac{1}{2}}E}$$

or more conveniently

$$\Pi_1 = \frac{\mu H^2}{\epsilon E^2}.$$

From the second of equations (25) we have

$$\alpha_2 = -2, \qquad \beta_2 = -1, \qquad \gamma_2 = 0,$$

whence

$$\Pi_2 = \frac{u}{\epsilon E^2},$$

and the equation which corresponds to (9) or (13) therefore has the form

$$\psi\left(\frac{\mu H^2}{\epsilon E^2}, \ \frac{u}{\epsilon E^2}\right) = 0. \tag{26}$$

Solving this for $u/\epsilon E^2$ and multiplying by ϵE^2 we have, finally,

$$u = \epsilon E^2 \varphi_1\left(\frac{\mu H^2}{\epsilon E^2}\right). \tag{27}$$

For the sake of illustration we have chosen to obtain this result by means of the general process described in the earlier sections: but the result is obvious without the aid of any such elaborate machinery. For since by equation (23) $[u] = [\epsilon E^2]$, it is evident directly from the principle of dimensional homogeneity that if u is to be expressed as a function of ϵ and E it can only be in the form ϵE^2 multiplied by a dimensionless number.

By taking $[H, \epsilon, \mu]$ as fundamental, instead of $[E, \epsilon, \mu]$ we should have got the obviously equivalent result

$$u = \mu H^2 \varphi_2\left(\frac{\epsilon E^2}{\mu H^2}\right).$$

Assuming that the complete formula is

$$u = \frac{1}{8\pi}(\epsilon E^2 + \mu H^2)$$

we have

$$\varphi_1(x) = \varphi_2(x) = \frac{1 + x}{8\pi}.$$

If the medium is not isotropic, certain angles which fix the directions of E and H with respect to the principal axes of ϵ and μ must also appear as arguments of the unknown functions φ_1 and φ_2.

(*B*) *Relation between Mass and Radius of an Electron.*—There is no object in limiting our considerations to a particular kind of disembodied charge moving in free space, and we may as well make the treatment more general.

Let T be the energy of a charge e of any fixed distribution and of a size specified by any one of its linear dimensions D, when it is moving at the speed S through a medium of permeability μ and inductivity ϵ. If r', r'', etc., are ratios of lengths which specify the distribution of the charge, and if we assume that, for a fixed distribution, T does not depend on any other quantities than those already named, we must have

$$f(T, e, D, S, \mu, \epsilon, r', r'', \cdots) = 0. \tag{28}$$

For measuring all these quantities, an absolute system requires four fundamental units, the only new electrical quantity not mentioned in equations (22) being e, which has the dimensions

$$[e] = [m^{\frac{1}{2}} l^{\frac{1}{2}} \mu^{-\frac{1}{2}}].$$

Since we wish finally to solve for T, this must be one of the $[P]$'s of equation (14) and for the $[Q]$'s we must take four of the five units $[e, D, S, \mu, \epsilon]$. The five different combinations of these units, taken four at a time are

$$D, S, e, \epsilon;\ D, S, e, \mu;\ D, S, \epsilon, \mu;\ D, e, \epsilon, \mu;\ S, e, \epsilon, \mu.$$

But since $[\mu\epsilon] = [S^{-2}]$ as is seen from equations (22) S, ϵ, and μ can not be used simultaneously as fundamental units, so that our choice is limited to the combinations

$$D, S, e, \epsilon;\ D, S, e, \mu;\ D, e, \epsilon, \mu.$$

The three separate sets of units are then as follows:

$$\left.\begin{array}{lllll} 1. & [D, S, e, \epsilon], & [\mu] = [S^{-2}\epsilon^{-1}], & [T] = [D^{-1}e^{2}\epsilon^{-1}], \\ 2. & [D, S, e, \mu], & [\epsilon] = [S^{-2}\mu^{-1}], & [T] = [D^{-1}S^{2}e^{2}\mu], \\ 3. & [D, e, \epsilon, \mu], & [S] = [\epsilon^{-\frac{1}{2}}\mu^{-\frac{1}{2}}], & [T] = [D^{-1}e^{2}\epsilon^{-1}], \end{array}\right\} \tag{29}$$

and we can get a solution by using any one of these sets in equation (14) for finding Π_1 and Π_2.

In any case, it is easily seen that $S^2\mu\epsilon$ will be one of the two Π's. The remaining Π is to be found from one of the equations

$$\begin{aligned} [\Pi] &= [D^{\alpha}S^{\beta}e^{\gamma}\epsilon^{\delta}D^{-1}e^{2}\epsilon^{-1}] = [1], \\ [\Pi] &= [D^{\alpha}S^{\beta}e^{\gamma}\mu^{\delta}D^{-1}S^{2}e^{2}\mu] = [1], \\ [\Pi] &= [D^{\alpha}e^{\beta}\epsilon^{\gamma}\mu^{\delta}D^{-1}e^{2}\epsilon^{-1}] = [1]. \end{aligned} \tag{30}$$

From these three equations we get successively

$$\Pi = \frac{D\epsilon T}{e^2}, \quad \Pi = \frac{DT}{S^2e^2\mu}, \quad \Pi = \frac{D\epsilon T}{e^2}.$$

Since the first and third solutions are identical we have only two different forms of equation corresponding to (13), and they are

$$\psi_1\left(\frac{D\epsilon T}{e^2},\ S^2\mu\epsilon,\ r',\ r''\cdots\right)=0, \tag{31}$$

$$\psi_2\left(\frac{DT}{S^2e^2\mu},\ S^2\mu\epsilon,\ r',\ r''\cdots\right)=0, \tag{32}$$

or solving for T,

$$T=\frac{e^2}{D\epsilon}\varphi_1(S^2\mu\epsilon,\ r',\ r''\cdots), \tag{33}$$

$$T=\frac{S^2e^2\mu}{D}\varphi_2(S^2\mu\epsilon,\ r',\ r'',\ \cdots). \tag{34}$$

It is interesting to consider the physical meaning of these results. We did not restrict T to being merely energy due to motion: it is the total energy and it must therefore reduce to T_0, the electrostatic energy of the charge, when $S=0$. Equation (33) accordingly gives

$$T_0=\frac{e^2}{D\epsilon}\varphi_1(0,\ r',\ r'',\ \cdots)$$

and this agrees, as it should, with the known fact that the work done in collecting a charge e, in a medium of inductivity ϵ, into a distribution of linear size D is proportional to $e^2/D\epsilon$, the proportionality factor depending on the shape of the distribution, *i. e.*, on the ratios r', r'', etc. Since equation (34) must give the same result, φ_2 must contain the factor $1/S^2\mu\epsilon$ in order to keep T finite as S vanishes. If this factor is taken out the two equations become identical.

If we had originally let T represent only T_s, the part of the energy due to motion or the work required to start the charge going from rest, nothing in the reasoning would have been changed and the resulting equations would have had the same form as above. If we define $T_s/S^2=m_s$ as the electrical mass of the moving charge, equation (34) gives us

$$m_s=\frac{e^2\mu}{D}\varphi_3(S^2\mu\epsilon,\ r',\ r''\cdots),$$

and if we set $\mu\epsilon=c^{-2}$, we have

$$m_s=\frac{e^2\mu}{D}\varphi_4\left(\frac{S}{c},\ r',\ r'',\ \cdots\right). \tag{35}$$

If we now let the charge be an electron of fixed quantity moving in free space so that ϵ, μ and c are constant, the equation takes the simpler form

$$m_s=\frac{1}{D}f(S,\ r',\ r'',\ \cdots) \tag{36}$$

or in words: the electrical mass of an electron of fixed distribution is inversely proportional to its linear dimensions; it also depends on the speed and on the distribution, but the nature of this dependence is not determinable from dimensional reasoning.

(*C*) *Radiation from an Accelerated Electron.*—Let a disembodied charge e be moving with the velocity S through a medium of permeability μ and inductivity ϵ. Let the distribution of the charge be specified by the length ratios r', r'', etc., and its size by some linear dimension D. Let the charge have a resultant acceleration a which makes an angle θ with S, θ being dimensionless like the r's. Let R be the total time rate of loss of energy by radiation.

So far as we can tell a priori, R may depend on all of the circumstances mentioned above, and if it does not depend on any others, we may write

$$f(R, D, S, a, e, \mu, \epsilon, \theta, r', r'' \cdots) = 0. \tag{37}$$

We have, in this instance, $n = 7$, $k = 4$ hence $i = 3$ and the equation will be reducible to the form

$$\psi(\Pi_1, \Pi_2, \Pi_3, \theta, r', r'', \cdots) = 0. \tag{38}$$

It shortens the process of solution to remark that $S^2\mu\epsilon$ and Da/S^2 are both dimensionless, so that as there can be at most only three such *independent* dimensionless products, only one remains to be found and these two can be used for two of the Π's, whatever form of solution we choose.

Since we wish ultimately to solve for R we must make $[R] = [P]$ in finding the third Π, and the four $[Q]$'s are to be selected from among $[D, S, a, e, \epsilon, \mu]$. Since the two relations $[S^2\mu\epsilon] = [1]$ and $[DaS^{-2}] = [1]$ are already given among the 6 units, these two combinations can not occur and there are only 6 combinations of 4 independent units each, instead of the 15 which there would otherwise be. Using these 6 combinations, together with $[R] = [P]$, successively in an equation of the form (14), we find that the resulting forms obtained for our third Π are

$$\Pi = \frac{e^2S^3\mu}{RD^2}, \quad \frac{e^2S}{RD^2\epsilon}, \quad \frac{e^2a^{\frac{3}{2}}\mu}{RD^{\frac{1}{2}}}, \quad \frac{e^2a^{\frac{1}{2}}}{RD^{\frac{3}{2}}\epsilon}, \quad \frac{e^2a^2\mu,}{RS}, \quad \frac{e^2a^2}{RS^3\epsilon}. \tag{39}$$

The 6 resulting forms of equation (38) are all equivalent, and it is sufficient to consider any one of them, *e. g.*,

$$\psi\left(\frac{e^2a^2\mu}{RS}, \ S^2\mu\epsilon, \ \frac{Da}{S^2}, \theta, ,r', r'', \cdots\right) = 0, \tag{40}$$

which we may put into the form

$$R = \frac{e^2a^2\mu}{S}\varphi\left(S^2\mu\epsilon, \frac{Da}{S^2}\theta, r', r'', \cdots\right). \tag{41}$$

Since D, a, and S appear in the unknown function, this equation gives us no definite information except that R is proportional to e^2, and we can not tell anything about how R may depend on a or S. It is therefore evident that we must assume a more limited range of dependence of R if we are to get any more definite results. If we assume that R does not depend on the linear size of the charge, φ must be independent of D, therefore of Da/S^2, and therefore of a. Hence on this hypothesis equation (41) reduces to

$$R = \frac{e^2a^2\mu}{S}\varphi_1\left(S^2\mu\epsilon, \theta, r', r'', \cdots\right). \tag{42}$$

If we assume that R is independent of S but make no assumption as to D, we can not eliminate a from the unknown function because S appears, in equation (41), in two of the independent arguments of φ. If we assume that R does not depend on either S or D, φ_1 of equation (42) must contain S as a factor and therefore $S\mu^{\frac{1}{2}}\epsilon^{\frac{1}{2}}$, so that the equation reduces to

$$R = e^2a^2\mu^{\frac{1}{2}}\epsilon^{\frac{1}{2}}\varphi_2(\theta, r', r'', \cdots). \tag{43}$$

Either (42) or (43) might have been obtained by making the necessary exclusion of variables from equation (37) and working the result out separately. It is, however, much more instructive to include, at the start, all the quantities which we can reasonably suppose might be of importance, and then carry out our exclusion after the general equation (41) has been obtained.

If we now restrict our considerations to a charge of fixed amount and of fixed shape and size; and if we further suppose it to move always in the same medium, e, D, μ, ϵ, r', r'', etc., are all constant and equations (41), (42), and (43) degenerate into

$$R = a^2f_1(S, a, \theta), \tag{41, a}$$
$$R = a^2f_2(S, \theta), \tag{42, a}$$
$$R = a^2f_3(\theta). \tag{43, a}$$

These are the equations for the radiation of an electron of specified shape and size moving in free space. As we see, the simple form (43, a) can not be obtained without assumptions which are far from plausible.

9. *Thermal Transmissivity.*—For variety, we may illustrate the use of the same general method by applying it to a thermal problem, namely that of the transmission of heat between the wall of a metal pipe and a

stream of fluid which is flowing through it and is hotter or colder than the pipe. Although a great many experiments have been made on this important practical subject, our information is still very incomplete, and the method of dimensions may be of service, both in planning experiments, and in analyzing and interpreting the results obtained.

Let the pipe be of uniform section and long compared with its greatest diameter. Let the shape of its section be specified by a number of length ratios, which we will represent by a single symbol r, and let D be any one dimension, such as the diameter if the pipe is round. Let S be the mean linear speed of the fluid at any section, as measured by the rate of discharge. Let θ be the absolute temperature of the wall surface at any section, and $\Delta\theta$ the difference between this and the mean temperature of the fluid at that section. There will be a flow of heat between the pipe and the fluid in one direction or the other, according to the direction of the temperature drop, and until the contrary is shown we must assume that this rate of heat transmission may depend on D, S, θ, $\Delta\theta$, and the properties of the fluid.

We shall suppose that the part played by radiation is negligible, thereby excluding the consideration of such cases as flame in boiler tubes; and the thermal properties of the fluid which need attention are then its thermal conductivity λ and its specific heat C. The rate of transmission will, in general, be affected by convection, so that we must take account of the mechanical properties which determine the nature of the motion of the fluid, namely its density ρ and viscosity μ. If the fluid is a gas, the compressibility may also need to be taken into consideration; but it appears that at speeds which are less than one half that of sound in the medium, this element may be disregarded, gases behaving sensibly like liquids of the same density and viscosity. We shall limit our considerations to these moderate speeds, so that such results as are obtained will not be applicable without modification to the transmission of heat between a steam-turbine nozzle and the jet flowing through it, or to similar cases where the speed is very high.

Let $\tau\Delta\theta$ be the heat transmitted per unit time through unit area of wall surface, τ being known as the transmission coefficient or "transmissivity." To obviate the need of introducing the mechanical equivalent of heat, we may suppose quantities of heat to be measured in absolute work units derived from the fundamental mechanical units $[m, l, t]$ which, together with the temperature unit $[\theta]$, will suffice for all the quantities with which we have to deal. We do *not* assume that the transmissivity τ is independent of $\Delta\theta$; that question is left open.

If we have not overlooked any of the circumstances which have a sensible effect on the heat transmission, we may now write

$$f(\tau, D, S, \Delta\theta, \rho, \mu, \lambda, C, \Delta\theta/\theta, r) = 0 \tag{44}$$

θ and $\Delta\theta$ being quantities of the same kind, so that only one of them appears in the list of variables while the other is represented by a ratio, $\Delta\theta/\theta$. The number of different kinds of quantity is $n = 8$; the number of fundamental units required is $k = 4$; hence $i = 4$ and the equation, whatever its precise form, must be reducible to

$$\psi(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Delta\theta/\theta, r) = 0, \tag{45}$$

We wish to find an expression for the transmissivity; hence in finding the Π's by means of equation (14), τ must be one of the P's. In the process of solution let us set

$$\begin{aligned} [\rho, D, S, \Delta\theta] &= [Q_1, Q_2, Q_3, Q_4], \\ [\tau, \mu, \lambda, C] &= [P_1, P_2, P_3, P_4]. \end{aligned}$$

The dimensions of these quantities on the $[m, l, t, \theta]$ system are

$$\begin{array}{ll} [\rho] = [ml^{-3}], & [\tau] = [mt^{-3}\theta^{-1}], \\ [D] = [l], & [\mu] = [ml^{-1}t^{-1}], \\ [S] = [lt^{-1}], & [\lambda] = [mlt^{-3}\theta^{-1}], \\ [\Delta\theta] = [\theta], & [C] = [l^2t^{-2}\theta^{-1}], \end{array}$$

and if we use these values in solving equations (14) for the four Π's, the usual routine procedure gives us the equation

$$\psi\left(\frac{\tau\Delta\theta}{\rho S^3}, \frac{\mu}{\rho DS}, \frac{\lambda\Delta\theta}{\rho DS^3}, \frac{C\Delta\theta}{S^2}, \frac{\Delta\theta}{\theta}, r\right) = 0 \tag{46}$$

or

$$\tau = \frac{\rho S^3}{\Delta\theta}\varphi\left(\frac{\mu}{\rho DS}, \frac{\lambda\Delta\theta}{\rho DS^3}, \frac{C\Delta\theta}{S^2}, \frac{\Delta\theta}{\theta}, r\right) \tag{47}$$

as a form to which the equation for τ must be reducible if our initial assumptions regarding the dependence of τ on the other quantities were correct.

Equation (47) conveys no definite information whatever, but we may give a few indications of how such an equation may, nevertheless, be utilized in supplementing incomplete experimental data or in planning new experiments.

Since dimensional reasoning can give us no further help, we turn to experiment. It is known that while at low speeds we may have streamline motion of a fluid through a smooth straight pipe, this form of motion becomes unstable at higher speeds and breaks up into turbulent motion. We shall suppose that the speed is high enough and the pipe sufficiently rough that the motion of the fluid is very turbulent. It is known, further, that under these conditions the mechanical behavior of the fluid and the nature of its motion are nearly independent of the value of the viscosity. And since in turbulent motion, convection will certainly play an important part in the phenomenon of heat transmission, and the nature of the fluid motion will therefore be important, it is legitimate to assume, as an approximation at all events, that μ does not appear in the equation for τ which applies to these conditions of flow. The variable $\mu/\rho DS$ will therefore be absent and equation (47) will assume the simpler form

$$\tau = \frac{\rho S^3}{\Delta\theta}\varphi\left(\frac{\lambda\Delta\theta}{\rho DS^3}, \frac{C\Delta\theta}{S^2}, \frac{\Delta\theta}{\theta}, r\right). \tag{48}$$

We must now resort to experiments on transmission, for information about the form of φ, varying the arguments of φ separately and determining corresponding values of τ. To vary one of these arguments we have to vary its separate factors, which are the physical quantities over which we have direct control; and it is usually most convenient in practice to vary these separate factors one at a time,—for instance, to find the relation of τ to S when everything else is kept constant. If we are to vary a single one of the arguments of φ by varying a particular one of the physical quantities in question, that quantity must appear in only one of the arguments. This is true, in equation (48), of ρ, λ, C, and θ, so that we could proceed at once to investigate the form of φ by making experiments on the relations of τ to these quantities separately. On the other hand, D, S, and $\Delta\theta$ appear in more than one argument, so that we could not at once interpret the results of experiments in which one of these quantities was varied.

Now it is not practicable to vary the density, conductivity, and specific heat of the fluid arbitrarily and independently, though we may keep them all constant by making all our experiments on the same fluid. Furthermore, while θ may be varied independently of D, S, and $\Delta\theta$, θ inevitably influences the properties of the fluid, which can not be kept entirely constant during variations of temperature; and, in addition, attempts to vary θ over a wide range may encounter formidable difficulties. The quantities ρ, λ, C, and θ are thus precisely the ones which we do not want to use as independent variables. In practice, the most natural and convenient mode of experimentation is to vary S or $\Delta\theta$; and if we have various pipes available, D also may be varied. Hence equation (48) is not at present in a suitable form for our purpose and the arguments of φ must be replaced by others which are still independent and dimensionless but in which D, S, and $\Delta\theta$ are, if possible, separated.

Before proceeding to this transformation we shall first limit our considerations to pipes of a particular shape, *e. g.*, round pipes, which are the the most important. The ratios r are then constant, and so long as it is understood that we refer only to round pipes, r may be omitted from the equations, the effect of varying shape being left for separate investigation, after the study has been completed for round pipes.

Since the speed S is the easiest of our quantities to vary arbitrarily, we attend to it first: it appears in two of the arguments of φ and we will therefore replace one of these by another which does not contain S. Since the form of φ is unknown, we may raise any one of its arguments to any desired power. We take the 2/3 power of the first and notice that

$$\left(\frac{\lambda\Delta\theta}{\rho D S^3}\right)^{\frac{2}{3}} = \frac{C\Delta\theta}{S^2}\left(\frac{\lambda^2}{\rho^2 C^3 D^2 \Delta\theta}\right)^{\frac{1}{3}}.$$

Let

$$\frac{\lambda^2}{\rho^2 C^3} = K,$$

K being then a quantity which involves only properties of the fluid and may be regarded as one of its characteristic constants. We may now write

$$\left(\frac{\lambda\Delta\theta}{\rho D S^3}\right)^{\frac{2}{3}} = \frac{C\Delta\theta}{S^2}\left(\frac{K}{D^2\Delta\theta}\right)^{\frac{1}{3}}.$$

But any function of xy and y may be expressed as a function of x and y or of x^m and y^n: hence we may replace (48) by the equivalent equation, referring to a fixed shape of cross section,

$$\tau = \frac{\rho S^3}{\Delta\theta}\varphi_1\left(\frac{K}{D^2\Delta\theta}, \frac{C\Delta\theta}{S^2}, \frac{\Delta\theta}{\theta}\right), \tag{49}$$

in which D, S and θ are separated, though $\Delta\theta$ remains involved in all three arguments. Equation (49) is suitable for the interpretation of experimental data on τ obtained by varying D, S, and θ separately, for the variations will vary the three arguments of φ_1 separately and so tell us how φ_1 varies with the *whole* of any one of its three arguments.

A sufficiently complete and accurate experimental investigation of this sort would, in principle, always enable us to find the complete form of the operator φ_1, and the use of dimensional reasoning has the advantage that it enables us to plan the experiments rationally. It may turn out that the form of φ_1 is so complicated and the investigation so laborious that a complete solution of the problem is virtually impossible, or the result to be obtained not of sufficient importance to warrant the labor involved. In such instances the use of the principle of similarity, *i. e.*, reducing the unknown function to a constant by keeping all its arguments constant, sometimes permits of our securing partial information which suffices for particular practical purposes, and this will be illustrated in the following section. On the other hand, especially in cases where the quantities in question can not be measured very accurately or where no great accuracy in the results is required, it may happen that, to the approximation needed, the form of the unknown function is very simple. The method of procedure in such an instance may be illustrated by continuing the consideration of transmissivity.

Returning to equation (49), let us consider variations of the speed S. It appears from experiment that the transmissivity is nearly proportional to the 0.8 power of the speed when other things are constant; and merely to illustrate how such a result might be used, we shall suppose this relation to be exact. It follows that S can not be involved in φ_1 except as a factor $S^{-2.2}$. And since S appears only in the argument $C\Delta\theta/S^2$, φ_1 must contain the factor $(C\Delta\theta/S^2)^{1.1}$. Equation (49) must therefore have the more specific form

$$\tau = \rho S^{0.8}C^{1.1}\Delta\theta^{0.1}\varphi_2(K/D^2\Delta\theta,\ \Delta\theta/\theta). \tag{50}$$

This is simpler than before and suitable for continuing the work by varying the diameter and wall temperature of the pipe, or by using various fluids so as to vary K, the values of ρ, λ, and C being assumed to be known for the fluids used.

We will suppose, however, that it is not practicable to vary θ through any wide range and that we prefer to make experiments with various values of the temperature drop $\Delta\theta$ before altering D, which requires the dismantling of the apparatus and the substitution of a new pipe of different diameter. We must then transform equation (50) in such a way that $\Delta\theta$, which we are to use experimentally as the independent variable, appears in only one of the arguments of the unknown function. This is evidently accomplished by writing

$$\tau = \rho S^{0.8}C^{1.1}\Delta\theta^{0.1}\varphi_3(K/D^2\Delta\theta,\ K/D^2\theta), \tag{51}$$

which is suited to the interpretation of experiments on the dependence of τ on $\Delta\theta$.

It is commonly assumed that so long as $\Delta\theta$ is small, τ is independent of $\Delta\theta$. If experiment were to show that this relation was a general one, it would thereby be proved that φ_3 must contain $(K/D^2\Delta\theta)^{0.1}$ as a factor, and equation (51) would receive the still more specific form

$$\tau = \frac{\rho S^{0.8} C^{1.1} K^{0.1}}{D^{0.2}} \varphi_4 \left(\frac{K}{D^2 \theta} \right). \tag{52}$$

Having reached this point, the investigation might be completed by varying D, or changing the fluid so as to vary the value of K, the two methods providing a mutual check. If θ also were varied, a second check would be provided.

If, to take a purely hypothetical case, it were found that, in pipes for which τ is sensibly proportional to $S^{0.8}$, and within temperature limits such that τ is sensibly independent of $\Delta\theta$, τ was also sensibly independent of the diameter D, we should know that within these limits φ_4 could be represented with sensible accuracy by $(D^2\theta/k)^{0.1}$ × constant and τ by the equation

$$\tau = \text{const.} \times \rho S^{0.8} C^{1.1} \theta^{0.1}.$$

Or if, to take another imaginary result, it were found that the transmissivity, beside being independent of the viscosity, proportional to the 0.8 power of the speed, and independent of the temperature difference, was also independent of the temperature of the wall surface, we should know that the expression for τ must be

$$\tau = \text{const.} \times \frac{\rho^{0.8} S^{0.8} C^{0.8} \lambda^{0.2}}{D^{0.2}}.$$

It would be out of place here to pursue this subject into an analysis of the numerous but unhomogeneous data which have been published concerning transmissivity. Enough has been said to illustrate the procedure and to show that the utility of the dimensional method is by no means confined to its applications to hydrodynamics or electromagnetic theory.

10. *An Illustration of Dynamical Similarity.*—The application of dimensional reasoning to mechanical problems is often useful in the interpretation of model experiments designed to furnish, at a comparatively small expense, information about the performance to be expected from full-sized machines. Advantage is then taken of the idea of dynamical similarity—a particular case of physical similarity in general. Since this subject seems to be less familiar to physicists than it deserves to be, a single illustrative example may, perhaps, be worth giving.

It was found, in section 5, that if the thrust F of a screw propeller of given shape and immersion can be assumed to depend only on the diameter D, the speed of advance S, the number of turns per unit time n, the density and viscosity of the liquid ρ and μ, and the acceleration of gravity g, we must have the relation given by equation (20) or

$$F = \rho D^2S^2\varphi\left(\frac{Dn}{S}, \frac{\rho DS}{\mu}, \frac{Dg}{S^2}, r', r'', \cdots\right) \tag{53}$$

in which the ratios r specify the shape and immersion of the propeller.

The principle of dynamical similarity states that in passing from one screw propeller to a second, in the same or in another liquid, any three kinds of quantity, such as (ρ, D, S), which can provide fundamental units, may be changed in any ratios whatever; and that the equation which connects the thrust with the other quantities will remain precisely the same if the values of the arguments of φ remain unchanged. This means, in simpler language, that if we find the value of the constant N in the equation

$$F = N\rho D^2S^2$$

from an experiment in which the arguments of φ have a certain fixed set of values, the same constant is applicable to any values of (ρ, D, S) if the values of Dn/S, $\rho DS/\mu$, Dg/S^2, and the r's are the same in the second case as in the first.

The simplest of the requirements for the useful application of equation (53) is that the r's shall be constant; hence the two propellers, whatever their diameters, must be geometrically similar and similarly immersed; and the smaller may be called the model while the larger is called the original. The next simplest condition is that Dn/S shall remain constant. Now πDn is the speed of the circumferential motion of a point on the tip of one of the blades, and $\pi(Dn/S)$ is the tangent of the angle between the actual helical path of such a point and the direction of advance of the screw as a whole, which is supposed to coincide with the axis of the screw. The blades being of a fixed shape, the condition that Dn/S shall be constant is the same as the condition that the "angle of attack" of the blades on the still water into which they are advancing shall be constant. If p is the pitch of the propeller so that pn is the so-called "speed of the screw" or the speed at which it would advance if the water acted like a solid nut, $(pn - S)$ is the "slip" and $(pn - S)/pn$ is the "slip ratio." It is easily seen that if Dn/S is constant for propellers of a given shape, the slip ratio is constant. Our two conditions may now be expressed by saying that for two screw propellers to be dynamically similar, they must first of all have the same shape and be run at the same relative immersion and at the same slip ratio.

When the foregoing preliminary conditions are fulfilled, equation (53) reduces to the form

$$F = \rho D^2S^2\varphi\left(\frac{\rho DS}{\mu}, \frac{Dg}{S^2}\right) \tag{54}$$

and the next question is whether we can obtain any information about the thrust to be expected from a screw of diameter D run at the speed S, by experiments on a model screw of diameter D', run at the speed S' and at the same immersion and slip ratio as the original. The answer depends on our ability to arrange matters so that $\rho DS/\mu$ and Dg/S^2 shall be the same in the model experiment as in the practical operation of the full-sized original, and we at once encounter difficulties. In the first place, the intensity of gravity g is sensibly constant so that D/S^2 must also be kept constant. But on the other hand, we are virtually limited to experimenting in water for which ρ/μ is sensibly constant. Hence DS as well as D/S^2 must be kept constant, so that neither D nor S can be varied: in other words, we can not, in practice, run a reduced-scale model screw propeller so that it shall be dynamically similar to its original. We must therefore limit ourselves to a less ambitious program and attempt to obtain an approximate result which may be of some value, even though it is recognized as incomplete; and to do this we must find a plausible pretext for omitting one of the two arguments of φ from equation (54).

This presents no difficulty. For it is apparent from various hydrodynamic experiments that when a fluid is in very turbulent motion its mechanical behavior is little influenced by viscosity, density being much more important. Now the motion of the water about the blades of a screw propeller at ordinary working speeds is certainly very turbulent indeed, so that we may safely assume that if μ, *i. e.*, $\rho DS/\mu$, occurs at all in equation (54), it is only in terms with very small exponents. It is therefore a legitimate approximation to omit it altogether and write the equation in the simpler form

$$F = \rho D^2 S^2 \varphi\left(\frac{Dg}{S^2}\right). \tag{55}$$

Since gravity is sensibly constant, we can now make two propellers dynamically similar, if they satisfy the preliminary conditions regarding shape, immersion, and slip ratio, by running them at speeds such that D/S^2 is constant. The condition for "corresponding speeds" is therefore

$$\frac{S'}{S} = \sqrt{\frac{D'}{D}},$$

accented letters referring to the model and unaccented to the original. When the two are run at corresponding speeds we therefore have, by equation (55),

$$\frac{F}{F'} = \frac{\rho}{\rho'}\left(\frac{D}{D'}\right)^3.$$

If the model is run in water of the same density as that in which the full-sized propeller is to run, $\rho = \rho'$ and we have

$$F = F'\left(\frac{D}{D'}\right)^2.$$

If a propeller is very deeply immersed so that no disturbance of the water surface is produced, the weight of the water can have no influence on the thrust and g can not appear in equation (55). The unknown function then degenerates into a mere constant and the equation reduces to

$$F = N\rho D^2S^2.$$

Any two propellers are then dynamically similar, whatever their speeds, if they have the same shape and are run at the same slip ratio, so that we have, for very deep immersion in a given liquid,

$$\frac{F}{F'} = \left(\frac{DS}{D'S'}\right)^2.$$

By disregarding viscosity we have, in effect, disregarded the effect of skin friction on the action of the propeller; and we have also left aside the question of cavitation. But without venturing further into the chaos of screw-propeller theory, the foregoing example will serve to illustrate the sort of use that may be made of dimensional reasoning in attacking mechanical problems which are—like most of those that occur in practical hydro- and aerodynamics—too difficult to be handled at all by ordinary methods.

11. *The Relation of the Law of Gravitation to Our Ordinary System of Mechanical Units.*—In our reasoning up to the present point, it has been assumed that three fundamental, *i. e.*, independent, units are required in an absolute system for measuring all the kinds of quantity needed in the description of purely mechanical phenomena, two more being required for thermal and electromagnetic quantities. If this assumption is permissible, a purely mechanical system may be kept similar to itself when any three independent kinds of mechanical quantity pertaining to it are varied in arbitrary ratios, by simultaneously changing the remaining kinds of quantity in ratios specified by equation (14), as described in section 6. We must now examine this assumption.

When we say that one quantity is derived from another or others which act as fundamental, we mean that by using or combining particular examples of these other kinds of quantitity in some specified manner, we can fix a quantity of the derived kind which has a particular definite magnitude. For instance, we derive a unit of force from independent fundamental units of mass, length, and time, by using these units in a certain way which is fixed by definition, and we thereby determine a definite force which is reproducible and may be used as a unit. Now by Newton's law of gravitation it is, in principle, possible to derive one of the three fundamental units of mechanics from the other two. Let two free masses be placed at rest at a distance apart which is very large compared with their linear dimensions. Let them be released and allowed to approach each other by a certain measured distance, and let the time required to cover this distance be observed. This interval of time is fixed by the masses and the distances: in other words, an interval of time can be derived from masses and lengths, and by adopting a suitable form of definition, a unit of time can be derived from the units of mass and length. It is, of course, immaterial which one of the three units is derived from the other two; the point is that if we utilize the law of gravitation, only two fundamental units are needed for mechanical quantities, instead of the three which physicists ordinarily use. By carrying out this process or some other equivalent to it, we should eliminate one of our three primary standards,—the international kilogram, the international meter, or the standard clock, namely the rotating earth which preserves the mean solar second. For practical purposes we should still use these three standards, but one of them would be reduced to the rank of a secondary or working standard.

One reason for not proceeding in this manner is that we do not yet know the value of the gravitation constant accurately enough to bring the proposition within the range of practicability. But since we must admit the theoretical possibility of such a procedure if we recognize the law of gravitation, it is incumbent upon us to consider what bearing this possibility may have on our dimensional reasoning and on our applications of the theorem of physical similarity; for the number of fundamental units needed is a matter of vital importance to our conclusions regarding any practical problem. For example; in treating the screw propeller, we assumed that $[m, l, t]$ were independent units and therefore that two propellers could be made to constitute dynamically similar systems when three quantities ρ, D, and S were varied in arbitrary ratios upon passing from one system to the other. The question now evidently presents itself: ought we not to have limited the arbitrary variations to two; are we not *bound* to treat mechanical quantities as derived from only two and not from three independent fundamental quantities?

To see the answer to this question, we may read over again the definition of physically similar systems given in section 6. It was found that a physical system remains similar to itself, as regards any relation among a number of kinds of quantity, when certain of these kinds—equal in number to the fundamental units required for the absolute measurement of all the quantities involved in the relation—are subject to variation in arbitrary ratios, if we fix the ratios in which the remaining kinds of quantity shall then change by imposing the condition that the Π's shall remain invariable. We now see that the answer to the question: how many fundamental mechanical units are to be used? *i. e.*, to the question whether we are or are not at liberty to ignore the law of gravitation, depends on the nature of the relation in question. If the relation with which we happen to be concerned refers to and characterizes some phenomenon which does not involve and is not affected by the form of the law of gravitation, we can carry out a complete investigation of the phenomenon and represent our results by a complete equation without ever knowing of the existence of the law of gravitation: this law does not concern us, and our knowledge of the phenomenon under investigation does not depend on our knowing the correct expression for the law of gravitation. We are therefore plainly at liberty to ignore it altogether, and if we do so, three fundamental units are indispensable because the only means of eliminating one of them is to use the law of gravitation. It is not necessary that the phenomenon be unaffected by the weight of material bodies, but merely that it be not sensibly dependent on the fact that weight is proportional to the mass of the Earth and to the inverse square of the distance from its center.

In the most general case, when we include within the field of our reasoning all kinds of physical quantity and *all possible relations* among them, we must admit our familiarity with the law of gravitation and limit ourselves to two fundamental mechanical units. But if for " all possible relations " we substitute " all relations that do not involve the law of gravitation," we may ignore the law and proceed as if it were non-existent.

With this single proviso all our foregoing reasoning retains its full validity. The limitation is seldom felt, because, in practice, physicists are seldom concerned with the law of gravitation: for all our ordinary physical phenomena occur subject to the attraction of an earth of constant mass and most of them occur under such circumstances that the variation of gravity with height is of no sensible importance. In precise geodesy and still more in astronomy, the observed phenomena do involve the operation of the law of gravitation in such a way that they can not be completely described without making explicit use of it. If the physical relations which characterize such phenomena are under discussion, we *must* recognize the law of gravitation, we *must* regard all mechanical units as derivable from two and not three independent fundamental units, and if a physical system is to remain similar to itself only four and not five arbitrary changes are possible, or if we exclude thermal and electromagnetic quantities, only two. The geodesist and the astronomer must therefore, in using dimensional reasoning, submit to one restriction from which the physicist is usually free, though this formal restriction is offset by the power of using the law of gravitation explicitly.

To take an illustration, let us suppose that we have to consider a phenomenon which involves mechanical and electromagnetic but not thermal quantities, and that the law of gravitation in its general form does not influence the phenomenon. The physical system in which this phenomenon occurs may remain similar to itself while four independent kinds of quantity Q are changed in any four arbitrary ratios, if all the other kinds P involved in the phenomenon are changed in the ratios specified by equation (14) taken with the arbitrary changes of the Q's. We may, for example, divide all lengths by x, divide all times by x, multiply all masses by x, and leave all electrical charges unchanged: the altered system will be similar to the original one as regards all phenomena that do not depend on the law of gravitation, if the remaining kinds of quantity are changed as shown by equation (14). But if the phenomenon involves the law of gravitation we can impose only three arbitrary ratios of change, of which one must refer to purely electromagnetic quantities: we can no longer impose arbitrary conditions on lengths, times, and masses but only on two of these kinds of magnitude. To put it in another way, and omitting electromagnetic quantities, which so far as we know have nothing to do with the case in hand, we may keep a gravitational system similar to itself while we change its size and its time intervals in any arbitrary ratios; but after the change, corresponding gravitational forces must stand in a determinate ratio which is not arbitrary. Or to make it less abstract, if we construct a miniature universe by multiplying all actual lengths by a, and if we change the densities in such a way that the mass of every volume element of the miniature universe is b times the mass of the corresponding volume element of the actual universe, then if the miniature universe is to be mechanically similar to the actual universe, the gravitational forces in the miniature universe must bear to the corresponding gravitational forces in the actual universe a ratio fixed by the law of gravitation. And if the speeds at which gravitational phenomena occur in the miniature universe are to have the same numerical values as corresponding speeds in the actual universe, the unit or time or speed can not be fixed arbitrarily but must have a particulaf relation to our actual unit.

Conclusion.—A convenient summary of the general consequences of the principle of dimensional homogeneity consists in the statement that any equation which describes completely a relation subsisting among a number of physical quantities of an equal or smaller number of different kinds, is reducible to the form

$$\psi(\Pi_1, \Pi_2, \cdots, \text{etc.}) = 0,$$

in which the Π's are all the independent dimensionless products of the form Q_1^x, Q_2^y, $\cdots$, etc. that can be made by using the symbols of all the quantities Q.

While this theorem appears rather noncommittal, it is in fact a powerful tool and comparable, in this regard, to the methods of thermodynamics or Lagrange's method of generalized coordinates. It is hoped that the few sample illustrations of its use which have been given will prove interesting to physicists who have not been in the habit of making much use of dimensional reasoning; but if this paper merely helps a little toward dispelling the metaphysical fog that seems to be engulfing us, it will have attained its object.

Bureau of Standards,
June 18, 1914.

THE VAPOR PRESSURE OF THE METALS PLATINUM AND MOLYBDENUM.

By Irving Langmuir and G. M. J. Mackay.

CONTINUING the line of investigation started by the determination of the vapor pressure of metallic tungsten,[1] the metals platinum and molybdenum have now been studied in a similar manner, and the results are given in this article.

The method involved is essentially the same as that described in the former paper, and consists in determining the loss of weight undergone by wires of the two metals maintained electrically at various temperatures for definite periods of time in glass vessels exhausted to an exceptionally good vacuum.

The calculation of the vapor pressure from the rate of loss of weight at a definite temperature is based upon the kinetic theory of gases and the Clausius-Clapeyron formula giving the relation between the vapor pressure of any substance and the temperature.

In the former paper the atomic heat of tungsten was assumed to be 6.8 at high temperatures. Recent work by Corbino and others, however, seems to indicate that the atomic heats of metals at temperatures as high as 2000° K., are considerably greater than would be expected according to the Dulong and Petit law. The most probable value for the atomic heats of platinum and molybdenum would therefore seem to be about 7.5 instead of the value 6.8 previously adopted. This gives, for the latent heat of evaporation of these metals,

$$\lambda = \lambda_0 - 2.5\,T. \tag{1}$$

Following the method of the previous paper, we thus obtain:

$$\log p = A - \frac{0.218\lambda_0}{T} - 1.26 \log T, \tag{2}$$

where A is a constant of integration, and

$$\log m = A' - \frac{0.218\lambda_0}{T} - 1.76 \log T, \tag{3}$$

where

$$A' = A - \tfrac{1}{2} \log M/2\pi R, \tag{3a}$$

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[1] Phys. Rev., N. S., II., 329 (1913).

p = vapor pressure in dynes per sq. cm.,
m = rate of evaporation in grams per square centimeter per second,
M = atomic weight of the metal,
λ_0 = latent heat of vaporization at 0° K. in calories per gram atom,
R = gas constant $8.32.10^7$ ergs per degree,
T = absolute temperature (Kelvin).

The assumptions involved in these derivations are:

1. The "accommodation coefficient" of the vapor of the metal is unity. That is, every atom which strikes the metal surface is condensed, no reflection taking place.

2. The volume of the metal is entirely negligible when compared to the volume of the vapor, so that the Clausius-Clapeyron formula reduces to

$$\frac{d\,lnp}{dT}=\frac{\lambda}{RT^2}.$$

3. The vapor of the metal is monatomic so that the specific heat of the vapor at constant pressure, is equal to $2.98 + R = 4.96$ calories per gram atom.

That the atoms of metal which evaporate from a hot wire are not reflected to any perceptible extent upon striking a surface, has been shown by many experiments in this laboratory. For example, if a single loop tungsten filament be heated in a highly exhausted bulb to such a temperature that the evaporation is fairly rapid, a dense black coating of the metal will be deposited uniformly over the glass. If, however, a screen, say of mica, be placed near the filament, that part of the bulb in the "shadow" of the obstruction will be perfectly clear with sharp boundaries. This shows that the atoms of tungsten travel in straight lines from the incandescent wire to the bulb, and that, at least from cold tungsten surfaces, the reflection of the tungsten atoms is negligible.

EXPERIMENTAL.

The molybdenum used was a sample of pure material made in this laboratory and consisted of wire 0.01549 cm. in diameter.

The platinum was obtained as wire 0.0254 cm. in diameter from Baker and Co., Newark, N. J. The temperature coefficient between 0°–100° C. was determined and found to be 0.0039 from 0° C. showing the material to be of the highest purity. Portions of this wire were drawn down to smaller sizes for the following experiments. The diameters ranging from 0.00572 to 0.0077 cm. were used.

The diameter of the molybdenum wire was measured carefully by a micrometer reading to 0.000025 cm.

The diameter of the platinum wire was calculated from the weight per centimeter using a density of 21.5. This is the more accurate way of obtaining the diameter, and would have been used with the molybdenum but the density of the latter is not so well known.

The general method of procedure was to mount a section of wire from 5 to 12 centimeters in length as a single loop on heavy nickel leading-in wires which was then sealed into a large straight sided lamp bulb about 9 cm. in diameter. The bulbs were carefully exhausted while heated to 360° C. by means of a mercury pump using liquid air to condense out water vapor and carbon dioxide. Before sealing off, the platinum or molybdenum filaments were heated electrically for about a minute to a bright red heat to drive off occluded gases and surface impurities. The bulbs were sealed off from the pump at a pressure of less than 0.001 mm. of mercury as read by a McLeod gage.

The wires were then set up to definite temperatures on the photometer bench, using the candle-power determination as the criterion of temperature as explained below, and maintained at constant temperature until the resistance had increased by from 5 to 10 per cent. The temperature was kept at the same value by maintaining the product of the voltage by the cube root of the amperes constant, this product being a function of the temperature only and independent of changing diameter.

After running in this way for a definite time, the bulb was opened, and the weight of the filament obtained and compared with the initial value. The final weight was the average weight per unit length found by weighing several pieces of the filament.

Weighings were made on a torsion balance reading directly to 0.01 milligrams.

The same formulas used in the paper on the vapor pressure of tungsten were used to calculate the rate of evaporation from the loss of weight and change in resistance. These are

$$m = \sqrt{\frac{\rho}{\pi}}\,\frac{\sqrt{w_0} - \sqrt{w}}{t} \tag{4}$$

and

$$m = \sqrt{\frac{\rho}{\pi}}\,\sqrt{w_0}\,\frac{1 - \sqrt{(R_0/R)}}{t}, \tag{5}$$

where w_0 = original weight of the wire per unit length,
w = weight per unit length after the time t,
ρ = density of the wire,
r = radius of the wire,
R_0 = initial resistance per unit length,
R = final resistance per unit length.

DETERMINATION OF TEMPERATURE.

The basis for the determination of the temperature of the wires was the same scale as used for tungsten in the preceding paper, modified by the ratio of the reflectivity of the metal in question to that of tungsten.

The temperature scale is based primarily on the following formula

$$T = \frac{11{,}230}{7.029 - \log H},$$

where H is the intrinsic brilliancy of the filament in international candle power per square centimeter of projected area. On this scale the melting point of tungsten is 3540° K.

In obtaining the candle power of the incandescent platinum and molybdenum, the wire was viewed through a calibrated slit, usually 2–4 cm. in width, which served the double purpose of cutting off light from that part of the filament cooled by the leads and of enabling an accurate determination of the length of the filament visible at the photometer head. Candle power per unit of projected area was thus readily obtained.

The emissivity of tungsten was taken as 0.514 and of molybdenum as 0.529[1], as compared to a black body. Hence in order to obtain the temperature from the curve of tungsten, the observed candle power for molybdenum was multiplied by the ratio 0.514/0.529.

The available published data on the emissivity for platinum varies so greatly, that a determination of the melting point was made as a check on the method. For this purpose three filaments of platinum, 0.0255 cm. in diameter, were carefully photometered, and quickly heated to the melting point and measurements of the volts, amperes and candle power were made at frequent intervals up to the burn-out point.

This method of obtaining the characteristics of a metal at its melting point has been found to be very reliable and reproducible in the case of tungsten, and the three measurements on platinum checked with each other within 5° C. The data obtained are as follows:

Bulb No.	Dia. Wire. Cm.	Length, Total. Cm.	Wire (Cm.), Visible.	Candle Power.	CP/ld
6,323–1	0.0255	19.0	10.50	4.46	16.6
6,323–2	0.0255	19.3	10.54	4.64	17.2
6,323–3	0.0255	18.6	10.56	4.54	16.9

Now the melting point of platinum is 1750–1755° C.[2] and assuming

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[1] H. v. Wartenberg, Ber. Dtsch. Phys. Ges. *12*, 105 (1910); W. W. Coblentz, Bull. Bureau of Standards, *7*, 198 (1911).

[2] Day & Sosman, Am. Journ. Sci., *29*, 161 (1910).

that we get within 5 degrees of the true melting point by the above method and taking the value 1750, we find the candle power per sq. centimeter at this temperature on the tungsten scale to be 30.2. The emissivity of platinum will then be equal to the product of the emissivity of tungsten 0.514 by the ratio of the candle powers 16.9/30.2, which is 0.288. This agrees quite closely with the value 0.278 determined by H. v. Wartenburg for block platinum.[1]

The temperatures of the three melting points as determined above are then 2022, 2027, and 2023 degrees Kelvin respectively if the value of 0.288 is taken for the emissivity.

RESULTS OF EXPERIMENTS.

The values of m, the rate of evaporation of the metal in grams per square centimeter per second, were calculated from the change in weight and also from the change in resistance by the formulas given above, and are tabulated in Table II.

TABLE II.

Rate of Evaporation of Platinum.

Identification Number.	Temp. of Wire °K.	$m \times 10^6$		$(1/T) \times 10^7$	Log m + 1.76 log T
		Weight.	Resistance.		
7,010–2	1,682	.00617	.0036	5,945	$\bar{3}.470$
6,916–3	1,710		.0052	5,848	
6,916–1	1,800	.0695	.049	5,555	$\bar{2}.572$
6,673–1	1,890	.373	.222	5,291	$\bar{1}.332$
7,010–3	1,912	.503	.363	5,230	$\bar{1}.482$
6,673–3	1,982	1.32	1.13	5,045	$\bar{1}.921$
6,673–5	2,000	1.88	1.35	5,000	.084

In order to determine whether m varied with the temperature according to the theoretical equation (3) the quantity $\log m + 1.76 \log T$ was plotted against $1/T$.

According to equation (3) the points should lie along a straight line. Reference to Figs. 1 and 2 will show that the agreement is excellent in both cases.

The values of m determined by change in resistance are in every case lower than those found from the change in weight. This is due to the fact that the elimination of impurities from the metal by volatilization and a slow sintering process tend to decrease the specific resistance and so counteract the increase in resistance due to evaporation. While affecting the resistivity of the metal these impurities are present in such small traces that they introduce no appreciable error in the determination

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[1] Ber. Dtsch. Physik. Ges., *12*, 105.

of loss of weight. Due to the low resistance of the heated wires, usually about 10 ohms, and the excessive cooling by the leading in wires, the values obtained from resistance measurements have been discarded in favor of these determined from change in weight. It will be noted however, that the quantities obtained by the two methods are not very different.

TABLE III.

Rate of Evaporation of Molybdenum.

Identification Number.	Temp. of Wire °K.	$m \times 10^8$		$(1/T) \times 10^7$.	Log m + 1.76 log T.
		Weight.	Resistance.		
6,975–8	1994	.00766	.00635	5,015	$\bar{3}.694$
6,975–3	2040	.0305	.0286	4,902	$\bar{2}.314$
6,911–3	2056		.081	4,864	
6,911–2	2112	.124	.118	4,735	$\bar{2}.953$
6,975–5	2121	.111	.104	4,715	$\bar{2}.905$
6,430–2	2220	.630	.560	4,505	$\bar{1}.689$
6,911–1	2287	1.74	1.56	4,372	$\bar{0}.150$
6,430–1	2312	3.29	2.70	4,325	$\bar{0}.447$
6,975–2	2326	3.65	3.24	4,299	$\bar{0}.492$
6,430–3	2350	6.49	5.14	4,255	.742
6.975–1	2373	8.47	7.07	4,214	.868

From the slopes of the lines in Figs. 1 and 2 we obtain for platinum

$$0.218\lambda_0 = 27{,}800,$$

whence

$$\lambda_0 = 127{,}600 \text{ grams calories per mol;}$$

and for molybdenum

$$0.218\lambda_0 = 38{,}600$$

whence

$$\lambda_0 = 177{,}000 \text{ gram calories per mol.}$$

From these values the heats of evaporation at any temperature are

Platinum.......................... 127,600–2.5 T.
Molybdenum...................... 177,000–2.5 T.

The values of A', in equation (3) may also be calculated from Figures 1 and 2, the result being

Platinum.............................. A 14.00
Molybdenum.......................... A 17.110

Equation (3) thus becomes

$$\log m = 14.00 - (27800/T) - 1.76 \log T \qquad \text{for platinum,} \qquad (6)$$

and

$$\log m = 17.11 - (38600/T) - 1.76 \log T \qquad \text{for molybdenum.} \qquad (7)$$

From (3*a*) we can now calculate A, equation (2) becoming

$$\log p = 14.09 - (27800/T) - 1.26 \log T \qquad \text{for platinum} \qquad (8)$$

and

$$\log p = 17.354 - (38600/T) - 1.26 \log T \qquad \text{for molybdenum.} \qquad (9)$$

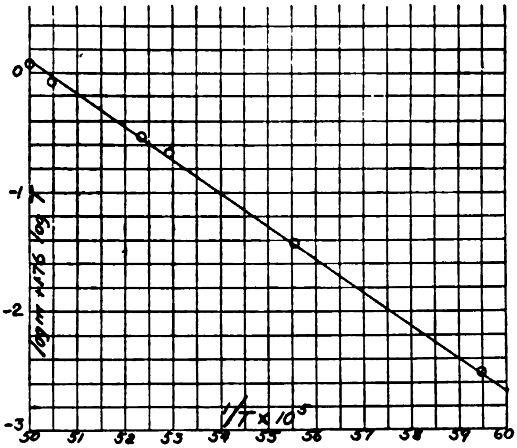

Fig. 1.

Rate of Evaporation of Platinum.

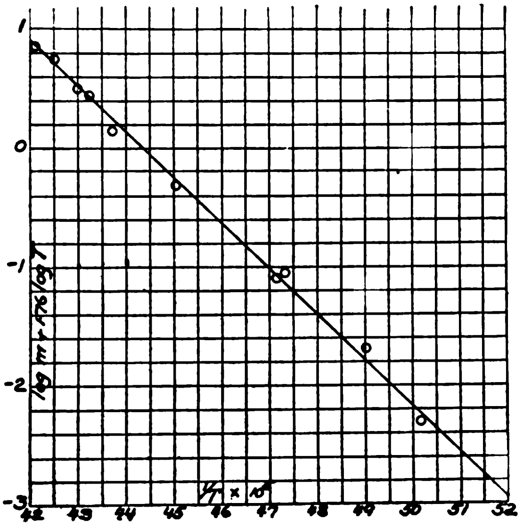

Fig. 2.

Rate of Evaporation of Molybdenum.

Since equations (6) and (7) represent the straight lines in Figs. 1 and 2, it is readily seen that this equation gives a very satisfactory method of calculating the rate of evaporation at any temperature.

Similarly, if it be assumed that the reflection of atoms of the metal is small, then equations (8) and (9) must give the vapor pressure with a fair degree of accuracy.

The rates of evaporation and vapor pressure at various temperatures as calculated from these equations is given in Tables IV. and V. Above the melting points, the vapor pressure would, of course, follow another curve. The boiling points calculated must therefore be looked upon as a lower limit. However, since the heat of fusion of metals is very small compared to their heats of vaporization the error is certainly not large, and it can therefore be safely concluded that the boiling points are in the neighborhood of the temperatures calculated.

TABLE IV.

Rate of Evaporation and Vapor Pressure of Platinum.

Absolute Temp.	Evaporation *g* per Sq. Cm. per Sec.	Vapor Pressure, Mm.
1000	8.32 10^{-20}	324.0 10^{-20}
1050	161 10^{-20}	6415 10^{-20}
1100	.238 10^{-16}	9.70 10^{-16}
1150	2.75 10^{-16}	115 10^{-16}
1200	26.0 10^{-16}	1110 10^{-16}
1250	204 10^{-16}	8910 10^{-16}
1300	.137 10^{-12}	6.11 10^{-12}
1350	.791 10^{-12}	36.2 10^{-12}
1400	4.01 10^{-12}	188 10^{-12}
1450	18.5 10^{-12}	867 10^{-12}
1500	75.4 10^{-12}	3610 10^{-12}
1550	.279 10^{-9}	13.7 10^{-9}
1600	.966 10^{-9}	48.4 10^{-9}
1650	3.11 10^{-9}	156 10^{-9}
1700	9.06 10^{-9}	465 10^{-9}
1750	25.4 10^{-9}	1300 10^{-9}
1800	66.7 10^{-9}	3500 10^{-9}
1850	168 10^{-9}	8800 10^{-9}
1900	.397 10^{-6}	21.3 10^{-6}
1950	.920 10^{-6}	49.5 10^{-6}
2000	1.95 10^{-6}	107 10^{-6}
2028	2.97 10^{-6}	164 10^{-6}
4180		760 mm.

CALCULATION OF THE VAPOR PRESSURE BY THE NERNST HEAT THEOREM.

According to Nernst's theorem, the vapor pressure of any substance is given approximately by the equation

$$\log p = -\frac{\lambda_0}{4.571T} + 1.75 \log T + C. \tag{10}$$

Here p is the pressure in atmospheres, λ_0 is the molecular heat of evaporation, and C is the chemical constant of the substance. Nernst finds for most substances that C has a value of about 3. If, then, we substitute $C = 3$, in the above equation, a roughly approximate relation between the vapor pressure and the heat of evaporation of a substance should be obtained.

Winterwitz[1] has shown that the previously published data on the vapor pressure of tungsten is in excellent agreement with such an equation.

It may be of interest, therefore, to compare the present data on platinum and molybdenum with the Nernst formula.

Table V.

Rate of Evaporation and Vapor Pressure of Molybdenum.

Absolute Temp.	Evaporation g per Sq. Cm. per Sec.	Vapor Pressure, Mm.
1800	0.0863 10^{-9}	6.43 10^{-9}
1850	0.318 10^{-9}	23.5 10^{-9}
1900	1.057 10^{-9}	80.8 10^{-9}
1950	3.72 10^{-9}	288 10^{-9}
2000	10.0 10^{-9}	789 10^{-9}
2050	0.0283 10^{-9}	2.24 10^{-6}
2100	0.0763 10^{-6}	6.12 10^{-9}
2150	0.196 10^{-6}	15.9 10^{-9}
2200	0.480 10^{-6}	39.6 10^{-6}
2250	1.13 10^{-6}	94.1 10^{-6}
2300	2.57 10^{-6}	214 10^{-6}
2350	5.64 10^{-6}	479 10^{-6}
2400	12.0 10^{-6}	1027 10^{-6}
2450	.246 10^{-4}	21.4 10^{-4}
2500	.490 10^{-4}	43.0 10^{-4}
2550	.959 10^{-4}	85.0 10^{-4}
2600	1.791 10^{-4}	160 10^{-4}
2650	3.30 10^{-4}	298 10^{-4}
2700	5.95 10^{-4}	542 10^{-4}
2750	10.50 10^{-4}	918 10^{-4}
2800	18.1 10^{-4}	1679 10^{-4}
3890		760 mm.

Let us calculate the value of C by (10) from the data on the vapor pressure of the three metals. Choosing a temperature in each case midway between the highest and lowest temperatures at which the rate of evaporation was measured, we obtain from the data of Table V. of the previous paper and from Tables IV. and V. of the present paper the following results:

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[1] Physik. Zeit., *15*, 397 (1914).

TABLE VI.

	T	p Mm.	p Atmos.	λ_0	C
W.........	2,700	6.9×10^{-6}	9.1×10^{-9}	218,000	3.5
Pt.........	1,850	8.8×10^{-6}	11.6×10^{-9}	128,000	1.5
Mo........	2,200	40×10^{-6}	53.6×10^{-9}	177,000	4.4

The chemical constant C in the last column was calculated from equation (10) by substituting in it the above values of T, p, and λ_0.

The value of C for platinum is unusually low, whereas that of molybdenum is considerably higher than the value 3.0 given by Nernst for the average of a large number of substances.

If, instead of calculating C, we assume it to be equal to 3.0 for each metal, and then choose a value of λ_0 which will give the best agreement with the experiments, we obtain

for W, $\lambda_0 = 210{,}000$,
Pt, $\lambda_0 = 136{,}000$,
Mo, $\lambda_0 = 162{,}000$,

instead of the values given in Table VI.

In view of the merely approximate form of the Nernst equation used, it is difficult to decide what weight should be given to the above results.

SUMMARY.

1. The rates of evaporation of heated platinum and molybdenum wires in high vacuum were determined over rather wide ranges of temperature.

2. From these data the vapor pressures of these metals were calculated. The results are given in equations (6) to (9) and in Tables II. to V. The latent heat of vaporization of these metals (in calories per gram atom) is found to be:

Pt, $128000 - 2.5\,T$,
Mo, $177000 - 2.5\,T$.

These results are found to be in moderately good agreement with an approximate equation derived by Nernst.

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THE SPECTRUM FROM MERCURY VAPOR IN AN ELECTRIC FIELD.

By C. D. Child.

IT has been shown that the vapor rising from a mercury arc is luminous and that the intensity of the light can be diminished by an electric field.[1] It has also been found that the relative intensity of the different lines in the spectrum of this light can be modified by an electric field. The following is an account of the latter phenomenon.

The apparatus used is shown in Fig. 1. *T* is a tube containing a mercury arc between the terminals *B* and *C*. *F* is a condensing chamber connected to the pump and drying chamber. The arc is in series with a dynamo *D* and a resistance *R*. In the tube *F* is a wire gauze filling the cross section of the tube and connected to the resistance *R* at a movable point, so that it can be given any desired potential.

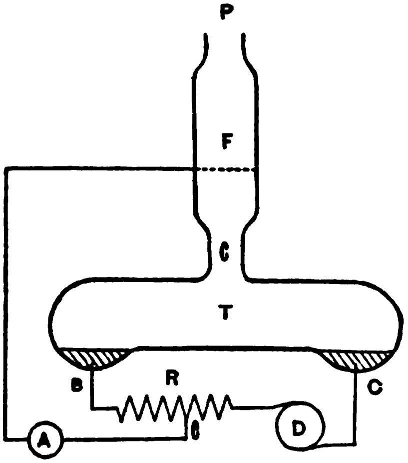

Fig. 1.

After the arc has been started luminous vapor passes from *T* to *F*. When this vapor begins to pass into *F*, its luminosity can be entirely destroyed by charging the gauze positively to a potential about 10 volts higher than that of the middle of the arc. The potential needed to produce this effect varies with the amount of vapor rising from the arc. When there is a large amount of vapor, it is impossible to destroy the luminosity entirely but it can be always diminished.

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[1] Phil. Mag. (6), 26, 906; 1913.

When the gauze is given a potential somewhat higher than 10 volts the luminosity is increased instead of being diminished. The color in that case changes from a reddish to a greenish tint. Photographs of these two colors did not show any lines different from those shown by the arc itself, but the relative intensity of the lines in the two spectra was somewhat different. This difference becomes more apparent, if the spectrum of the region above the arc is examined by the eye, and the gauze is charged so as to change the color while the spectrum is being examined.

The principal lines in the visible spectrum of the mercury arc are two yellow lines (5,790.5 and 5,769.5 microns) which according to Hagenbach and Konen[1] do not belong to any series; a bright green line (5,460.0) which belongs to the second sub-series of triplets; one in the greenish-blue (at about 5,000. I have not been able to identify it definitely), and two in the indigo (4,358.0 and 4,046.8) which belong to the same series as the green line.

When the region above the arc is changed from a red to a green tint, but with a field too weak to increase the intensity of the light appreciably, the yellow lines and the greenish blue are distinctly less bright, while the green and the others of that series are distinctly brighter. With a still stronger field all of the lines become brighter, but the change in the green and the violet is always greater than in the yellow. This phenomenon is quite possibly allied to one observed by Lenard,[2] who found that the arc in which there is some metal, as for example sodium, can be divided into three parts, the different parts showing the different series of the spectrum of the metal in different degrees of intensity.

It is indeed no new thing to have the relative intensity of the lines of a given spectrum vary under different conditions, but I believe that it is unusual to have so simple a change in the spectrum produced by so definite a change in the conditions. With no field we have the yellow lines strong; with a comparatively weak field we have no lines; with a stronger field we have the green lines intensified.

It is probable that ionization in the region above the arc begins when the light changes from a red to a green tint. Certainly this change is accompanied by an increase in conductivity. Thus in one case where much vapor was passing into F the color was distinctly red with a current of .04 ampere and a potential difference of 7 volts between F and the middle of the arc, and was distinctly green with a current of .09 ampere and a potential difference of 9.5 volts. The conductivity in the latter case was nearly twice what it was in the former.

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[1] Atlas of Emission Spectra, p. 21.

[2] Ann. d. Phys., 11, 636; 1903.

The color of the light from mercury vapor was also examined in the following cases. A discharge from an induction coil was passed through the tube shown in the figure after it had been heated by the arc so as to be filled with mercury vapor. The color of the discharge was distinctly greener than the discharge above the arc had been at any time. With capacity in the circuit the light was slightly greener than without it.

It was also found that the light from a mercury vapor lamp when the arc is started is much greener than it is when the current has been running for a few minutes. This is quite noticeable when one attempts to compare by means of a Lummer-Brodhun photometer the candle power of a newly started mercury arc with a similar arc which has been running for some time.

Explanation of the Change in the Spectrum.—We may sum up these facts as follows. When there is no electric field and no ionization by impact of the electrons, the yellow line is the stronger. When the electric force is comparatively small and the density of the gas is large, so that the mean free path of the electrons is small and the potential difference through which the electrons pass between one collision and another is small, the green is slightly intensified. When the mean free path is greater and the electric force is also increased, the green is still more intensified. The second of these conditions exist in the arc proper, especially after it has been running for some time. When the arc is first started the condition is somewhat more like the third case, the density of the gas being less and the color greener. Again in the region above the arc the density of the vapor is less than in the arc itself and the green is more intense when ionization occurs in that region. Finally in the discharge from an induction coil the density of the gas is least of all, the electric force is greatest, and the color is greener than in any other cases.

These facts are quite in harmony with the explanation of the different spectra of mercury which has been given by Horton,[1] although I am stating the explanation in a slightly different form from that given by him. According to this the light is caused by the recombination of positive ions with electrons. Some of the positive ions lack one electron and some two or more. The recombination of an electron with a positive ion that lacks two electrons, for example, gives rise to a different kind of vibration from that of an electron with a positive ion lacking only one electron.

It is but reasonable to suppose that the relative number of the positive ions having but one charge should be greater in the region above the

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[1] Phil. Mag. (6), 22, 214; 1910.

arc than in the arc itself. The ions having two charges would be apt to combine quickly with electrons, since their attractive force would be twice as great as that of positive ions having but one charge. Moreover the number of ions having but one charge would be augmented continually by the combination of ions having two charges with electrons.

If we assume that the yellow lines are given by the combination of positive ions having one charge with electrons, we may expect this to be relatively intense in the region above the arc. If the green light comes from the recombination of ions having two charges with electrons, this would be less intense, which is in accordance with the observations.

Again where the mean free path is through the greatest potential difference, there is the greatest chance for the ionizing electrons to knock off two electrons from the molecules leaving positive ions with a double charge and consequently for the green light to be the more prominent. This occurs when a discharge from an induction coil is passed through mercury vapor at a comparatively low pressure and to a somewhat less extent when ionization occurs in the region in the condensing chamber and also in the mercury lamp when it is first started.

Conclusion.—The relative intensity of the lines in the mercury spectrum may be changed by changing the potential difference covered by the mean free path of the electrons. The greater this potential difference the more prominent the green line as compared with the yellow ones.

This may be explained by assuming that certain lines are produced by the union of electrons with positive ions which lack one electron, while other lines are produced by the combination of electrons with positive ions lacking more than one electron.

COLGATE UNIVERSITY,
June, 1914.

THE APPROXIMATE DETERMINATION OF THE RESISTANCE OF AN IRREGULAR CONDUCTOR.

By J. F. H. Douglas.

In determining the resistance of an irregular-shaped conductor, one must at times be content with approximate values. Lord Rayleigh has developed a method[1] which enables one to obtain a superior and an inferior limit to the resistance. This consists of imagining thin insulating (or conducting) sheets embedded in the conductor which either cramp the flux causing it to follow parallel to these partititions, or else maintain artificial equipotential surfaces.

This principle, when applied graphically leads to a method developed by L. F. Richardson[2] for drawing a field of force or of current, and estimating its resistance. Since this connection apparently was not mentioned in the paper referred to, it will be pointed out here. If a field of force is divided into checkerwork and the cells in any row are combined first in parallel, and then the rows are connected in series, we are in effect imagining infinitely thin conducting sheets to be embedded in the conductor between the rows. As a consequence the resistance as computed is less than the actual value, unless by chance the surfaces were assumed correctly. On the other hand, if we combine the cells in any column first in series, and then the different columns in parallel, we are in effect imagining infinitely thin insulating partitions inserted into the conductor between the columns. As a consequence the resulting resistance is too high, unless lines of force were assumed correctly. The criterion of a correct choice of checkerwork in the paper referred to, was that each cell form a curvilinear square, in other words, that the resistance of each cell should be the same. If this is secured, it is obvious that, whether first combined in series or in parallel, the result must be the same. Thus the resistance so obtained must be equal to the true resistance of the conductor.

At present, when an exact analytical solution is not known, it is frequently assumed that the lines of force are arcs of circles, straight lines,[3] involutes,[4] or other such curves which remain parallel. In this way the

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[1] Theory of Sound, Vol. 2, Sect. 305, p. 175. See also V. Karpen, Comptes Rendus, Vol. 134, pp. 88–90.

[2] Phil. Mag., Series 6, Vol. 15, Feb., 1908, p. 237.

[3] Forbes, Jour. Soc. Teleg. Eng. and Elec., Vol. 15 (1886), p. 551.

[4] Pohl, Jour. Inst. Elec. Eng., Vol. 52, Jan. 1, 1914, p. 170.

conductance of each filament is easily calculated. However the field is so cramped by this rather arbitrary assumption, that is to be expected that the resulting resistance is much too high. It is to be shown here how to compute the resistance of an irregular, two-dimensional conductor, when divided by any assumed flux or equipotential lines (which can be expressed analytically).

In Fig. 1 let $y = F(m, x)$ be the equation of the assumed lines of force; m being a variable parameter. Let $y = F(m_1, x)$ and $y = F(m_2, x)$ be the equations of the extreme lines of force. Let also $y = \phi_1(x)$ and $y = \phi_2(x)$ be the equations of the extreme lines of equipotential. Then considering the small element ΔS, ΔL, of which the resistance is R_Δ, we have the following relations.

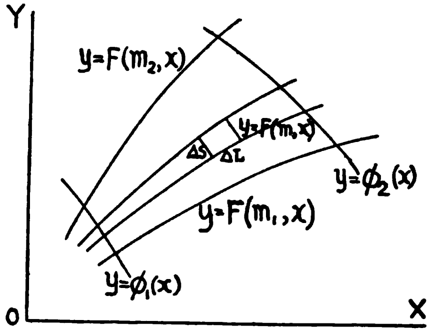

Fig. 1.

$$\Delta y = \frac{dF(m, x)}{dm}\Delta m + \frac{dF(m, x)}{dx}\Delta x. \qquad (1)$$

When (m) is constant we have,

$$\Delta y = \frac{dF(m, x)}{dx}\Delta x,$$

$$\Delta L = \sqrt{1 + \left(\frac{dF(m, x)}{dx}\right)^2}\,\Delta x. \qquad (2)$$

In order to evaluate ΔS we note that ΔS is perpendicular to ΔL and hence,

$$\frac{\Delta y}{\Delta x} = -1 \bigg/ \frac{dF(m, x)}{dx}$$

or

$$\Delta x = -\frac{dF(m, x)}{dx}\Delta y. \qquad (3)$$

Substituting for ΔX in Eq. (1) we have,

$$\Delta y = \frac{dF(m, x)}{dm}\Delta m - \left[\frac{dF(m, x)}{dx}\right]^2 \Delta y.$$

$$\Delta S = \sqrt{\Delta y^2 + \Delta x^2} = \sqrt{1 + \left(\frac{dF(m, x)}{dx}\right)^2}\,\Delta y.$$

Hence,

$$\Delta S = \frac{dF(m, x)}{dm}\Delta m \Bigg/ \sqrt{1 + \left[\frac{dF(m, x)}{dx}\right]^2} \tag{4}$$

Now, by definition,

$$R_\Delta = \rho\frac{\Delta L}{\Delta S} = \frac{1 + \left[\dfrac{dF(m, x)}{dx}\right]^2}{\dfrac{dF(m, x)}{dm}}\,\frac{\Delta x}{\Delta m}. \tag{5}$$

Integrating the resistances in series in one column, we obtain

$$\Sigma R_\Delta = \frac{\rho}{\Delta m}\int_{X_1}^{X_2} \frac{1 + \left[\dfrac{dF(m, x)}{dx}\right]^2}{\dfrac{dF(m, x)}{dm}}\,\frac{dx}{\Delta m}, \tag{6}$$

where X_1 and X_2 are obtained by solving simultaneously the equations $y = F(m, x)$ and $y = \phi_1(x)$ or $y = \phi_2(x)$. Combining now the various columns in parallel by integrating the X conductance with regard to m we obtain the total conductance G.

$$G = \frac{1}{R} = \int_{m_1}^{m_2} \frac{dm}{\rho(\Sigma R)_\Delta}. \tag{7}$$

The analytical process outlined above serves, with a few obvious modifications, for finding a lower limit to the resistance when the shape of the equipotential lines is assumed. This process of double integration is very apt to lead to complicated analytical forms. Thus in every case tried by the writer, the second integration had to be performed graphically. If a particular case only is to be studied, there is no objection in making both integrations graphical, though, if a curve of resistance is desired, the first integration should be analytically performed if possible.

As an example for the illustration of this method, we will take the leakage field of force between the poles of a dynamo-electric machine. This is a case where the present methods of calculation are very rough and need improvement. In Fig. 2 is shown a section of the field system of a multipolar machine. While the field of force is not two-dimensional, a solution on that basis will give us at least a lower limit to the magnetic conductance or permeance. It is hoped that the result will be nearer the truth than present values. In Fig. 2 the lines of force shown below the horizontal are as assumed at present for calculating the leakage permeance.[1] They consist of straight lines and involutes. Above the horizontal line is drawn another possible assumption of lines of force. In this case the assumption is that they are sine curves, of the form

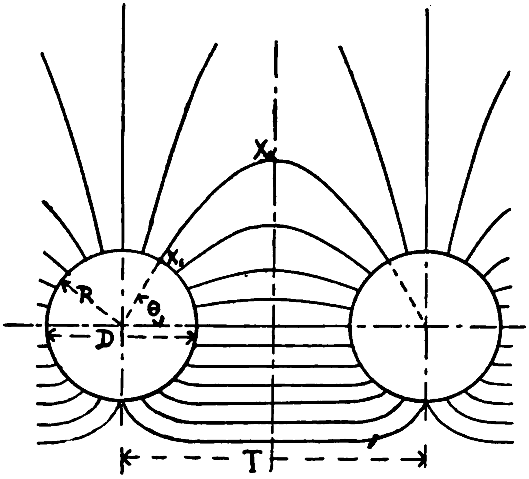

Fig. 2.

$$y = \frac{T}{\pi} \tan \theta \sin \pi \frac{x}{T}, \tag{8}$$

where θ is the parameter of the curve, and incidentally the angle subtended by the curve at the origin. Let $R = D/2$ be the radius of the poles, and T the pole pitch. The boundaries of the irregular conductor are determined by the conditions as follows. Limits of the parameter θ, $\theta_1 = m_1 = 0$; $\theta_2 = m_2 = 2\pi$. The superior limit of X is determined by the equations $y = \phi_2(x)$ and $y = F(m, x)$

$$X_2 = T/2.$$

The inferior limit of X is determined by the equations $y = \phi_1(x)$ which in this case is $y = \sqrt{R^2 - X^2}$ and equation (8); the result is,

$$\tan^2 \theta \sin^2 \frac{\pi X_1}{T} + \left(\frac{\pi X_1}{T}\right)^2 = \left(\frac{R\pi}{T}\right)^2. \tag{9}$$

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[1] Pohl, loc. cit.

Substituting now the various values into Eqs. (6) and (7) we obtain.

$$G = \int^{2\pi} \frac{d\theta}{\rho \displaystyle\int_{x_1}^{\pi/2} \frac{1 + \tan^2\theta \cos^2 \dfrac{\pi x}{T}}{\dfrac{\pi}{\pi} \sec^2\theta \sin \pi \dfrac{x}{T}} dx}$$

$$= \frac{4}{\rho} \int_0^{\pi/2} \frac{d\theta}{\text{Log} \cot \dfrac{\pi}{2} \dfrac{x_1}{T} - \sin^2\theta \sin \dfrac{\pi x_1}{T}}. \qquad (10)$$

Equation (10) was integrated graphically for various values of the ratio (R/T). The results are given in the table following, together with

TABLE I.

Values of Leakage Permanence from Circular Poles.

	Ratio (D/T).						
	0.10	0.20	0.30	0.40	0.50	0.60	0.70
Permeance, New Method	2.4	3.24	4.20	5.10	6.40	8.10	10.7
Pohl's Formula	0.6	1.30	2.20	3.30	4.70	6.30	9.1

the corresponding values given by Pohl's formula. The usual proportion of (D/T) in dynamo electric machinery is approximately 0.3, hence there is an error of at least 90 per cent. in present methods of computing permeance.

As a conclusion, we may say, that the method of computing resistance which is outlined here, is flexible and permits us to approximate to a given case flux distribution more closely than we have often been able to do previously.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

A NEW TURBIDIMETER.[1]

BY P. V. WELLS.

A SYSTEMATIC study of turbidity for the purpose of defining a proper standard is much needed. The paper describes an instrument in which the turbidity is measured by the light diffracted from the particles. A collimated beam from an intense source such as a Nernst or tungsten filament passes through a variable thickness of the turbid medium and is totally reflected by a prism, thence forming a uniform beam in one field of a photometer. The light diffracted by only a small angle is not totally reflected, but is refracted by the prism into the other aperture of the photometer. Thus the reading is a function of the ratio of the intensities of the light scattered and transmitted, which, in turn, varies with the turbidity.

The instrument is adapted to liquids, gases and solid plates. Minute traces are measureable with photometric precision, while the range is widened by varying the thickness of the medium. The characteristics of a preliminary instrument constructed at the Bureau of Standards are discussed.

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[1] Abstract of a paper presented at the Washington Meeting of the Physical Society April 24, 1914.

NEW BOOKS

Light, Radiation and Illumination. Translated from the German of **Paul Högner.** By **Justus Eck.** New York: D. Van Nostrand Company, 1913. Pp. ix + 88. Price, $2.50.

The best possible summary of the contents of the present volume may be quoted from its own preface.

"This book is intended to assist electrical engineers, architects and others in planning lighting installations, and deals especially with the problems arising in arc lighting. The first part of the book deals with the Radiation of Light from Surfaces and Bodies. Some examples are given showing how the spherical light radiation, the quantity of light as well as the mean lighting power, can be calculated from the form of the surface, and the brightness of the source.

"In the next part, dealing with Illumination, rules are given for lamp distribution, also worked-out tables for determining the average illumination, together with examples of the use of the same for the ordinary problems arising in practical work.

"Later, the problem of unidirectional or ordinary street lighting is dealt with in considerable detail; and, finally, new methods for the calculation of street and surface illumination."

E. F. N.

Einführung in die theoretische Physik in zwei Banden. Erster Band. By **Dr. Clemens Schaefer.** Leipzig: Veit & Comp., 1914. Pp. xii+925. Price, Mk. 20.

Fortunate are those students of mathematical physics of today who can have it boiled down into as thoroughgoing and clearly written a treatise as the present, by one of the most competent writers of the younger German school. Beginning with the fundamental notions of space and time, the subject is carried through both the kinematics and dynamics of points, rigid bodies, and continuous media, including the important questions of the vibrations of strings and membranes, rods and plates. The treatment is clear and up to date in every particular, as may be seen by the frequent introduction of integral equations in connection with problems of vibration. In the subject of hydrodynamics, the very latest researches on resistance of liquids by Kármán and others, are taken account of. If the next volume is as good as this, its appearance may be eagerly awaited.

A. G. W.

Neuebahnen der Physikalischen Erkenntnis. By DR. MAX PLANCK. Leipzig: Johann Ambrosius Barth, 1914. Pp. 1 + 28. Price, M. 1.

This pamphlet gives the text of a rectorial address given by Professor Planck at the opening of Berlin University in October, 1913. It contains an illuminating, non-technical discussion of the present state of our theoretical knowledge as affected by the relatively recent and brilliant discoveries in the field of experimental physics. Three hypotheses, upon which later facts and speculations have cast doubt, namely: the unchangeable character of the chemical atom, the mutual independence of space and time, and the continuity of all dynamic actions are given special consideration and emphasis.

E. F. N.

Propriétés Cinématiques Fondamentales des Vibrations. By A. GUILLET. Paris: Gauthier-Villars, 1913. Pp. 1+465. Price, 16 Fr.

This book is an excellent elementary treatment of the subject of vibrations, and would make an excellent source of exercises for college juniors and seniors. Not only the properties of vibrating points are taken up, but application to the kinematical theory of diffraction is treated, and the second part of the book is devoted to the treatment of waves in continuous media with applications to sound and light. The book is very clear and attractive in appearance, and although rather thick, considering its content, this will be an advantage rather than a disadvantage for the student. Our college students should be encouraged to read this kind of book.

A. G. W.

La Theorie du Rayonnement et Les Quanta. By MM. P. LANGEVIN and M. DE BROGLIE. Paris: Gauthier-Villars, 1912. Pp. 1+461.

This book is an absolute necessity to everyone who wishes to be up to date regarding those ideas which are producing a *bouleversement* in our physical notions. To review it is useless. All the authorities on the subject are represented, being gathered to the remarkable international scientific undertaking instituted by the generosity of M. Ernest Solvay at Brussels in 1911. One must regret the European war, if for no other reason than for its upsetting such a beneficent undertaking as the present.

Worthing ?

A. G. W.

Der Geist des Hellenentums in der modernen Physik. By ARTHUR ERICH HAAS. Leipzig: Veit and Comp., 1914. Pp. 1+32. Price Mk. 1, 20 pf.

This is Professor Haas' Antrittsvorlesung upon taking the chair of the history of physics in the University of Leipzig. It is chiefly a critical, though highly appreciative review of the fundamental physical conceptions underlying the writings of Thales, Pythagoras, Empedocles, Anaxagoras and Democritus. To these philosophers the author credits the creation of the fundamental conceptions and ideas upon which modern physics is built, namely, the ideas of the unity of nature, the constancy of matter, the existence of chemical elements, of physical forces, of an ether, and of atoms. He credits the Greeks also with having first believed in a world governed completely by law, and with having first conceived the plan of objectivizing physics and reducing it to mechanics. A physics then whose aim is not merely to describe but to rationally *explain* nature the author regards as a Greek discovery.

R. A. M.

An Elementary Treatise on Calculus. By WILLIAM S. FRANKLIN, BARRY MACNUTT and ROLLIN L. CHARLES. South Bethlehem, Pa.: Published by the Authors, 1913. Pp. ix+253. Price $2.00.

A very instructive and suggestive book. The shackles of conventionalism rest lightly, if at all, and throughout the treatment is original and vivid. In the introductory note calculus is defined as a mathematical method of thinking, and the contrast between the method of differential calculus and the method of integral calculus is at once presented clearly and definitely. This definition and the contrast are emphasized continually in the text. The fundamental principles of both differential and integral calculus are developed in the first chapter. In subsequent chapters these fundamental principles and the special formulæ for differentiation and integration which may be deduced by means of them are applied to the solution of typical problems in physics and elementary electrical engineering. In this way chapters on ordinary differential equations, the partial differential equations of wave motion, and vector analysis are introduced naturally. The text contains numerous practical problems worked out in detail in connection with the development of the theory, the same problem frequently serving as the vehicle for more than one idea, and other problems of a similar character to be worked out by the student. An interesting analogy is drawn between the motion of a ship when starting and stopping and the building up and falling off of an electric current. Purely formal problems are relegated to an appendix. Other commendable features are the early introduction of the concepts of the definite integral and of infinitesimals of the second and higher orders, the comparison between differentiation by development and differentiation by rule, the clear exposition of MacLaurin's and Demoivre's Theorems, and the inclusion of Fourier's Theorem. Not the least valuable feature of the book is the frequent reference to standard works throughout the text and the annotated list of these works in appendix C. The book should appeal to the interest of students of physics not only because it gives the method for solving the particular problems included in the text but also, and especially, because it presents calculus—the mathematical method of thinking—as a powerful instrument for the solution of problems which cannot be solved by observation alone.

WORCESTER, MASS.

A. ST. J.

Electromagnetic Radiation. By G. A. Schott. Cambridge: The University Press, 1912. Pp. xxii+330.

Although this book appeared two years ago, it is far too important not to receive mention, being not only an Adams Prize essay in the University of Cambridge, but also almost the first extended treatment in English of the newer aspects of Lorentz's electromagnetic theory. The book is very mathematical, and is not easy to read. The fundamental equations of the electron theory are plainly stated, and their solutions given as definite integrals convenient for treatment. Many illustrative examples of the motion of single electrons, as well of groups, are given, and their mathematical consequences worked out. The question of the various kinds of stiff or deformable electrons and their mechanical consequences is thoroughly treated. This book will well repay careful perusal, if one possesses the by no means small mathematical skill required.

A. G. W.

Electromagnetic Theory. Vol. III. By Oliver Heaviside. New York: D. Van Nostrand Company, 1912. Pp. ix+519.

The preceding review (Schott's Electromagnetic Radiation) will do almost verbatim for the present third volume of Heaviside's remarkable work, although the methods of the two books are very dissimilar. All the remarkable originalities, both mathematical and conversational, of this brilliant genius are here to be seen in full force. It is a great pity that the methods of Heaviside have not received the attention they deserve from the professional mathematician. In the words of the hymn, we stand " lost in wonder, love and praise."

A. G. W.

Second Series. *November, 1914* *Vol. IV., No. 5.*

THE

PHYSICAL REVIEW.

ON THE ABSORPTION OF HYDROGEN BY SODIUM-POTASSIUM ELECTRODES.

By R. C. Gowdy.

MEY[1] has shown that the liquid alloy of sodium and potassium, when used as a cathode for the glow discharge, absorbs all except the inert gases. He found that with hydrogen and nitrogen the hydrides and nitrides of the metals were formed and that the rate of absorption was proportional to the current.

Skinner[2] and Cunningham[3] have experimented on the evolution of gas from various electrodes and have found that most metals liberate gas for a certain period at a rate equal to that defined by Faraday's law for electrolytic decomposition. Chrisler[4] has observed the electrolytic rate for the absorption of hydrogen by various anodes, among them, anodes composed of sodium-potassium alloy.

This alloy then presents the anomaly that it may absorb hydrogen when operated as cathode or as anode.

The object of the present experiments was to investigate the behavior of sodium-potassium electrodes in glow discharge in hydrogen.

Apparatus and Experimental Method.

Discharge tubes of the form shown in Fig. 1 were used.

The alloy was introduced into the bulb *B*, which was afterwards sealed off at *F*.

After evacuating the apparatus and washing out with dry hydrogen, the tube was tilted and the alloy allowed to run into the discharge tube. A trap, *D*, prevented specks of oxide from being carried over. A clean

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[1] Mey, Annalen der Physik, 11, 1903.

[2] Skinner, Phys. Rev., XXI., p. 1 and p. 169. Phys. Zeit., 6, p. 610. Phil. Mag. (6), Vol. 12, p. 481.

[3] Cunningham, Phil. Mag. (6), IX., 1905, p. 193.

[4] Chrisler, Phys. Rev., XXIX., Nov., 1909.

surface could thus be obtained. A large glass spiral, later replaced by a mercury-sealed ground joint, provided the necessary flexible connection between the discharge tube and the pump and gauge. The auxiliary electrode, *E*, was used when a preliminary cleansing discharge was sent through the tube to free the upper electrode of gas.

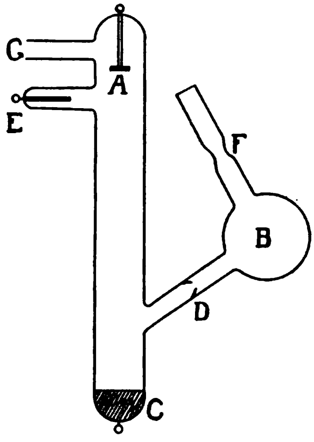

Fig. 1.

The absorption was determined by observing the change in pressure as indicated by a McLeod gauge. The apparatus was calibrated by admitting a known volume of gas and noting the change in gauge reading.

The rates of absorption were determined by plotting curves with pressures as ordinates and total duration of discharge as abscissæ, and determining the slopes of these curves at the required points.

The hydrogen used was produced by electrolysis of barium hydroxide and was carefully dried with sulphuric acid and phosphorus pentoxide.

CATHODE ABSORPTION.

Tables I. and II. give the initial rates of absorption of hydrogen by K-Na cathodes for various current strengths. The results in Table I. are for a cathode 30 mm. in diameter, those in Table II. for one of 16 mm. diameter.

TABLE I.

Diam. of Tube 30 Mm.	
Current in milliamps.	Rate of absorption in mm.³ per Sec.
0.20	0.141
0.40	0.323
0.60	0.541
0.93	0.885
1.00	0.973
1.20	1.14
1.48	1.44
1.60	1.64
1.91	2.12
2.20	2.40
2.30	2.73
2.50	2.88
2.80	2.84
3.20	2.82
4.20	2.86
4.70	2.82

TABLE II.

Diam. of Tube 16 Mm.	
Current in milliamps.	Rate of absorption in mm.³ per Sec.
0.20	0.21
0.44	0.33
0.48	0.38
0.64	0.45
0.70	0.68
1.10	0.59
2.00	0.50

These rates were determined by finding the slope of the pressure-time curve at the point where it meets the pressure axis and therefore represent the initial rates of absorption. When the surface is clean the rate of absorption is a maximum, but decreases rapidly as the film of hydride forms on the surface. The rate of absorption by a clean surface can not be found directly from the observations but must be determined by graphic extrapolation from the curves.

The film of hydride was decomposed and the surface cleaned before each set of observations, by running the alloy as anode at a pressure of about 0.03 mm.

The curves in Fig. 2 show graphically the results given in Tables I. and II.

The curves show that the rate of absorption increases a little more rapidly than the current up to a certain current strength, above which the rate of absorption is constant.

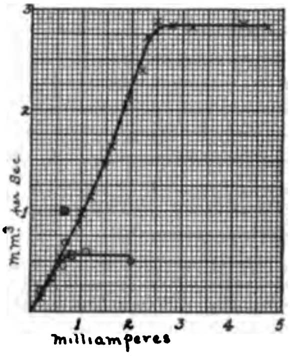

Fig. 2.

The total areas of the electrodes were in the proportion of about 3½ : 1, but the edge of the alloy for about 2 mm. in from the sides of the tube is practically inactive, as shown by the fact the surface shows little or no discoloration in this region. The active electrode areas were in the ratio of about 4.7 : 1. This is very nearly the ratio of the current strengths at which the rates of absorption become constant, which indicates that this takes place at a certain current density,—about 0.5 milliamps. per cm.^{2}

The rate of absorption below this current density is nearly ten times the electrolytic rate.

Two experiments were also tried to determine the effect of the temperature of the cathode on the rate of absorption.

The lower part of the tube containing the alloy was immersed in oil and heated to 140° C. A current of 0.66 milliamps. was used and the rate of absorption was found to be 1.0 mm.^{3} per sec. This is shown on the curve in Fig. 2 by a point marked with a square, and is much greater than the rate at room temperature for the same current. The cathode was frozen with solid carbon dioxide and probably reached a temperature of at least —20° C. The rate of absorption at 0.8 milliamps. was found to be 0.58 mm.^{3} per sec. This is also shown on the curve in Fig. 2 and lies considerably below the curve taken at room temperature.

ANODE EVOLUTION.

At pressures of one to two millimeters the K-Na anode absorbs hydrogen and at low pressures evolves it. There must be some equilibrium pressure at which it neither evolves nor absorbs. A number of experiments were made to determine this pressure, but while it is sharply defined in any one experiment, it varies from time to time, being dependent on the history of the tube,—primarily on the quantity of hydride available for decomposition: the more hydride present the higher will be the equilibrium pressure. It was found generally to lie below 0.160 mm., this pressure being obtained with a thick coating of hydride. A thin, almost imperceptible coating gave an equilibrium pressure of about 0.06 mm. By continued discharge the supply of hydride may be so far diminished that the equilibrium pressure may be reduced to one or two hundredths of a millimeter. It could probably be reduced still further were it not for the fact that the tube, when operated with an induction coil, tends to reverse at low pressures, becoming in fact, a more efficient rectifier than the " valve " tube with which it was connected.

Until a method was found to account for the quantity of hydride present, measurements of the relative variation of the equilibrium pressure with the current could not be made.

It seemed probable that the decomposition of the hydride was due to the impact of the cathode rays. To test this hypothesis an electromagnet was so placed that a field could be made to pass transversely through the tube a few centimeters above the surface of the alloy. Equilibrium pressure was established at 0.145 mm. with the magnetic field off. On applying the field, the discharge current being kept constant, the pressure in the tube decreased, finally reaching a steady value at 0.053 mm. This could not have been due to any effect arising from the deflection of the rays against the side of the tube, for when the field was reversed, projecting the rays against the opposite wall, the presure remaining unaltered. On removing the magnetic field the pressure rose to its original value.

That the decomposition of the hydride is due to the impact of the cathode rays, is indicated by the following experiment. A thin film of hydride was formed on the surface of the alloy, which was then insulated. A discharge was then passed from the auxiliary electrode, *E*, as anode, to the electrode *A* as cathode. A strong magnetic field traversed the tube below *E*, preventing the cathode rays from reaching the alloy. The pressure was so low that the tube glowed with a bright green fluorescence. The discharge was passed for five minutes with no change in the appearance of the tube. On removing the field and allowing the cathode rays to strike the alloy, the pressure rose rapidly, the green fluorescence disappearing in a few seconds. A second application of the field produced no further change in pressure.

The gas liberated by the anode was repeatedly tested spectroscopically; only hydrogen was found.

We have now a means of arriving at some more definite results con-concerning the equilibrium pressure. By applying the magnetic field during the discharge the pressure may be reduced below the normal equilibrium point without removing hydrogen from the tube, and experiments on the rate of evolution and the equilibrium pressure repeated as often as desired with the assurance that at a given pressure there is always the same quantity of hydride pressent.

An induction coil was used to supply the current, since at low pressures sufficient potential could not be obtained from the batteries. It was doubtful whether the shunted galvanometer, calibrated on constant current, gave a proper indication of the average current from the coil. A small silver voltmeter was placed in series with the tube and a current of one milliampere, as indicated by the galvanometer, was passed for one hour. The deposit of silver weighed 4 mg., indicating a current of 0.99 millimaps. and showing that the average current was registered with sufficient accuracy by the galvanometer.

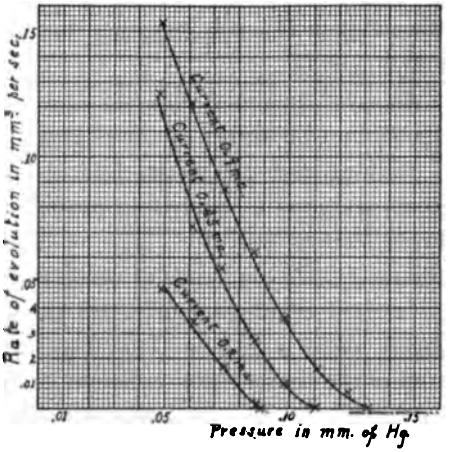

Fig. 3.

TABLE III.

Anode Evolution. Diam. of tube 30 mm.

Current = 0.9 m. amp.		Current = 0.45 m. amp.		Current = 0.20 m. amp.	
Pressure in Mm. of Hg.	**Rate of Evolution in Mm.³ per Sec.**	**Pressure in Mm. of Hg.**	**Rate of Evolution in Mm.³ per Sec.**	**Pressure in Mm. of Hg.**	**Rate of Evolution in Mm.³ per Sec.**
0.049	0.153	0.048	0.085	0.049	0.048
0.061	0.121	0.061	0.072	0.062	0.035
0.074	0.087	0.073	0.056	0.074	0.017
0.086	0.062	0.086	0.027	0.087	0.002
0.098	0.036	0.097	0.010	0.088	0.000
0.111	0.016	0.110	0.000		
0.123	0.006				
0.131	0.000				

The rates of evolution at various pressures for given current strengths are given in Table III., and shown graphically in Fig. 3.

The equilibrium pressure and the rate of evolution at any given pressure both increase with the current.

DISCUSSION.

Direct electrical action is insufficient to explain the high rate of cathode absorption observed. We have seen that this rate is considerably affected by the temperature in a sense indicating that the absorption may be largely due to a heating effect.

We have observed that K-Na anodes absorb hydrogen at high pressures, evolve hydrogen at low pressures, if the hydrides are present on the surface, and that, under this condition also, there is an equilibrium pressure at which neither absorption nor evolution takes place. We have also shown that the hydrides are decomposed by the direct impact of the cathode rays.

Apparently these phenomena are due to the rise in temperature produced by the impact of the charged particles.

Sodium and potassium hydrides are both formed at a temperature of about 350° C. and it is probable that in both cases decomposition begins below 400° C.[1] The combination therefore takes place within relatively narrow limits of temperature.

In the foregoing experiments the average temperature of the alloy was rarely above 40° C. and it is therefore unlikely that the average temperature of the surface was sufficient to form the hydrides. The temperature at the point struck by a charged particle of relatively small velocity may, however, be very high and the average temperature of the neighboring molecules might easily be of the order of three or four hundred degrees. Dunoyer[2] has suggested that this local heating effect may be sufficient to expel electrons from the electrode and even to vaporize the metal. In fact a simple calculation shows that a charged particle falling through ten volts possesses sufficient energy to raise ten atoms of sodium through 400° C. of temperature. We may therefore assume that at the point of impact the temperature is usually above the decomposition temperature and that only in a given zone surrounding this center the conditions exist favorable to the formation of the hydride. Thus at a given instant there is a given fraction of the surface of the electrode active is forming the hydride. When the charged particles fall so close together that the zones of favorable temperature begin to overlap, the active fraction of the surface will not be increased by further increase in the density of the

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[1] Moissan, Compt. Rend., 134, p. 18 and p. 71, 1902.

[2] Dunoyer, "Sur la formation des rayons cathodiques," Le Radium, VII., Oct., 1910.

impinging particles. This may be the explanation of the maximum rate of absorption attained at a current density of 0.5 milliamps. per cm.^{2} On this theory we should expect it to be possible to increase the area where dissociation is being produced until it would exceed the area where the hydride is being formed, *i. e.*, with a sufficiently strong discharge, the cathode, if coated with hydride, could be made to liberate hydrogen. This fact has been observed by Chrisler[1] who used a strong discharge in helium as a means of cleansing sodium cathodes, in the same manner that the writer used the anode discharge in hydrogen at low pressure to produce the dissociation of the hydrides. In the case mentioned the decomposition taking place at the hot center is in no way affected by the presence of the helium, while the chances of combination taking place in the zones where the temperature is favorable are greatly diminished, owing to the presence of the inactive gas; a resultant evolution of hydrogen therefore takes place.

The behavior of the K-Na anode is readily explained in the same manner. In fact, from the point of view of heating effects, there is little difference between the two cases, since the energies of the positive and negative particles are comparable in magnitude. It is necessary however to remark on certain differences in the conditions in the two sets of experiments:

1. Much higher potentials were used in the experiments on anode evolution,—due both to the lower pressure and to the valve action of the tube. There was therefore greater heating effect at the surface of the metal.

2. There was much less hydrogen in the neighborhood of the alloy when it was operated as anode,—due to the reduced pressure and to the fact that the hydrogen tends to accumulate near the cathode.

The existence of an equilibrium pressure may be explained as follows; Suppose that at a given pressure and current density, combination and decomposition are taking place at the same rate, due to an effective equality between the portions of the surface that are at a temperature favorable to the formation of the hydride and those that are too hot. If the pressure be raised the velocity of the particles is reduced and the heating effects lessened. There will therefore be a decrease in decomposition and a resultant absorption will take place, reducing the pressure until equilibrium is again established. If the pressure be reduced, the velocity of the particles is increased, the decomposition overbalances the absorption and the pressure is again raised to an equilibrium value.

The equilibrium pressure must also depend on the quantity of hydride

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[1] Chrisler, loc. cit.

present. The less hydride, the less chance there is for decomposition and the more for combination. To bring about a balance between the two will require an increased area of high temperature, necessitating higher velocity of the impinging particles and consequently, reduced pressure.

The rise of equilibrium pressure with the current may be explained very simply if it be remembered that at equilibrium the areas of formation and decomposition must be effectively equal. If by increasing the current we increase the number of centers of dissociation there will be a resultant liberation of hydrogen which will continue until the increase in pressure diminishes the velocity of the particles sufficiently for equality to be again established.

The curve, Fig. 2, shows that the cathode absorption increases more rapidly than the current. This is what would be expected considering that, under the conditions of the experiment, an increase in current meant an increase both in the number of particles and in their velocity.

SUMMARY.

1. The rate of absorption of hydrogen by K-Na has been measured for various currents. At current densities less than 0.5 milliamps. per cm.^{2} the rate of absorption is approximately ten times that at which hydrogen is liberated by electrolysis with an equal current. With greater current densities the rate of absorption is constant. The rate of absorption increases with the temperature of the cathode.

2. An " equilibrium " pressure has been found at which K-Na anodes neither absorb nor evolve gas. This equilibrium pressure depends on the current and on the quantity of hydride on the surface of the alloy.

3. The rate of evolution by anodes below equilibrium pressure has been measured.

4. It has been shown that sodium and potassium hydrides are decomposed by the cathode rays.

The cathode absorption and anode evolution are regarded as being due to the formation or decomposition of the hydrides by the local heating effects of the impinging charged particles, according as the temperature is favorable to one or the other of these processes.

These experiments were carried on in part at the Cavendish Laboratory and the writer wishes in conclusion to tender his sincere thanks and appreciation to Professor Sir J. J. Thomson for his interest and helpful suggestions during the course of the work, and for his kindness in putting the facilities of the laboratory at the writer's disposal.

UNIVERSITY OF CINCINNATI,
DEPARTMENT OF PHYSICS,
July, 1914.

THE RELATION BETWEEN ALPHA-RAY ACTIVITIES AND RANGES IN THE ACTINIUM SERIES, WITH NOTES ON THE PERIOD AND RANGE OF RADIOACTINIUM

By Herbert N. McCoy and Edwin D. Leman.

It now seems well established that the number of ions, and therefore the ionization current in air due to a single particle of range R is very closely proportional to $R^{2/3}$.[1] In a recent paper,[2] it has been shown that the activity of each α-ray product of the thorium series is proportional to the 2/3 power of its range, account being taken of the fact that ThC is complex. The activity of ThC found was in excellent agreement with that calculated for a multiple disintegration in which C_1 furnished 65 per cent. and C_2 35 per cent. as many α particles per second as the equilibrium amount of each of the other products of the thorium series, these being the percentages found in other ways by Marsden and Barratt[3] and Barratt.[4] The only discrepancy which appeared in the course of the work on thorium was traced to a large error in the accepted value for the range of ThX. But after the true range of ThX had been found, there was very good agreement between the observed activities and those calculated from the ranges of the various products. This led to the conclusion that there were no other α-ray products in that series than those already known; and that in each change, excepting that of the ThC, a single α particle is expelled from each exploding atom, while in the case of ThC the conclusions of Marsden and Barratt were fully confirmed.

In the present paper we wish to report the results of an analogous study of the actinium series, for which Table I. gives the latest available data.

We have prepared radioactinium (Rn) free from actinium itself as well as from actinium X, and subsequent products and measured its increase of alpha-ray activity with time, due to formation of AcX and its products. If f_1 is the fraction of the original Rn left after an interval of t days and f_2 is the fraction for AcX, unit amount being the equilibrium

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[1] Geiger, Proc. Roy. Soc., *A*, *82*, 486 (1909). Taylor, Phil. Mag., *21*, 371 (1911). McCoy, Phys. Rev., *1*, 393 (1913).

[2] McCoy and Viol, Phil. Mag., *25*, 333 (1913).

[3] Proc. Lond. Phys. Soc., *24*, 50 (1912).

[4] Le Rad., *9*, 81 (1912).

quantity corresponding to the initial amount of Rn; then the activity, A, at time t, resulting from unit initial quantity of Rn is given by the equation:

$$A = f_1 + f_2 x, \tag{1}$$

where x is the alpha-ray activity of the equilibrium quantity of AcX plus products for unit quantity of Rn. Strictly speaking, for intervals less than 3 or 4 days one should take into account the fact that the products of AcX are not in equilibrium with the latter; practically, however, for longer intervals, the simple equation (1) is entirely sufficient. If λ_1 and λ_2 are the decay constants of Rn and AcX respectively, then $f_1 = e^{-\lambda_1 t}$ and

$$f_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}).$$

The periods being 18.8 days and 11.35 days respectively, $\lambda_1 = 0.0369$ and $\lambda_2 = 0.0611$.

TABLE I.

The Actinium Series.

	Symbols.	Periods.	Rays.	Ranges, Cm.
Actinium	Ac	?	—	—
Radioactinium	Rn	18.88 days[1]	$\alpha+\beta$	4.40[3]
Actinium X	AcX	11.35 days[2]	α	4.40
Emanation	Em	3.9 sec.	α	5.70
Actinium *A*	*A*	0.002 sec.	α	6.50
Actinium *B*	*B*	36 min.	β	—
Actinium *C*	*C*	2.1 min.	α	5.40
Actinium *D*	*D*	4.71 min.	$\beta+\gamma$	—

The actinium used in this work had been found to be free from all but negligible traces of radium and thorium and their active products.[4] It contained, however, an appreciable amount of ionium. To the solution of the actinium bearing material in dilute hydrochloric acid we added a few mg. of thorium nitrate and then precipitated the latter with hydrogen peroxide. The filtrate contained all the actinium and was apparently free from Rn and Io; but to be certain that no trace of the latter remained

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[1] This paper.

[2] McCoy and Leman, Phys. Zeit., *14*, 1280 (1913). Hahn and Rothenbach found 11.6 days; Phys. Zeit., *14*, 409 (1913). The previously accepted periods of radioactinium and actinium X were 19.5 days and 10.2 days respectively.

[3] The range of radioactinium found by Geiger was 4.60 cm.; the value given above is that which we have found, as described in this paper.

[4] McCoy and Leman, loc. cit.

in the Ac, the treatment with Th and hydrogen peroxide was repeated three times. To the purified actinium solution 1 c.c. of 5 per cent. aluminium chloride solution was added and ammonia gas (free from carbon dioxide) passed in. The aluminium hydroxide was filtered out, dissolved in hydrochloric acid and precipitated with ammonia once more. The filtrate from the first aluminium precipitate contained two thirds of the AcX; that from the second the larger part of the balance; while 6 per cent. of the AcX remained in the third aluminium precipitate which contained practically all of the Ac. The precipitations with ammonia had freed the Ac from the minute amount of mesothorium introduced with the few milligrams of Th used. This Ac solution stood for several months during which time a large amount of Rn formed. The Rn was then separated from this solution, which had a volume of 5 c.c. by adding 3 drops of a 2 per cent. solution of thorium nitrate, which had just been freed from mesothorium, and precipitating the Th by hydrogen peroxide. The precipitate was filtered out, dissolved in dilute hydrochloric acid and potassium iodide, and precipitated by hydrogen peroxide four times more. The 10 c.c. of acid solution of the last Th precipitate which contained all the Rn but was entirely free from AcX and its products was treated with oxalic acid. The precipitate of 3 or 4 mg. of thorium oxalate contained all of the Rn. It was separated from the solution by a centrifuge, and washed thoroughly with water and then with alcohol. A small portion was then spread on a metal plate in an exceedingly thin film, the activity measurements of which are given in Table II.

TABLE II.

The Alpha-Ray Activity Curve of Radioactinium Initially Free from its Products. Film No. 4.

Interval in Days.	Activity.	x
0.000	1.000	
5.101	1.933	4.54
6.025	2.045	4.53
15.76	2.597	4.55
16.08	2.620	4.59
17.07	2.623	4.59
17.80	2.629	4.60
20.05	2.593	4.56
23.10	2.527	4.54
27.09	2.397	4.54
31.10	2.281	4.63
36.04	2.053	4.60
39.97	1.881	4.61
206.0	.007	4.573 mean

In the above table x is the activity of AcX + Em + A + B + C + D in terms of the activity of the equilibrium amount of Rn as unity.

The activity at the end of 206 days (Table II.) was found to be 0.007; of this 0.001 was calculated to be due to remaining Rn and products; the balance, 0.006, represents the constant activity of the minute amount of thorium present. The activities given in Table II. have been corrected for this constant thorium activity. The activity measurements were made in the alpha-ray electroscope previously described[1] having an ionization chamber 19.5 cm. square. The active films were placed 6.5 cm. below the charged electrode, thus allowing all rays to reach their full ranges. The activities of the films varied between half and one and one half that of the 35 sq. cm. standard film of uranium oxide. Sufficient potential, 500 to 600 volts, was used to insure practically complete saturation currents for the weak ionization measured.

Three other series of measurements like those represented by Table II. were made, with films made from two additional lots of radioactinium, each prepared and purified separately. The results for the four films are summarized in Table III.

TABLE III.

No. of Film.	No. of Observations.	x
1	18	4.59
2	14	4.57
3	15	4.55
4	12	4.57
		Mean 4.57

The value of x thus found is subject to a correction, which we estimate at 2.5 per cent., due to the effects of β rays, recoil, and loss of emanation. The corrected value is 4.68. By direct measurement we found that the β rays, largely from actinium *D*, increased the value of x 0.5 per cent. for the alpha-ray electroscope used in our measurements. To find the loss of activity due to recoil and loss of emanation we placed a brass plate charged to a negative potential of 110 volts one mm. above the radioactive film in a closed vessel and measured the activity which deposited on the plate during the first 16 days, following the preparation of the radioactinium film. Almost exactly half of the activity of the deposit was due to AcC; the balance decayed with the period of AcX, the presence of the latter substance being due to recoil. Assuming that all of the AcC lost by the film was deposited on the plate, the correction to x on this account would amount to 2 per cent. The correction for AcX, which is accompanied of course by Em and A, is also apparently 2 per cent., but should be taken somewhat less by reason of the probable

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[1] McCoy and Ashman, Amer. Jour. Sci., *26*, 521 (1908).

extensive escape in the ionization chamber of the Em and A from the surface of the brass plate into the space above, where they would cause greater ionization than if they remained wholly on the plate. We estimate that the true correction should be 1.8 per cent. instead of 2 per cent. A further correction should be made because of the volatilization of Em and therefore of A from the principal film in the ionization chamber of the electroscope. From the amount of C in the active deposit, we estimate that 2 per cent. of the whole Em is present in gaseous form in the electroscope and that this would cause the value of x as found to be 0.8 per cent. too high. The combined effect of these corrections is $2 + 1.8 - 0.5 - 0.8 = 2.5$ per cent. Taking x uncorrected as 4.57, the corrected value becomes 4.68.

Table IV.

	Ranges at 15°.	$R^{\frac{2}{3}}$.	Percentage Activity.	
			Calculated.	Found.
Radioactinium	4.40	2.69	17.8	17.6
Actinium X	4.40	2.69	17.8	82.4
Emanation	5.70	3.19	21.1	
Actinium A	6.50	3.49	23.0	
Actinium C	5.40	3.08	20.3	
			100.0	

Table IV. shows the ranges, R, at 15° and 76 cm., $R^{\frac{2}{3}}$, and the percentage activity of each alpha-ray member of the series; these percentages are taken proportional to the corresponding values of $R^{\frac{2}{3}}$, it being assumed that the equilibrium amount of each member of the series produces the same number of alpha particles per second.[1] The values given in the last column of Table IV are those found by experiment: for radioactinium the "activity found" is $1/(1 + x) = 17.6$ per cent. of the whole. The value of $\Sigma R^{\frac{2}{3}}$ for AcX + Em + A + C divided by $R^{\frac{2}{3}}$ for Rn gives 4.63 which is therefore the theoretical value of x. The close agreement of this result with that found by experiment, 4.68, is further evidence, if such were necessary, that the roll of the alpha-ray members of the actinium series is now complete and that the constants given in Tables I. and IV. may be accepted with considerable confidence as being at least close approximates to the true values.

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[1] While it is probable that AcC is complex, the number of α-particles of longer range than 5.4 cm. compared with those of this range is so small—0.15 per cent.—that we need not take them into account here. See Marsden and Perkins, Phil. Mag., 27, 700 (1914).

Note One: The Period of Radioactinium.

Hahn's early work[1] on radioactinium indicated a period of 19.5 days. This value has been accepted during the past eight years and was thought to be confirmed by Rothenbach[2] during the past year, working under the direction of Professor Hahn. In determining the period of a radioactive substance, which produces products of considerably shorter life, it is usually practicable to consider that the rate of decay is exponential after a sufficient lapse of time. It was by this method of calculation that the period of radioactinium was determined by Hahn and by Rothenbach. However, in case the period of one or more of the products is of the same order of magnitude as that of the mother substance, this simple treatment of the problem is not accurate.

The familiar equation which we have used in the form

$$f_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

may be written

$$f_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1} f_1[1 - e^{-(\lambda_2 - \lambda_1)t}] = \frac{p\lambda_2}{\lambda_2 - \lambda_1} f_1$$

if for $1 - e^{-(\lambda_2 - \lambda_1)t}$ we write p.

The equation

$$A = f_1 + f_2 x$$

then becomes

$$A = f_1 \left[1 + \frac{p\lambda_2 x}{\lambda_2 - \lambda_1}\right].$$

Now the limiting value of f_2 is given by

$$f_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1} f_1$$

and to calculate the period of Rn by the simple exponential equation is to assume that f_2 has approached this limiting value sufficiently closely. But this is by no means the case, even after a much longer time than that over which Hahn's and Rothenbach's observations were carried. After 115 days the activity of a Rn film is about 6 per cent. less than it would be if the limiting ratio of AcX were present. Let us represent by A' the activity of the film if this limiting ratio of AcX (+ products) were present: then

$$A' = f_1 \left[1 + \frac{\lambda_2 x}{\lambda_2 - \lambda_1}\right]$$

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[1] Phil. Mag., *12*, 244 (1906); *13*, 165 (1907).

[2] Dissertation, Berlin, 1913.

and

$$\frac{A'}{A} = \frac{\lambda_2 - \lambda_1 + \lambda_2 x}{\lambda_2 - \lambda_1 + p\lambda_2 x},$$

which expression we may call q, therefore $A' = qA$. It is obvious that the quantity A' will decrease with time strictly exponentially with the true period of Rn.

In order to find the period of Rn, we made a film from the larger part of the material used for the measurements recorded in Table II. This film had an initial activity nearly three times that of our uranium standard (about 35 sq. cm. of a thick film of U_3O_8). We made very careful measurements of its initial activity, extrapolating from the first measurements made 45 min. after the precipitation of the Rn with the thorium oxalate, to find the true activity at time zero. We allowed this film to decay for nearly four months and then made the measurements shown in Table V. between the 115th and 206th day, by which latter time the activity had fallen to 0.0469 of the uranium standard. Of this final activity we calculated that 0.0284 was due to the thorium present. We got this result by considering that of the activity observed at the 164th day, that due to Rn + products decayed exponentially with the period of 18.8 days, account being taken of the q factor. All of the observed activities given in Table IV. have been corrected for the constant activity of the thorium present. While a knowledge of the value of x is required for the calculation of q, any probable error in x is unimportant: thus, if x were 5.0 instead of 4.57, q would be changed by only 0.1 per cent. for the 115th day.

TABLE V.

Age of Film in Days.	Interval Since 115th Day.	Activity Found.	Activity Calculated.	q	Period of Radioactinium.
0.0	——	2.828			
115.1	0.00	0.4809	0.4804	1.060	——
117.1	2.02	0.4476	0.4473	1.058	18.94
127.9	12.02	0.3118	0.3129	1.045	18.68
132.0	16.91	0.2605	0.2630	1.040	18.60
139.1	24.00	0.2005	0.2032	1.033	18.56
145.9	30.82	0.1562	0.1592	1.028	18.59
152.9	37.80	0.1213	0.1238	1.024	18.67
160.0	44.89	0.0948	0.0954	1.020	18.84
164.1	49.01	0.0824	0.0823	1.018	18.97
206.0	90.87	0.0185	(0.0185)	1.007	(18.80)
			Mean Period found......		18.74

From the consideration of all of our results on the change of activity of Rn films with time, we are led to conclude that the period of Rn is very close to 18.8 days, although the results in Table V. lead to a mean of 18.74 days. We believe that better results for this constant would have been obtained if we had begun measurements before the activity of the film had become so small that the inevitable errors of measurement made appreciable errors in the period found. Using the value 18.8 days, we have calculated the activities to be expected from the activity, A_0, at time zero, by the strictly accurate equation

$$A = A_0(f_1 + f_2x),$$

taking $A_0 = 2.828$ and $x = 4.57$. The results given in the column headed "Activity Calculated" were obtained in this way. It is seen that the agreement between these values and those found are quite as close as one could expect for films of such small activity. We have also made calculations of the activities at the various times upon the assumption that the period of Rn is 19.0 and 19.1 days. If the period of Rn is 19.1 days, the mean difference between the activities found and calculated is 5.9 per cent.; for a period of 19.0 days the difference is 3.7 per cent., while for a period of 18.8 days, the difference is only 0.8 per cent. It is therefore perfectly clear that the old value 19.5 days is far from the true value and can no longer be supported.

We find that the difference between our value for the period of radioactinium and that of Hahn and Rothenbach is wholly due to the fact that in the latter case the formula used in the calculation was not sufficiently accurate. Our experimental results which are in satisfactory agreement with theirs, indicate also a period of 19.5 days, if we make the calculation by the simple exponential equation used by Hahn and Rothenbach. The latter's calculations were based on measurements made between the 115th and 174th day. Now a simple calculation will show that if the period as found by the simple exponential equation is 19.5 days for the interval between 120 and 139.5 days, then by use of the correct formula used by us, the same data show the true period to be 18.89 days.

Since the foregoing paragraphs were written we have completed four additional series of measurements of four new radio-actinium films and have obtained the results shown in Table VI.

TABLE VI.

Age of Film at First Measurement.	Age of Film at Last Measurement.	Number of Measurements.	Period.
60.1 days	109.2 days	11	18.88
60.1	118.0	12	18.88
104.0	120.0	7	18.75
92.2	119.2	6	18.88

The average difference between single results and the mean for the four series was 0.08 days. The low result, 18.75 days, for the third series was caused by three very low values found on successive measurements on the 9th, 12th and 14th days, viz: 18.56; 18.57; and 18.67. On both the 15th and 16th days, however, the measurements gave 18.84 days; and we are therefore inclined to think that these low results were caused by some undiscovered error in the activity measurement. If these low results are omitted from the mean, we obtain for this series 18.87 days, a value in good agreement with the means of the other three series.

These newer measurements indicate that the period of radioactinium is a little higher than 18.8 days, which value was used in finding the value $x = 4.68$. If, as is very probable, the period is 18.88 days, our experimental results give $x = 4.67$; the difference is not great and the lower value of x is in even closer agreement with the value calculated from the ranges, viz.: 4.63.

Note Two: Determination of the Range of Radioactinium.

The value found for the activity of the products of radioactinium compared with radioactinium itself led us to suspect either that the accepted range of radioactinium was too great or that of AcX too small. The values found by Geiger[1] for Rn and AcX at 15° and 760 mm. were 4.60 and 4.40 cm. respectively. Furthermore, by Geiger's law relating ranges and decay constants, the range of Rn should be less than that of AcX. As it was a comparatively easy matter to determine the range of Rn, we carried out this measurement, making use of a modification of Geiger's spherical flask apparatus. The flask, which was chosen from a large stock, was almost perfectly spherical, except in the region near the neck. Its internal radius was 6.7 cm.

The radiothorium was prepared by the same method as that used in preparing this substance for the activity measurements. A large quantity of actinium was used, and but one drop of a 0.4 per cent. thorium nitrate solution, = 0.1 mg. of ThO_2. The thorium was precipitated three times with hydrogen peroxide, and converted into oxalate, which was separated by means of a centrifuge and washed with dilute hydrochloric acid and finally with alcohol. A small portion of this oxalate was made into a very thin film, about 1 mm. in diameter on the center of the brass disc of the range apparatus. The range measurements were completed within two or three hours from the time of the oxalate precipitation; from which it follows that the maximum amount of AcX formed did not exceed 0.25 per cent. of the equilibrium amount. The

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[1] Phil. Mag., 24, 653 (1912).

temperature was read at frequent intervals during the measurement and always remained constant within one degree.

Fig. 1.

The results of one determination are shown graphically in Fig. 1, which is drawn to scale. The break comes at 504 mm. of mercury and as the temperature was 20°, this gives as the range of radioactinium 4.17 cm. at 760 mm. and 0°, or 4.40 cm. at 15°. We made two other determinations, each with separately prepared samples of radioactinium. One gave 4.38 cm., the other 4.43 cm. at 15°; the mean of the three results is 4.40 cm. for 15°. This is 0.20 cm. lower than Geiger found for radioactinium and is identical with Geiger's value of actinium *X*. The periods of radioactinium and actinium *X* being 18.8 and 11.35 days respectively, if the range of the former is 4.40 cm., that of the latter should by Geiger's law be 4.45 cm., which differs from the value found by Geiger by scarcely more than the experimental error of the measurements.

SUMMARY.

1. Methods are described for the chemical separation of radioactinium and also of actinium *X* from actinium and from one another and the preparation of each in radioactively pure form.

2. The period of radioactinium has been found to be 18.88 days instead of 19.5 days as accepted for many years. The old higher value resulted from an inaccurate method of calculation.

3. The period of actinium *X* has been found to be 11.35 days instead of 10.2 days. This result first published some months ago,[1] is in fair agreement with that found by Hahn and Rothenbach,[2] a few months earlier, viz.: 11.6 days.

4. The range of radioactinium at 15° and 76 cm. was found to be 4.40 cm. instead of 4.60 cm., as found by Geiger. The new value is in closer agreement than the old with Geiger's Law relating ranges and periods. This agreement is also improved by our results showing the period of radioactinium to be shorter and that of actinium *X* to be longer than the values formerly accepted. Complete agreement would result if the range of actinium *X* were 4.45 cm. instead of 4.40 cm. as given by Geiger.

5. Careful and extensive measurements of the change of the alpha ray activity of radioactinium with time has shown that the alpha activity of AcX + Em + A + C is 4.67 times that of the equilibrium amount of radioactinium alone. If in each atomic disintegration one and only one alpha ray is expelled, and if the ionization produced by any alpha ray is proportional to the 2/3 power of its range, we should expect this ratio to be 4.63. The good agreement between experiment and theory indicates the correctness of the latter. The important revised data for the actinium series are given in Tables I. and IV.

Kent Chemical Laboratory,
University of Chicago,
June 20, 1914.

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[1] Phys. Zeitschr., *14*, 1280 (1913).

[2] *Ibid.*, *14*, 409 (1913).

DETERMINATION OF THE VALUE OF "e," BY MILLIKAN'S METHOD, USING SOLID SPHERES.

By John Yiubong Lee.

Introduction.

THE value of the elementary electric charge was very carefully determined by Millikan[1] by his well-known method. By varying the size of his drops and the pressure, he also obtained empirically a correction term for the law of Stokes on the rate of fall of spheres in viscous media. The following experiments were undertaken for the sake of seeing (1) whether solid spheres would give a value of e in any way different from that obtained with the liquid drops; and (2) whether the correction term to Stokes's law is the same for both solid and liquid drops.

The apparatus used was that employed by Millikan, with but slight modifications. The substance in liquid form is blown into very minute particles by means of a commerical atomizer (*A*, Fig. 1) into the dust free chamber *D*, where the drop solidifies and falls slowly to the top of the plate condenser near the bottom of the chamber. Some of the particles fall though the small holes (*h*) in the middle of the upper plate into the space between, where on account of the charge which they carry, they may be pulled upwards against the action of gravity by applying an electric field of sufficient strength and of the proper sign to the condenser plates.

The force acting on the particle while falling under the action of gravity is simply mg, while the force pulling it upwards is $Fe_n - mg$, where F is the field strength and e_n the charge on the particle. According to Stokes's law, the force acting on a solid sphere in a viscous medium is

$$x = 6\pi\eta a v, \tag{1}$$

where η is the viscosity of the medium, a, the radius of the particle, and v, the velocity of the fall. Now if v_1 represents the downward velocity and v_2 the upward velocity, we have

$$\frac{mg}{Fe_n - mg} = \frac{v_1}{v_2}. \tag{2}$$

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[1] R. A. Millikan, PHYS. REV., N. S., Vol. II., p. 109–143, Aug., 1913.

Combining equations (1) and (2) and substituting for m, $(4/3)\pi a^3(\sigma-\rho)$, where σ is density of the particle, and ρ that of the medium, we obtain

$$e_n = \frac{4}{3}\left(\frac{9\eta}{2}\right)^{\frac{3}{2}}\left(\frac{1}{g(\sigma-\rho)}\right)^{\frac{1}{2}}\left(\frac{v_1+v_2}{F}\right)v_1^{\frac{1}{2}}. \tag{3}$$

By repeatedly changing the charge on a given particle by allowing it to capture ions from the air, a series of values of v_2, and hence of e_n, may be obtained. The values of e_n for any given drop come out to be exact multiples of an elementary unit, e_1, which shows that the electric charge is atomic in structure. This value of e_1 is not the same, however, for small particles of varying sizes, a fact which led Millikan to the conclusion that Stokes's law breaks down when the dimensions of the particles are comparable with the mean free path of a gas molecule. Accordingly a corrected form of Stokes's law was set up; viz.

$$v_1 = \frac{2}{9}ga^2\frac{(\sigma-\rho)}{\eta}\left\{1+A\frac{l}{a}\right\}, \tag{4}$$

in which A is a constant to be determined; l, the mean free path of a gas molecule, and a, the radius of the particle. From the variations of e_1 with the different values of l/a, A was shown to be determinable empirically by a graphical method.[1]

THE APPARATUS.

The arrangement of the apparatus is shown in Fig. 1. D is a heavy brass vessel about 30 cm. high, and 35 cm. in diameter. It has three glass windows (only two are shown in the figure), a and b, 180° apart, and c, 28° from b in the same horizontal plane with a and b. The light from an arc lamp A enters from the window a to illuminate the particles. An X-ray bulb is placed in front of the window b to ionize the air inside the chamber, and c is the observation window. The whole vessel is immersed in an oil bath, which is kept constant to within 0.1° C. for several hours before and during the observation. The heat rays from the arc are absorbed by means of a water cell 80 cm. long and a cupric chloride cell 4 cm. thick. The condenser plates are 22 cm. in diameter and are separated by three ebonite rods, 15.98 cm. in length. A thin ebonite strip entirely encircles the sides of the condenser, with the exception of three glass windows, which are directly opposite the three windows in the vessel D. The pressure is measured by a very carefully made barometer, which has been compared with a standard weather bureau barometer, and found to be correct. The atomizer is made entirely of glass,

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[1] L. c.

and is cemented to one half of a union joint, by means of which it is attached to the vessel *D* immediately in front of the valve *W*. The space above the reservoir is connected to the vessel *D* also by means of a glass tube, so that the pressure at the nozzle and above the liquid in the reservoir is the same no matter what the pressure inside the vessel may be. This atomizer may be detached from the rest of the apparatus for cleaning without interfering with the vacuum in *D*, by simply closing the valve *W*. The electric field is produced by about 2,500 storage cells which gives a potential difference of about 5,000 volts. By means of a specially designed switch *k*, containing eight mercury cups and six movable contacts, the condenser may be charged, short circuited, or reversed, merely by changing the position of a single handle. The observing telescope has an objective of 28 mm. aperture, and 12.5 cm. focal length, and an eyepiece of 12 mm. focal length. The eyepiece is kept at a fixed distance from the objective (about 25 cm.), and focusing is done by moving the whole telescope forwards or backwards, so that the magnification of the optical system remain the same at all times. The scale in the eyepiece has eighty divisions, and the distance between the extreme divisions of the scale corresponds to an actual distance of 1.022 cm. at the object. The time is measured by a Hipp chronoscope which reads to .002 second, except the very slow times (longer than 120 seconds) which are measured by a stopwatch reading to .2 second.

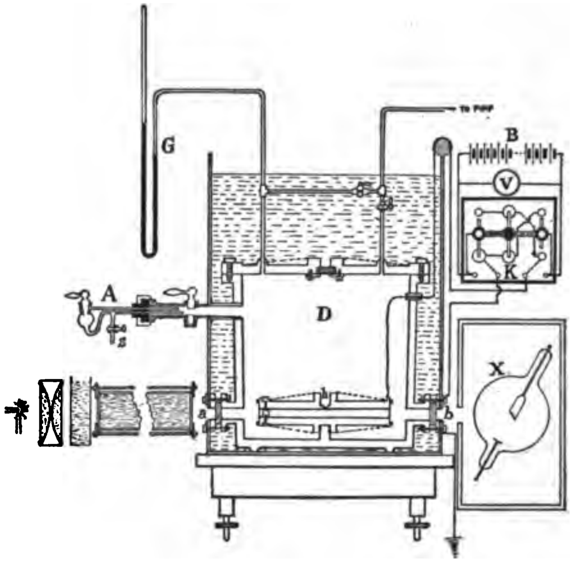

Fig. 1.

The substance used for making the spheres is a carefully filtered solution of shellac in alcohol. This solution can be made very thin so that exceedingly small spheres may be made. It is more convenient to use than sulphur, wax, or paraffin, etc., in that heat is not necessary to keep it in the liquid state for atomizing. The alcohol evaporates very quickly, and the density of the spheres reaches a constant value in a very short time. Particles caught on a microscopic slide placed on top of the condenser plates, have been repeatedly examined under a microscope (at about 1,000 diameters) and show perfect sphericity except a few which are distorted by particles of dust or other foreign substances adhering to them, or by being flattened against the microscopic slide before they were completely solidified. Although the radius of the spheres ranges from .0012 to .0030 mm., there is no evidence of lack of homogeneity, as the results of these experiments show. To measure the density of the shellac, a thin coat of it is sprayed on a watch crystal with the atomizer, allowed to dry, and weighed in air and water. Six determinations were made by two observers at different stages of the experiment with varying quantities of the substance, and the results were found to check to within one half of one per cent. of the mean, as is shown by the following table:

Observer.	Quantity in Grams.	Density.
L.	.3083	1.154
W.	.1626	1.151
L.	.3089	1.149
L.	.0995	1.155
W.	.1339	1.144
L.	.4830	1.148
	Mean.....	1.150

This mean value, 1.150, is also within one half of one per cent. of the density of the shellac flakes, from which the solution was made.

The Observations.

Before and after the taking of an observation on every particle, the potential of the batteries was measured in groups of about 400 cells each by means of a Kelvin and White electrostatic voltmeter having a range of 300 to 900 volts. This voltmeter showed remarkable constancy and all the readings could be duplicated to within 0.5 volt. It was carefully calibrated toward the close of these experiments by comparison with a Weston Laboratory Standard Voltmeter, which was calibrated by the Bureau of Standards just a few days previously, and all voltmeter readings in these experiments have been corrected by the new calibration curve. The drop of potential of this entire set of storage batteries of 5,000 volts is on the average only 10 volts for an observation of an hour's duration. The potential drop is assumed to be a linear function of the time during an observation so that the potential at any time during the observation may be found by interpolation.

A few seconds after the shellac solution is blown into the chamber with the atomizer, the large particles may be seen in the field of the telescope; but the particles used in these experiments were of such size that it required from 2 to 5 minutes for them to reach the condenser. A careful selection was made to obtain a particle of the proper size and one which showed no flickering, since a particle which flickers is presumably not spherical.

A little shutter (controlled by an electro magnet) above the condenser was closed over the small holes as soon as a particle had been selected, and the electric field was then turned on and off alternately to keep the selected particles in the field of the telescope while all the other particles were allowed to fall to the bottom plate. The speed of the spheres under gravity was then measured by timing it between the two extreme divisions of the scale in the eyepiece of the telescope. It was then brought back to the top and the timing was repeated from 5 to 20 times depending on the speed and the consistency of the observations. The speed of the particle under the force of the elecric field was measured in the same way, except that instead of allowing it to fall under gravity alone after each measurement, the field on the condenser was reversed so that the particles fell under the force of gravity and that of electric field combined. By keeping the electric field on either one way or the other all the time all stray ions, as soon as they are formed, are removed to the plates thus preventing the particle from changing its charge, until a sufficient number of observations of that particular speed have been taken. The charge on the particle was changed by turning on the X-rays, directly behind the window *b*, for a short time. Five or six different speeds in the field are usually taken on each particle, after which its speed under gravity is again taken to check the readings taken at the beginning of the experiments. Tests for convection currents in the space between the condenser plates were made frequently by timing the spheres through the first half and then the second half of the distance and in some cases, particularly in the case of slowly moving particles, timing is done for every one of the eight large divisions in the scale of the eyepiece; but no evidence can be found to support the conjecture that the air in the chamber is not stagnant.

Tables I., II., and III., are typical examples of the observations taken.

Table I.

Particle No. 22.

P.D. = 5023.0 volts.
Temp. = 26.8° C.
Pres. = 74.40 cm. Hg.
$1/t_g$ = .043280.
a = .0001774 cm.
l/a = .05507.
$1/pa$ = 75.77.
$e_1^{\frac{2}{3}}$ = 64.41 × 10^{-8}.

t_g	t_f	Mean t_f.	$\frac{1}{t_f}$	n	$\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_f}\right)$
23.276					
23.090					
23.192					
23.112					
23.110					
23.042					
23.150					
23.122					
23.042					
23.112	24.164	24.188	.041342	10	.008462
	24.268				
	24.226				
	24.122				
	24.162				
	62.588	62.588	.015976	7	.008465
	30.270	30.340	.032960	9	.008471
	30.376				
	30.374				
	40.926	40.899	.024450	8	.008466
	40.872				
	20.108	20.073	.049818	11	.008463
	20.080				
	20.030				
	20.088				
	20.082				
	20.054				
23.114	20.160				
23.042					
23.142					
23.104					
22.940					
22.934					
22.131					
23.050					
23.130					
23.105 = Mean				Mean = .008465	

Column 1 shows the time of fall under gravity (t_g); Column 2 shows the time of rise in the field (t_f) and Column 3 is the mean of the different t_f's in column 2. Column 4 shows the reciprocals of the number is column 3, and are therefore directly proportional to the velocity in the field.

TABLE II.

Observation No. 26.

P.D. = 5009.5 volts.
Temp. = 23.45° C.
Pres. = 59.27 cm. Hg.
$1/t_g$ = .049541.
a = .0001879 cm.
l/a = .06439.
$1/pa$ = 89.81.
$e_1^{\frac{2}{3}}$ = 65.14 × 10^{-8}.

t_g	t_f	Mean t_f.	$\frac{1}{t_f}$	n	$\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_f}\right)$
20.144					
20.142					
20.144					
20.154					
20.188					
20.178					
20.160					
20.244	64.868	64.868	.015416	8	.008120
	42.244	42.222	.023684	9	.008136
	42.200				
	25.006	24.987	.040022	11	.008142
	25.010				
	24.938				
	24.992				
	15.494	15.518	.064441	14	.008142
	15.550				
	15.518				
20.232	15.512			Mean......	.008135
20.086					
20.134					
20.228					
20.228					
20.168					
20.218					
20.228					
20.216					
20.185					

Table III.

Observation No. 53.

P.D. = 1660.2 volts.
Temp. = 20.6° C.
Pres. = 12.19 cm. Hg.
$1/t_g = .040329.$
$a = .0001458.$
$l/a = .3986.$
$1/pa = 562.48.$
$e_1^{\frac{2}{3}} = 87.30 \times 10^{-8}.$

t_g	t_f	Mean t_f.	$\frac{1}{t_f}$	n	$\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_f}\right)$
24.790					
24.854					
24.862					
24.706					
24.716					
24.712					
24.802					
24.992	39.446	39.538	.025291	14	.004687
	39.630				
	25.478	25.355	.39440	17	.004692
	25.226				
	25.360				
	62.700	62.700	.015949	12	.004690
	48.320	48.379	.020669	13	.004692
	48.438				
	33.368	33.412	.029929	15	.004684
	33.596				
24.856	33.260				
24.756	33.424				
24.714				Mean......	.004689
24.812					
24.828					
24.840					
24.748					
24.750					
24.796					

The Calculations.

It may be seen that equation 3 may be written in the form

$$e_n = K\frac{(v_1+v_2)v_1^{\frac{1}{2}}}{F} \tag{5}$$

or more conveniently

$$e_n = K'\frac{\left(\frac{1}{t_g}+\frac{1}{t_f}\right)\left(\frac{1}{t_g}\right)^{\frac{1}{2}}}{F}. \tag{6}$$

Now if the particle caught say n' additional charges, its time of rise may now be indicated by t_f', so that equation 6 becomes

$$e_{n+n'} = K' \frac{\left(\frac{1}{t_g} + \frac{1}{t_{f'}}\right)\left(\frac{1}{t_g}\right)^{\frac{1}{2}}}{F} \qquad (7)$$

and subtracting equation 6 from 7, we have

$$e_{n'} = K' \frac{\left(\frac{1}{t_f'} - \frac{1}{t_f}\right)\left(\frac{1}{t_g}\right)^{\frac{1}{2}}}{F}. \qquad (8)$$

Now since $K'/F(1/t_g)^{\frac{1}{2}}$ remains constant throughout the experiment, the successive values of the charge on the particles are proportional to the successive values assumed by the quantity $(1/t_f' - 1/t_f)$ and the value of the elementary charge is obviously equal to the constant $[K'/F(1/t_g)^{\frac{1}{2}}]$ multiplied by the greatest common divisor of all the successive values of $[1/t_f' - 1/t_f]$. When this common divisor is known, the value of n or n' is determined, since n is usually a small number (never more than 30 in these experiments), so that the value of the elementary charge e_1 may now be obtained by dividing equations 6 and 7 by n and $n + n'$ respectively; thus

$$e_1 = \frac{K'}{F} \frac{\left(\frac{1}{t_g}\right)^{\frac{1}{2}}\left(\frac{1}{t_g} + \frac{1}{t_f}\right)}{n}, \qquad (9)$$

$$e_1 = \frac{K'}{F} \frac{\left(\frac{1}{t_g}\right)^{\frac{1}{2}}\left(\frac{1}{t_g} + \frac{1}{t_{f'}}\right)}{n + n'}. \qquad (10)$$

The mean of the e_1's thus obtained is taken as the true value of the e_1 for the particular particle.

The coefficient of viscosity of air at 23° C. is taken as .0001824. This value was adopted by Millikan for his oil drop experiments, after a very careful study of all the experimental results that are available.

DETERMINATION OF THE CONSTANT A.

It will be observed from equation 3, that e is proportional to the 3/2 power of v, so that if Stokes's law is to be corrected by the term $\{1 + A(l/a)\}$, e_1 must be divided by $\{1 + A(l/a)\}^{\frac{3}{2}}$ in order to give the correct value of e; *i. e.*,

$$e = \frac{e_1}{\{1 + A(l/a)\}^{\frac{3}{2}}} \quad \text{or} \quad A = \frac{e_1^{\frac{2}{3}} - e^{\frac{2}{3}}}{l/a} \div e^{\frac{2}{3}}. \qquad (11)$$

If l/a is known for each particle, we can plot the values of $e_1^{\frac{2}{3}}$ as ordinates, and l/a as abscissæ, and if A is a constant, the line joining the values of e_1's would be a straight line, and A would equal to the slope of this line divided by the y-intercept; for obviously, when $l/a = 0$, $e_1^{\frac{2}{3}} = e^{\frac{2}{3}}$. In order to determine this slope as accurately as possible, e_1 should be measured through large ranges of l/a. This is done by decreasing the value of a, that is by using small particles, or by increasing the mean free path by decreasing the pressure of the air in the chamber.

THE VALUE OF THE RADIUS, "a."

To obtain the value of a, the radius of the particle we may substitute for m, its value $4/3[\pi a^3(\sigma - \rho)]$ in equation 2, and solve for a.

$$a = \sqrt[3]{\frac{3Fe}{4\pi g(\sigma - \rho)} \frac{v_1}{v_1 + v_2}}. \qquad (12)$$

It will be seen from this equation, that all the quantities except e are directly measured in each experiment. It will be seen, also, that e enters only in the 1/3 power in the determination of a, so that a relatively large error in e would not appreciably affect the correct value of a. In these calculations, e is taken as 4.774×10^{-10} e.s.u. (Millikan's value), and the results are shown in column 2 of Tables VII. and VIII.

In the graphs (Figs. 9 and 10), $e_1^{\frac{2}{3}}$ is plotted against $1/pa$ instead of l/a, because of the uncertainty of l, due to the fact that k in the equation $n = knm\bar{c}l$ has not been exactly determined. The correction term to Stokes's law then becomes $(1 + b/pa)$. The value of l/a however is computed for every particle, and is shown in column 8 of Tables IV. and V. For these computations, k is taken as .3502.

SUMMARY OF RESULTS.

Table IV. contains the results of observations on 19 particles, all of which were taken under atmospheric pressure, in the summer of 1913. In this group of experiments, the cover was placed only loosely over the vessel, and no particular care was taken to keep the air dry. Table V. contains the results of 39 observations taken in the following fall, and these last observations were made in a sealed chamber, which was frequently washed with dry air. The pressure varied from atmospheric to 9.65 cm. of mercury, as is shown in column 5 of Table V. The value of $1/pa$ and $e_1^{\frac{2}{3}}$, are shown in columns 6 and 8 of Tables IV. and V., and are plotted in the graphs shown in Figs. 2, 3, and 4. From the slope of this line and its y intercept, $e^{\frac{2}{3}}$ is determined to be 61.03×10^{-8}, and b,

.0007627. The value of A of equation 4, corresponding to this value of b, is 1.067.

TABLE IV.

No.	t_g	$a \times 10^4$ cm.	P. D.	n	P cm. Hg.	$1/pa$	l/a	$e_1^{2/3}$	$e^{2/3}$
1	8.682	2.915	4905.8	25–30	75.13	45.66	.03286	63.03	60.91
2	9.226	2.820	4897.4	22–26	74.54	47.56	.03399	63.07	60.86
3	10.854	2.598	4911.3	18–24	75.05	51.28	.03670	63.39	61.01
4	12.055	2.471	4913.0	16–25	74.98	53.98	.03898	63.36	60.75
5	13.743	2.303	4912.9	13–16	75.05	57.87	.04136	63.55	60.87
6	14.830	2.216	4916.4	12–18	75.09	60.10	.04313	63.76	60.99
7	14.955	2.205	4912.7	12–16	75.00	60.45	.04354	63.97	61.16
8	16.126	2.126	4912.5	9–15	75.39	62.40	.04500	63.93	61.04
9	16.688	2.081	4951.0	10–15	75.05	64.04	.04570	64.05	61.08
10	17.916	2.030	4904.6	9–15	74.89	66.31	.04760	63.93	60.85
11	19.753	1.910	4900.0	8–13	74.58	70.21	.05011	64.22	60.97
12	22.308	1.795	4916.2	6–13	75.20	74.20	.05351	64.48	61.04
13	25.061	1.689	4888.0	6–9	75.18	78.79	.05664	64.69	61.03
14	28.208	1.594	4087.3	5–12	74.91	83.78	.06007	64.75	60.70
15	30.650	1.527	4913.2	4–9	75.13	87.15	.06253	64.87	60.83
16	31.935	1.500	4910.7	4–7	74.98	88.94	.06432	64.99	60.87
17	36.936	1.384	4085.3	4–9	74.91	96.43	.06927	65.64	61.17
18	40.300	1.325	4919.2	3–6	75.39	100.14	.07220	65.64	61.00
19	46.278	1.235	4902.6	4–6	74.89	108.08	.07758	65.74	60.72
							Mean.......		60.939

The question may be raised as to whether the alcohol evaporated from the shellac solution may not effect the pressure of the air in the chamber, and hence the slope of the line in Figs. 2, 3, and 4. That the alcohol vapor has no appreciable effect on the observations with the shellac

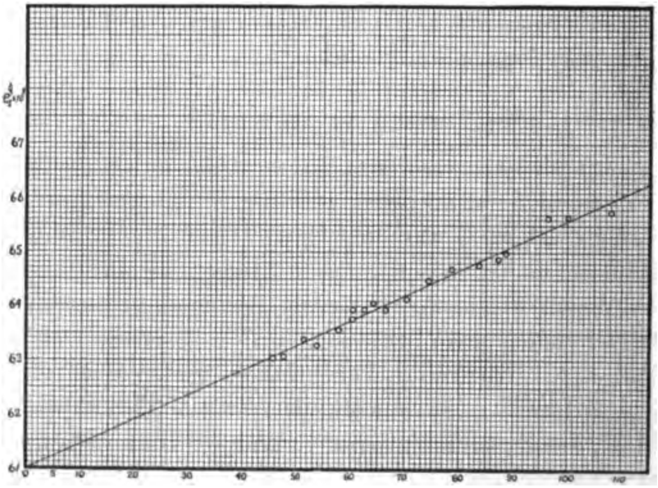

Fig. 2.

Table V.

No.	t_g	$a \times 10^4$ cm.	P. D.	n	P	$1/pa$	l/a	$e_1^{\frac{2}{3}}$	$e^{\frac{2}{3}}$
20	13.300	2.347	5006.7	14–19	74.40	57.28	.04162	63.64	60.98
21	17.271	2.051	5014.0	10–17	75.66	64.43	.04616	63.91	60.92
22	23.105	1.774	5023.0	7–11	74.40	75.77	.05507	64.41	60.90
23	27.453	1.612	5013.7	5–11	75.66	81.99	.05892	64.82	61.02
24	19.556	1.909	5006.6	7–13	59.86	87.52	.06236	65.14	61.08
25	30.682	1.527	5014.4	5–8	74.40	88.00	.06396	65.17	61.09
26	20.185	1.879	5009.5	8–14	59.27	89.81	.06439	65.14	60.97
27	34.810	1.427	5013.4	4–7	75.66	92.62	.06653	65.54	61.24
28	25.850	1.787	5014.7	6–9	54.25	103.06	.07388	65.65	60.87
29	43.932	1.267	5005.0	3–7	74.50	105.93	.07596	65.70	60.78
30	22.279	1.779	4186.0	9–14	48.89	114.97	.08273	66.16	60.82
31	55.020	1.122	5004.7	2–6	75.28	118.41	.08484	66.98	61.49
32	21.600	1.792	4176.0	9–14	43.73	127.63	.09142	67.08	61.16
33	22.096	1.769	4176.0	8–12	40.76	138.69	.1000	67.38	60.94
34	20.960	1.805	4178.8	9–12	39.35	140.76	.1007	67.84	61.31
35	19.164	1.895	4181.8	9–13	35.45	148.87	.1071	67.51	60.60
36	20.787	1.821	4192.0	8–12	34.97	157.06	.1130	67.90	60.61
37	22.655	1.727	4180.8	7–11	33.93	170.63	.1220	68.53	60.61
38	25.829	1.604	3317.5	7–13	36.17	172.33	.1216	69.06	61.06
39	19.380	1.810	3348.4	11–16	29.00	180.57	.1294	69.76	61.38
40	20.140	1.852	3343.8	12–16	28.15	196.25	.1406	70.27	61.16
41	20.428	1.791	3352.4	10–14	26.56	210.19	.1505	70.74	60.98
42	21.850	1.715	3334.4	9–13	25.18	231.64	.1660	72.22	61.47
43	11.758	2.326	3243.5	21–25	18.08	237.74	.1685	72.35	61.31
44	19.500	1.814	3347.2	11–17	22.82	241.58	.1732	72.33	61.11
45	23.366	1.629	3260.0	8–12	21.33	287.88	.2035	74.18	60.82
46	17.656	1.847	2488.2	15–19	16.50	328.14	.2333	76.62	61.39
47	23.134	1.592	2500.0	9–14	16.46	381.52	.2719	78.79	61.08
48	29.924	1.394	2486.1	7–13	18.23	394.49	.2810	79.38	61.07
49	25.080	1.523	3220.0	7–18	16.40	400.33	.2834	79.07	60.48
50	20.220	1.671	2523.7	12–18	14.50	412.68	.2951	81.14	62.04
51	28.800	1.405	3150.0	6–15	16.43	433.10	.3064	80.87	60.86
52	19.557	1.607	2494.5	10–15	12.34	481.57	.3414	83.17	60.81
53	24.796	1.458	1660.2	12–17	12.19	562.48	.3986	87.30	61.19
54	27.398	1.390	2482.3	6–13	12.78	562.98	.4015	87.36	61.22
55	28.360	1.449	2509.0	6–10	10.74	644.67	.4619	91.05	61.12
56	21.602	1.535	1663.6	14–22	10.04	649.00	.4596	90.45	60.32
57	19.855	1.593	1661.4	14–26	9.65	650.60	.4607	91.36	61.16
58	25.550	1.406	2483.3	7–11	10.75	661.63	.4733	91.74	61.02

Mean $e^{\frac{2}{3}} = 61.024$

spheres is shown by the fact that *drying materials* placed in the chamber produced no different result in the observations, and that experiments with beeswax[1] and other substances containing no volatile vapors gave a line of the same slope as shown in Figs. 2, 3, and 4.

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[1] Beeswax was used for making these experiments several months before shellac was adopted, and fair consistency of results is obtained. The main difficulty is the uncertainty of the density of the small particles employed. One would naturally think that particles of

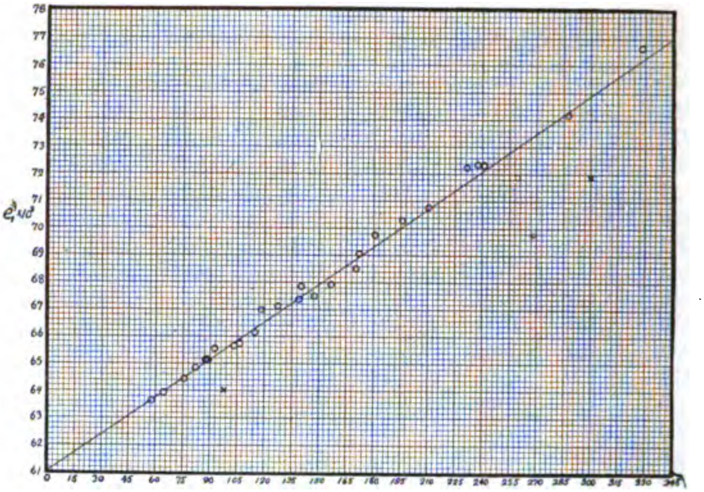

Fig. 3.

Tables IV. and V. include all the observations that were taken and calculated in the period of about seven months, except two observations (shown as crosses in the graph), which were so far from the line joining the rest of the observations that there is every reason to believe that they were dust particles or other foreign substances. There was also a

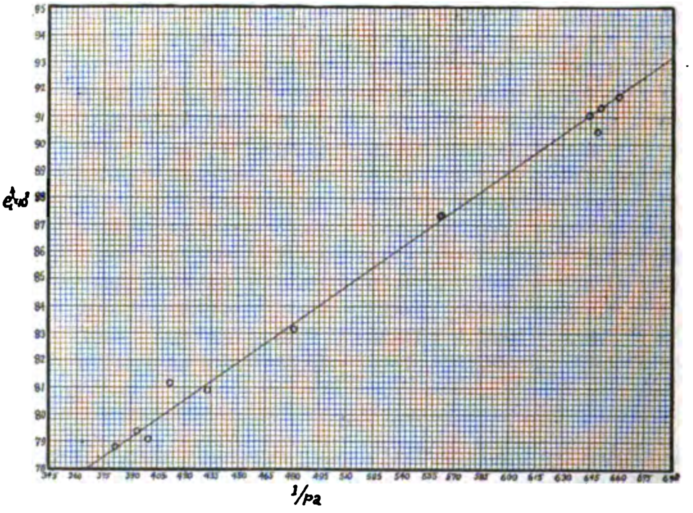

Fig. 4.

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the order of .002 mm. in diameter would have the same density as the cold solid mass, a short time after they were blown from the atomizer. As a matter of fact, their density does not reach a constant value for an hour or more after they were blown, as may be shown by suspending one of them between the condenser plates with the electric field, and measuring its rate of fall under gravity, every now and then.

group of observations which appeared very erratic, and which upon investigation was found to be due to some oil, from the constant temperature bath, leaking into the chamber through the valve between the atomizer and the vessel, so that some of the observations were on oil drops instead of on shellac spheres. For the sake of certainty, however, all the observations taken from the first appearance of the lower values, to the overhauling of the apparatus, 16 in number, were discarded.

A more accurate method to determine the value of $e^{\frac{2}{3}}$ than the graphical evaluation, is to reduce each of the observations on $e_1^{\frac{2}{3}}$ to $e^{\frac{2}{3}}$, by means of the above value of b and the equation

$$e^{\frac{2}{3}}\{1 + (b/pa)\} = e_1^{\frac{2}{3}}. \tag{13}$$

The results of this reduction are shown in column 9 of Tables IV. and V. The mean of the values of $e^{\frac{2}{3}}$ obtained from observations 1 to 19 inclusive, is 60.939×10^{-8} making $e = 4.758 \times 10^{-10}$, while observations 20 to 58 give $e^{\frac{2}{3}} = 61.024 \times 10^{-8}$, and $e = 4.767 \times 10^{-10}$. The mean of all observations is

$$e = 4.764 \times 10^{-10} \text{ e.s.u.}$$

This value agrees to within one-fifth of one per cent. with the value obtained by Millikan with the oil drops and the uncertainty in the density of the shellac is one half of one per cent. so that the difference is well within the limits of experimental error. The value of A, however, is about 20 per cent. higher than Millikan's value, and this can be explained only by assuming that the coefficient of slip between oil and air is different from that between shellac particles and air.

M. Jules Roux[1] has obtained a value of 1.64 for this constant A by measuring the rate of fall of sulphur spheres first in air and then in a liquid (xylol). He assumed that Stokes's law holds for the motion of the spheres in the liquid, and thus calculated the radius a. Then with this value of the radius, he calculated the speed which the particle should have in air if Stokes's law held, and by comparing this speed with the actual observed speed, he found the correction term $\{1 + A(l/a)\}$. The error in the determination of A is thus the error in the difference between these two speeds. Now the time intervals in Roux's measurements were from 3 to 5 seconds—and the differences between the calculated and the observed speeds were about 0.4 second. Further he made but one single time measurement upon each drop; he records observations upon but 5 drops in all, and his measurements were all made with a stopwatch, which read to .2 second, so that his possible error would be .2 second for starting

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[1] J. Roux, Annal. de Chim. et de Phys., 8th series, Vol. XXVIII., 69–132, May, 1913.

and .2 second for stopping, making .4 second in all, *or just enough to cover the entire value of the constant which he was trying to determine.* Since then his possible error was 100 per cent. it is evident that his results are in entire agreement, *within the limits of observational error*, with the present work which locates the value of A with an uncertainty of say 2 or 3 per cent. As to Roux's determination of e with sulphur spheres by Millikan's method it need only be said that it is based on the value of A just described. Further his mean value (4.17×10^{-10}) is obtained from observations on only eight particles, and the results, aside from involving the incorrect value of A, are not consistent enough to justify the belief that they are comparable in accuracy with the results of the oil drop experiments.

I wish to express my sincere thanks to Professor Millikan for his kind assistance and counsel throughout these experiments, and I also take pleasure in acknowledging the able assistance of Mr. C. W. White, and Mr. H. L. Wilson in making some of the above observations.

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DIURNAL AND ANNUAL VARIATIONS IN OVERLAND RADIOTRANSMISSION.

By A. H. Taylor.

IT is a fact well known among radio operators that the winter range of a wireless station greatly exceeds its mid-summer range.

This is partly due to the prevalence of " strays " during the summer season, especially on the long wave-lengths, and partly due to a real difference in transmission, which may perhaps be ascribed to a difference in absorption brought about by different conditions of earth and atmosphere during summer and winter.

Most quantitative experiments so far in long distance work have, for obvious reasons, been with oversea transmission, since a ship station at definite adjustment may be sent out to great distance from a land station sending at constant power, thus making possible, as in the experiments of Austin,[1] a series of measurements under similar conditions at the receiving station, varying only the distance between the two. This method of procedure is obviously impossible for long distance overland transmission, although it is possible in shorter distance field tests. Austin represents the oversea transmission as being quite constant in the daytime, and following the law

$$i_2 = \frac{K i_1 \epsilon^{-\frac{KD}{\sqrt{\lambda}}}}{\lambda D},$$

when i_2 is the received current, to the square of which the audibility may be taken as proportional. The constant K depends on the heights to center of capacity of sending and receiving antenna, the type of antenna used, and the equivalent resistance of the receiver, while i is the sending current and λ the wave length in kilometers. D is the distance between stations, also in K.M. Application of this formula to the 3,500 m. wave so frequently used this spring by Arlington (NAA) in correspondence with Key West (NAR) indicates that it should suffer a thirty-six fold loss in audibility due to absorption in traversing an oversea distance equal to the overland distance between Grand Forks and Washington. Some roughly quantitative results calculated from observations taken here on this wave indicated that it was received nearly 120 times stronger

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[1] Bull. Bur. Standards, Vol. 7, No. 3.

at midnight than at noon. This estimate is based on shunted telephone readings, using a single 800 foot aerial wire of seven strands of No. 22 phosphor bronze, far end 120 feet in height, near end 65 feet. An audion detector was used and a 3,200Ω New Navy type telephone receiver. The signals were barely distinctly audible at noon, but the best midnight (or thereabouts) readings showed shunts of as low as 14Ω. Bearing in mind Braun's[1] recent criticism of shunted telephone readings, the writer can hardly give these results weight enough to attempt to calculate the absorption coefficient for this wave, especially since it is far from safe to assume the midnight transmission to be unabsorbed. They do indicate, however, that the average daylight transmission was fully four to five times as much absorbed as it should have been in an equal oversea distance, according to the Austin formula. Furthermore, as anticipated in a previous paper,[2] the daylight transmission overland is most certainly affected by the weather and the time of year, and is by no means as constant as oversea transmission.

In this connection it is of interest to note that the noon time signal from N A A at 2,500 m., $i. = 87A$, $p = 52$ K.W. could not be received here on the single-wire aerial without the use of a second audion used to amplify the first, while the 3,500 m., $i = 104A$, $P = 58$ K.W., an hour later was distinctly audible, with one audion. This shows that the shorter wave suffers even greater absorption.

I am greatly indebted to Captain Bullard, U. S. N., for data on the 3,500 m. wave.

At the present time, June 20, in order to get the 11:00 A.M. (Central time) or noon time signal at 2,500 m., we are obliged to use a 3-wire 800 foot aerial, although in midwinter a small 150 foot aerial sufficed.

I have no hesitation, therefore, in asserting that the daylight transmission between here and Washington is many times stronger in winter than in summer, this statement being based on comparative shunted telephone readings on the two aerials, long and short, indicating that for Sayville, N. Y., 2,800 meters, the long aerial is seven times as effective as the short one. The long aerial having been recently installed, to cope with summer conditions, no winter observations have yet been taken on it. This long aerial points S. E. and is, therefore, unfavorably directive, else it would be even more efficient.

From observations on Arlington, Va., 1,400 miles, and Key West, Fla., 1,840 miles, it would seem that annual variations are dependent on the character of the country traversed. In midwinter the 2,500 m. wave $i = 57A$, $P = 52$ K.W. of Arlington, received on the smaller aerial,

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[1] F. Braun, Yahrbuch d. drahtlosen Teleg. und Telephonie, April, 1914.

[2] Taylor, PHYS. REV., May, 1914.

equivalent resistance 40Ω, was very little stronger, and indeed frequently not as strong as the 1,800 m. $i = 37\,A$, $P = 23$ K.W. wave of Key West. (The equivalent aerial resistance for 1,800 m. wave 27Ω) I am indebted to Captain J. L. Jayne, U. S. N., for the data on these waves. I am not informed as to the height to the center of capacity at Key West, but for the purpose of argument, assuming it to be the same, the relative intensities of these two waves should be as

$$\frac{i^2}{\lambda^2 D^2 \cdot R} = 2.7 \times 10^{-6}$$

for N A A (Arlington), and $n = 1.8 \times 10^{-6}$ for N A R, Key West, neglecting absorption. With absorption at 1,800 meters in all probability greater than at 2,500 m. N A A should have been heard much louder than N A R. These observations were taken between 9:00 and 9:45 P. M., Central time. As the spring came on, a marked change in this relation came about. Both stations averaged weaker and weaker, finally frequently necesstating the use of the double audion detector or amplifier, but N A R faded much faster than N A A, becoming at that hour seldom audible after April 1, so that from approximate equality in intensity, the signals of Key West were in the course of three months reduced to less than one tenth the intensity of those of Arlington. The writer can scarcely believe that this difference can be attributed entirely to difference in wave-lengths, unless at the same time we assume that the absorption in the Mississippi Valley, which offers a remarkably preferential highway for radio-transmission in winter, increased much more rapidly than that over the more northerly path between here and Washington.

It has occurred to the writer that the loosening up of the ground by plant growth in the spring, and even the growth itself, may be accompanied by a certain amount of ionization from radioactive sources, thus accounting for the almost sudden increase of absorption of electromagnetic waves which occurs in the spring. This effect would be more noticeable over a southerly route.

The preferential winter transmission in the direction of the Gulf of Mexico to Grand Forks, may be due to high soil conductivity in the lower half of the Mississippi Valley, brought on by abundant rainfall. The generally level nature of the country is also unquestionably an aid.

Several investigators mention in recent papers the fact that river valleys, such as the Rhine and Rhone, offer preferential paths. Among these might be mentioned M. Reich,[1] who mentions a distinct difference in transmission between Göttingen and Köln, and Göttingen and Strass-

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[1] M. Reich, Phys. Zeitschrift, 14, 1913.

burg, while he attributes to the fact that a valley favors, for a great part of the distance, the transmission from Strassburg to Göttingen. The writer does not, therefore, advance the idea of the preferential Mississippi Valley path as anything new, but wishes rather to call attention to the fact that, judging from the Key West signals, this preference, so striking in the winter, disappears quite completely in the spring.

Since it usually happens that the transmission rises rapidly after sunset, it might be urged that the whole matter might be accounted for by the difference in time of sunset between Washington and Key West.

This difference is, however, only about one fourth the total difference between Washington and Grand Forks, and seems insufficient to cause any great difference. In order to be sure of this point, the Key West signals were usually followed, when possible, to nearly 10:30, Central time, with marked increase in intensity, although not nearly enough increase even then to compare favorably in the spring with the 9:00 Arlington signals.

It must be noted, in reference to the inconstancy of daylight transmission, that Austin distinctly states that for oversea transmission the daylight transmission is exceedingly constant.

His experiments over long distances would seem to prove this quite conclusively, but his experiments have not, to the knowledge of the writer, been repeated across equally long distances entirely overland. The investigations of H. Mosler,[1] on the annual and daily variations in the intensity of signals from Nordeich and the Eiffel tower led him to conclude that the intensity in the daytime was essentially constant, and varied little if any during the year. This absolutely erroneous conclusion is due to the short distance of only a few hundred kilometers, over which his signals were received. If the absorption is, as seems to be generally accepted, a logarithmic function of the distance, which holds also for Sommerfeld's surface wave, then variations scarcely perceptible at 300 K.M. will be very great at 3,000 K.M. As for the daily variation at Grand Forks of the Arlington 11:00 A.M. Central signal, while it is by no means as great as the variation from summer to winter, yet variations of threefold in the intensity are now being observed, and an attempt is being made to correlate these diurnal variations with weather conditions.

During the month of April Arlington was not only heard frequently on 3,500 m. at 1:00 P.M. but also at 6:00 P.M., and almost invariably with greater intensity, in the ratio of two or three to one. This may be due to the fact that twilight had already begun at Arlington.

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[1] H. Mosler, Electrot. Ztschr., 35, 1913.

Summary.

1. Overland daylight absorption of electromagnetic waves is several times greater than oversea absorption calculated from the Austin formula.

2. Overland daylight absorption is, unlike oversea absorption, not constant.

3. Overland daylight transmission on long waves is much stronger between Grand Forks and Washington (1,400 miles) in winter than in summer.

4. Overland nocturnal transmission varies with the time of year to a different degree, depending on the nature of the country traversed.

5. The Mississippi Valley shows a very wide variation in summer and winter transmission.

Radio Station 9YN,
University, N. D.
July 15, 1914.

A DETERMINATION OF AVOGADRO'S CONSTANT N FROM MEASUREMENTS OF THE BROWNIAN MOVEMENTS OF SMALL OIL DROPS SUSPENDED IN AIR.

BY HARVEY FLETCHER.

INTRODUCTION.

In a previous paper[1] equations were developed which showed the effect of the so-called Brownian movements upon a spherical particle moving under the influence of a constant outside force. In these formulæ there occurred a factor k which was a function of l/a, the exact dependence being unknown at the time the paper was written. For this reason the experimental work was directed toward determining Ne, which process did not involve k. Since that time Prof. Millikan, by a series of careful experiments, has determined the form of the function[2] k; and has thus made possible a fairly exact determination of N by the Brownian movement method as applied to gases.

The former equations have been modified so that the task of finding the Brownian movement effect is now much less laborious than before. Also the apparatus has been improved, making it possible to obtain from 1,000 to 6,000 observations on a single particle.

§ 1. DEDUCTION OF THE EQUATION FOR THE DETERMINATION OF N.

It was shown in the article cited above that the law of distribution of the times of fall through a constant distance of a small spherical particle which is subjected to the bombardment of gas molecules is

$$n = \frac{\mathfrak{N}}{2}\sqrt{\frac{h}{\pi}}\int_{t_2}^{t_1}(bt^{-\frac{3}{2}} + Vt^{-\frac{1}{2}})e^{-h/t(b-Vt)^2}dt, \tag{1}$$

where

$$h = \frac{9\pi\mu ak}{4\epsilon} = \frac{3}{2}\frac{\pi\mu akN}{RT}. \tag{2}$$

In this formula n is the number of times, out of $\mathfrak{N}$ observations, that the particle will fall a constant distance b in a time which lies between t_1 and t_2; V is the constant velocity due to gravity and may be found

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[1] PHYS. REV., Aug., 1911, p. 81.

[2] PHYS. REV., Vol. II., p. 109, 1913, gives the method and PHYS. REV., Vol. I., p. 219, 1913, gives the results on k. See also Le Radium, *10*, p. 15, 1913.

experimentally as shown later; h is a constant depending upon the number of molecules N in a gram molecule of the gas, the viscosity coefficient μ, the gas constant R, the absolute temperature T, and the function k as indicated in equation 2. The method of finding a and k will be outlined in another section.

Let the time of fall due to gravity,

$$\frac{b}{V} = t_g. \tag{3}$$

Now the average value of all the observed times of fall below t_g will be given by

$$t_a = \frac{1}{2}\sqrt{\frac{h}{\pi}}\int_{t_g}^{\infty} t(bt^{-\frac{3}{2}} + Vt^{-\frac{1}{2}})e^{-h/t(b-Vt)^2}dt.$$

Change the variable from t to u by the relation $b = Vt + ut$ and we get

$$\overset{-}{t}_a = 2\sqrt{\frac{h}{\pi}}\int_0^{\infty}\frac{2u^2 + 4bV - 2u\sqrt{u^2 + 4bV}}{4V^2}e^{-hu^2}du. \tag{4}$$

Similarly, the average value of all the observed times of fall above t_g is given by

$$\overset{+}{t}_a = 2\sqrt{\frac{h}{\pi}}\int_{-\infty}^{0}\frac{2u^2 + 4bV - 2u\sqrt{u^2 + 4bV}}{4V^2}e^{-hu^2}du. \tag{5}$$

Evaluating (4) and (5) we have

$$\overset{+}{t}_a = \frac{b}{V} + \frac{1}{4hV} + \frac{1}{V^2}\sqrt{\frac{h}{\pi}}\int_0^{\infty} u\sqrt{u^2 + 4bV}e^{-hu^2}du,$$

$$\overset{-}{t}_a = \frac{b}{V} + \frac{1}{4bV} - \frac{1}{V^2}\sqrt{\frac{h}{\pi}}\int_0^{\infty} u\sqrt{u^2 + 4bV}e^{-hu^2}du.$$

Adding we have

$$\frac{\overset{+}{t}_a + \overset{-}{t}_a}{2} = \frac{b}{V} + \frac{1}{4hV^2}. \tag{6}$$

Theoretically this equation in connection with equation (2) could be used to determine N, but the difference between $\left(\frac{\overset{+}{t}_a + \overset{-}{t}}{2}\right)$ and b/v is so small that it is not well adapted for this purpose.

Let $\tau = \left(\frac{\overset{+}{t}_a - \overset{-}{t}_a}{2}\right)$, then

$$\tau = \frac{1}{v^2}\sqrt{\frac{h}{\pi}}\int_0^{\infty} u\sqrt{u^2 + 4bv}e^{-hu^2}du. \tag{7}$$

If the variable is changed by the relation $hu^2 = x$, and the substitution

(8) $$z^2 = 4hbv$$

is made, there results

$$\tau = \frac{2}{\sqrt{\pi}} \frac{t_g}{z^2} \int_0^\infty \sqrt{x + z^2} e^{-x} dx.$$

If we expand the integrand and integrate the resulting series we obtain

$$\tau = \frac{t_g}{z^2} \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-x} \left(1 + \frac{1}{2} \frac{x}{z^2} - \frac{1}{8} \left(\frac{x}{z^2} \right)^2 + \frac{1}{16} \left(\frac{x}{z^2} \right)^3 \cdots \right) dx$$

$$= \frac{t_g}{z} \frac{2}{\sqrt{\pi}} \left(1 + \frac{1}{2z^2} - \frac{1}{4z^4} + \frac{3}{8z^6} + - \right)$$

or

(9) $$z = \frac{t_g}{\tau} \frac{2}{\sqrt{\pi}} \left(1 + \frac{1}{2z^2} - \frac{1}{4z^4} + \frac{3}{8z^6} \right).$$

This expresses z in terms of itself; but inasmuch as the term $1/2z^2$ is usually of the order .02, the formula

$$z = \frac{2}{\sqrt{\pi}} \frac{t_g}{\tau}$$

may be used to get an approximate value of z, and if then this value is substituted in the brackets a more accurate value may be obtained which may again be substituted in the brackets. We thus arrive at a value of z in terms of t_g and τ which can be determined from experiment. Comparing (9) with (8) and (2) we see that

(10) $$N = \frac{RT}{6\pi\mu} \frac{z^2}{akV^2 t_g}.$$

For air this reduces to

$$N = 7.12 \times 10^{12} \{ 1 + .0008(t - 20) \} \frac{p^2}{akV^2 t_g}.$$

Comparing (2), (6) and (9), we get

(11) $$\frac{\overset{+}{t}_a + \overset{-}{t}_a}{2} = t_g + \frac{1}{3} \frac{t_g}{z^2}.$$

This equation is of theoretical interest only and can be checked by experiment. In the observations recorded below, it is nearly always found that $(\overset{+}{t}_a + \overset{-}{t}_a)/2$ is greater than t_g by a quantity of the order of magnitude indicated in this equation.

All of the quantities of the right-hand member of equation (10) can be measured by experimental methods to be described later.

§ 2. METHOD OF CALCULATING a AND k.

Prof. Millikan gives[1] for the value of k,

$$k = \left\{ 1 + \frac{l}{a}(.874 + .32e^{-1.54a/l}) \right\}^{-1} \tag{12}$$

and deduces l from the Boltzman equation $\mu = .3506\rho\bar{c}l$. If we combine this with the pressure equation $p = \frac{1}{3}\rho c^2$ and remember that $\bar{c} = .921c$ we get for air the equation

$$l = \frac{\mu}{.919p\sqrt{\frac{3\rho_0}{p_0}}} = \frac{70.5}{p} \times 10^{-5}\{1 + .00446(t - 20)\}, \tag{13}$$

where p is expressed in centimeters of mercury and t is the number of centigrade degrees above 20° C.

When gravity is the only force acting, we have from Stokes's law of fall

$$mg = 6\pi\mu a k V.$$

Write equation (12) in the form

$$k = \left(1 + \frac{B}{a}\right)^{-1} \tag{14}$$

where

$$B = l(.874 + .32e^{-1.54a/l}),$$

and substitute in the above and solve for a. Then

$$a = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \frac{9}{2}\frac{\mu V}{\rho g}} = -\frac{B}{2} + \sqrt{\frac{B}{2} + .921 \times 10^6 V}. \tag{15}$$

To determine a from (14) and (15) first neglect the exponential term to get an approximate value of B; substitute this in (15) to obtain an approximate value of a; then substitute this in (14) thus giving a more accurate value of B, and repeat the process until the desired accuracy is obtained. In most cases this need only be repeated once and at most twice. a and B are then substituted in 13 to obtain k.

§ 3. DESCRIPTION OF APPARATUS.

The apparatus is similar to Professor Millikan's and to that used in my previous experiments on the Determination of Ne for Gaseous Ioniza-

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[1] Phys. Rev., 1, p. 218, 1913. Also Le Radium, 10, p. 15, 1913.

tion with some important modifications. It consists essentially of four parts, the lighting system, the observing system, the timing system, and the vessel containing the small oil drops suspended in air. The lighting system is the same as used before. The light from a 15-ampere direct current arc is passed through a condensing lens, then through a column of water to absorb the heat, and then between the condenser plates to illuminate the drop (See Fig. *B*, p. 93, PHYS. REV., Aug., 1911). The arc is held in position by means of two tangent screws at right angles, which enables one to adjust it to any desired position.

The observing telescope, which was made by Wm. Gaertner and Company especially for this experiment, is mounted on a cathetometer. In addition to the two tangent screws ordinarily found on this instrument, namely one for changing the heighth and the other for changing the azimuth, the telescope is mounted on a ratchet and pinion which enables one to move it back and forward without changing the magnifying power, *i. e.*, without changing the distance between the objective and eye piece. The objective is corrected for the distance for which it is used, namely about ten inches. In the eye piece is mounted a scale having ten large divisions each divided into five equal parts. After each set of observations this scale is calibrated by comparison with a standard centimeter.

The time is taken by means of a kymograph. The paper which is to receive the record is mounted on two slowly revolving drums which are placed about three feet apart, and it is then covered with lamp black. By means of a signal key the time when the small oil droplet crosses each division in the eye-piece is recorded on the kymograph. Fig. 1 shows a photograph of a very small part of the record after it has been fixed with shellac.

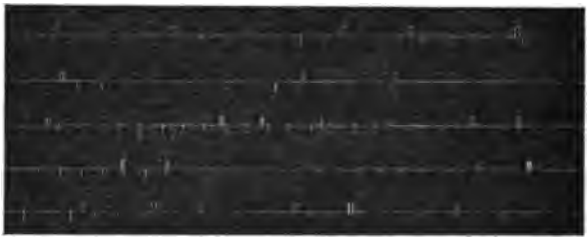

Fig. 1.

The complete record measures 90×8 inches. The time marker was compared with a standard clock and found to agree to one fortieth per cent.

Fig. 2 shows a vertical cross section of the vessel in which the small

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[1] PHYS. REV., Aug., 1911.

oil drops are suspended. The chamber *B* is exhausted to the desired pressure and the stopcock C_2 is closed. A fine spray of liquid vaseline is produced in the bottle *A* by means of an ordinary atomizer. The cock is then opened, permitting part of the spray to pass along the tube into the chamber *B*, and to finally fall through the small hole in the top plate of the condenser into the position indicated in the figure. Here the spray is illuminated and observed as described above. During the observation the three-way stopcock C_3 is turned so that the vessel and pressure gauge *M* are in communication. The entire vessel containing the condenser plates is closed at the top with a bell jar and immersed in a bath of transformer oil. This serves two purposes; it stops small leaks which might exist at the different joints, and keeps the air in the vessel at a constant temperature—thus preventing convection currents.

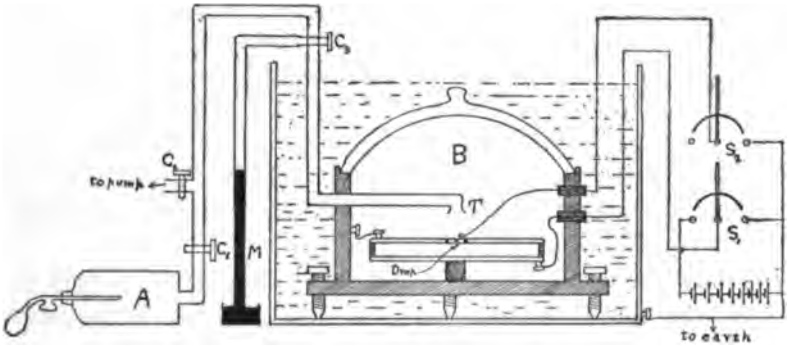

Fig. 2.

In order to control the drop with electric forces, it is necessary that it be charged. To prevent it from remaining uncharged in case it should catch an ion from the air an X-ray bulb is arranged so as to produce intense ionization between the plates making it possible for the drop to become quickly recharged. However it was found that for pressures under 20 cm. of mercury that the drop very seldom caught ions from the air. In fact only two of the drops recorded in this paper changed their charge, although some of them were under observation six hours.

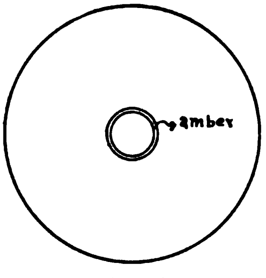

Fig. 3.

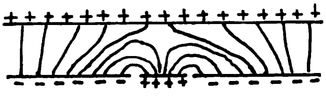

Fig. 4.

To enable one to raise the drop after it had fallen, a condenser of dimensions like that used by Professor Millikan in all the oil drop work was employed. It was found that the small droplets, due to Brownian movements, would sometimes drift to the right or left and finally go out of the beam of light and be lost. To prevent this the top plate was cut into two concentric circular plates which were insulated from each other by a thin strip of amber as indicated in Fig. 3. If a negatively charged drop should drift in any direction whatever from the center of the condenser, the switch S_1, is thrown to the right, and the switch S_2 to the left, producing an electrostatic field similar to that shown in Fig. 4. The drop would be forced back again into the center of the beam of light. The switches would just be reversed for a positively charged drop. This precaution was justified for the drop was shifted from three to ten times during the progress of a set of observations.

With these precautions it was found that a drop could be held under observation for almost an unlimited time. One drop was under observation six hours and if we had so desired we could have held it twice that long as the seeing was just as good at the end as it was at the beginning.

§ 4. THE DATA.

The density of the liquid vaseline was found to be .845 gm./cm.^{3} at 20° C.; the viscosity coefficient of air was taken as .0001826 at 23° C.; and the gas constant R, as 82.9×10^6. Because of the large amount of space that would be taken the complete data is not given but only the final values. Table I. shows a sample of the original data as it was read from the kymograph record. It is the first 100 observation on drop 1. In the first column is recorded the time at which the drop crossed each scale division. It was found impracticable to permit the drop to fall the full fifty divisions, as it would drift slightly out of focus and in the process of refocusing the scale would be shifted. So that the breaks in the column represents periods of refocusing or some other difficulty, while from one break to the next represents a continuous falling of the drop under the influence of gravity. For example, the first time it fell 24 divisions in 71.0 seconds, and was then brought back by the electric field. It then fell 23 divisions in 72.1 seconds; 45 divisions in 131.3 seconds; and 25 divisions in 75.6 seconds; etc. The time of fall t_g for one division due to gravity was obtained by dividing the total distance of fall by the number of divisions crossed, *i. e.*, in this case

$$t_g = \frac{71.0 + 72.1 + 131.3 + 75.6 + \text{etc.}}{24 + 23 + 45 + 25 + \text{etc.}} = 3.092.$$

In the second, third, and fourth columns are recorded the times necessary to travel each scale division, each two scale divisions, and each three scale divisions respectively. These times are derived from the first column in a very obvious manner. The average of all of the values in the first column above 3.09 is found to be 3.587, while the average of those below 3.09 is 2.661. These values are designated $\overset{+}{t}_a$ and $\overset{-}{t}_a$ and are tabulated in Table II. Then

$$\tau = \frac{3.587 - 2.661}{2} = .463 \text{ second.}$$

z can now be computed from equation 9 and is found to be 7.58. If this value is substituted in (10) there results for N, 60.9×10^{22}. The velocity V is simply the fall distance .00916 divided by 3.09 or .002962 cm./sec. From this value a and k can be computed as explained in Section 2. The final values for this drop are tabulated in Table II. It is interesting to note that z for the three different fall distances is proportional to 1, 1.408, and 1.697 *i. e.*, to 1, $\sqrt{2}$, and $\sqrt{3}$ as it should be according to equation (10).

The amount of labor that was necessary to obtain a value of N according to the original method of calculation can be partly realized when it is pointed out that Table I. contains only one seventh of the observations on this drop (Drop 12 has five times as many observations) and that each observation required a separate calculation[1] to obtain a value of u, and finally an average of all of these values of u had to be found. It still requires considerable time to work up the data, but by using the adding machine one can add 1,000 numbers in a comparatively few minutes.

Tables II. to XIII. contain the final values obtained from observations taken during the last three months. This represents all of the data taken during this time with the exception of two drops, the first being discarded because it was lost when only a few observations were taken; the second being discarded because the seeing was poor and consequently inaccurate timing resulted. At the top of each table are given the constants a and k obtained from the velocity V. In the different columns are given the average values obtained from Brownian movements.

Particular attention is called to Drop 12, the last drop which was observed. As is seen from the table we obtained 5,950 observations on the time of fall through ten different fixed distances. In order to bring out more clearly the fact that z is proportional to $\sqrt{t_g}$ the value of $z/\sqrt{t_g}$ is recorded in the 7th column. Although there is some little

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[1] $u = \dfrac{V(t_g - t)}{\sqrt{t}}$ equation 28 of the art. already cited.

TABLE I.

Time.	1 Division.	2 Division.	3 Division.	Time.	1 Division.	2 Division.	3 Division.
				36.10	2.30		8.90
0.20				38.40	2.80	4.90	
3.20	3.00	5.40	7.70	41.20	2.10		
5.60	2.40			43.30	3.90	6.80	
7.90	2.30	5.70	9.7	47.20	2.90	5.60	9.70
11.20	3.40			50.10	2.10		
14.80	3.50	6.30		52.20	3.20		
17.60	2.80		8.40	55.70	3.30	6.20	
20.70	3.10	5.60		59.00	2.90		10.00
23.20	2.50			61.90	3.00		
26.00	2.80	5.50	9.60	64.90	2.90	7.00	
28.70	2.70			67.80	4.10	6.70	9.20
32.30	3.60	6.90		71.90	3.60	6.70	
35.60	3.30		9.70	75.50	3.10		
38.70	3.60	7.40		78.60	2.50	5.30	9.10
43.00	4.30			81.10	2.80		
45.30	2.30	4.80	7.50	83.90	3.00	6.30	
47.80	2.50			86.90	3.30		6.70
49.90	2.10	5.00		90.20	2.70	4.70	
52.80	2.90		8.20	92.90	2.00		
56.10	3.30	5.90		94.90	2.00	4.30	8.60
58.40	2.30			96.90	2.30		
61.00	2.60	6.40	10.20	99.20	3.80	6.30	
64.80	3.80			103.00	2.50		9.00
67.70	2.90	6.40		105.50	3.20	6.30	
71.20	3.50		10.90	108.70	3.10		
				111.80	2.70	4.90	8.10
20	3.20			114.50	2.20		
3.40	3.50	6.70		116.70	2.60	5.90	
6.90	3.50		9.50	119.30	3.30		9.00
10.40	3.90	7.40		122.60	2.90	6.20	
14.30	3.70			125.50	3.30		
18.00	2.20	5.90	9.50	128.80	2.80	4.70	6.90
20.20	3.60			131.60			
23.80	2.60	6.20					
26.40	3.30		8.00	70	1.90	5.00	
29.70	3.60	6.90		2.60	2.90		
33.30	2.30			5.50	2.10	6.00	8.20
35.60	2.80	5.10	9.80	7.60	2.70		
38.40	2.60			10.30	3.30	5.00	
41.00	2.60	5.20		13.60	2.20		9.20
43.60	3.90		10.20	15.80	2.80	6.40	
47.50	3.30	7.20		18.60	3.00		
50.80	2.60			21.60	3.40	6.70	9.00
53.40	3.20	5.80	8.70	25.00	2.80		
56.60	3.10			27.80	3.90		
59.70	3.90	7.00		31.70	3.30	6.30	10.30
63.60	3.10		8.90	35.00	3.00		

Table I.—*Continued.*

Time.	1 Division.	2 Division.	3 Division.	Time.	1 Division.	2 Division.	3 Division.
66.70	3.50	5.60		38.00	3.80	6.10	
70.20	2.10			41.80	2.30	6.90	9.60
72.30		6.00		44.10	4.20		
			9.90	48.30	2.70		
30	3.90			51.00	2.80	6.90	
4.20	2.40			53.80	4.10		10.80
6.60	2.60	5.00	8.40	57.90	4.50	71.0	
9.20	2.80	6.00		62.40	2.60		
12.00	3.20			65.00	3.70	6.40	7.60
15.20	3.90	7.00	8.60	68.70	2.70		
19.10	3.60			71.40	2.20	4.90	
22.20	2.20	5.30		73.60	2.70		
24.40	3.10		7.20	76.30			
27.50	2.80	6.20					
30.30	3.40						
33.70	2.40	4.70					

variation from a constant it seems quite remarkable to think that an apparently haphazard set of numbers unite on the average according to such a definite law.

Table II.

Drop No. 1.

Pressure = 24.8 cm. Temperature = 18.8° C.
V = .00296 cm./sec. $a = 4.04 \times 10^{-5}$ cm. k = .611.

b	t_g	$\overset{+}{t}_a$	$\bar{t}_a$	τ	s	$N + 10^{22}$	No. Obs.
1 × .00916	3.92	3.587	2.661	.463	7.58	60.9	707
2 × .00916	6.694	6.811	5.515	.648	10.81	61.9	355
3 × .00916	9.276	10.052	8.444	.804	13.03	60.2	239

Weighted mean $N = 61.0 \times 10^{22}$.

Table III.

Drop. No. 2.

Pressure = 24.8 cm. Temperature = 18.8° C.
V = .002770 cm./sec. $a = 3.93 \times 10^{-5}$ cm. k = .604.

b	t_g	$\overset{+}{t}_a$	$\bar{t}_a$	τ	s	$N + 10^{22}$	No. Obs.
1 × .00916	3.306	3.945	2.893	.526	7.16	65.0	312
2 × .00916	6.612	7.589	5.901	.844	9.00	51.6	150
3 × .00916	10.926	9.057	9.057	.934	12.02	61.2	95

Weighted mean $N = 60.7 \times 10^{22}$.

TABLE IV.

Drop. No. 3.

Pressure = 31.7 cm. Temperature = 19.5° C.

V = .001465 cm./sec. $a = 2.79 \times 10^{-5}$ cm. k = .577.

b	t_g	$\overset{+}{t}_a$	$\overset{-}{t}_a$	τ	z	$N + 10^{22}$	No. Obs.
1 × .00918	6.27	8.11	4.85	1.63	4.44	64.7	208
2 × .00918	12.54	14.80	10.48	2.16	5.62	72.1	102
3 × .00918	18.81	22.44	16.38	3.03	7.08	54.8	70

Weighted mean $N = 64.8 \times 10^{22}$.

TABLE V.

Drop. No. 4.

Pressure = 12.25 cm. Temperature = 20.0° C.

V = .00303 cm./sec. $a = 3.12 \times 10^{-5}$ cm. k = .348.

b	t_g	$\overset{+}{t}_a$	$\overset{-}{t}_a$	τ	z	$N + 10^{22}$	No. Obs.
1 × .00918	3.033	3.834	2.401	.716	4.87	55.8	338
2 × .00918	6.06	7.108	5.237	.936	7.47	65.8	168
3 × .00918	9.09	10.41	7.97	1.220	8.47	56.5	111

Weighted mean $N = 58.8 \times 10^{22}$.

TABLE VI.

Drop. No. 5.

Pressure = 13.52 cm. Temperature = 19.3° C.

V = .00393 cm./sec. $a = 3.99 \times 10$ cm. k = .442.

b	t_g	$\overset{+}{t}_a$	$\overset{-}{t}_a$	τ	z	$N + 10^{22}$	No. Obs.
1 × .00914	2.325	2.735	1.997	.369	7.19	58.2	1010
2 × .00914	4.650	5.172	4.146	.516	10.22	58.9	508
3 × .00914	6.975	7.57	6.32	.635	12.65	60.2	336

Weighted mean $N = 58.8 \times 10^{22}$.

TABLE VII.

Drop No. 6.

Pressure = 15.65 cm. Temperature = 18.6° C.

V = .00321 cm./sec. $a = 3.69 \times 10^{-5}$ cm. k = .460.

b	t_g	$\overset{+}{t}_a$	$\overset{-}{t}_a$	τ	z	$N + 10^{22}$	No. Obs.
1 × .00914	2.844	3.410	3.410	.500	6.51	60.7	968
2 × .00914	5.688	6.404	4.962	.721	8.97	57.8	481
3 × .00914	8.532	9.450	7.700	.875	11.05	58.4	319

Weighted mean $N = 59.0 \times 10^{22}$.

TABLE VIII.

Drop No. 7.

Pressure = 19.4 cm. Temperature = 19.1° C.

V = .00332 cm./sec. $a = 4.11 \times 10^{-6}$ cm. k = .551.

b	t_g	$\overset{+}{t}_e$	$\overset{-}{t}_e$	τ	s	$N + 10^{22}$	No. Obs.
1 × .00918	2.77	3.19	2.40	.395	8.00	65.8	561
2 × .00918	5.54	6.15	5.00	.575	10.88	60.6	283
3 × .00918	8.31	9.14	7.67	.735	12.93	57.2	188
4 × .00918	11.08	11.80	10.36	.780	16.05	66.1	142
5 × .00918	13.85	14.87	13.00	.935	16.75	57.7	109
6 × .00918	16.62	17.77	15.73	1.020	18.40	58.0	94

Weighted mean $N = 62.2 \times 10^{22}$.

TABLE IX.

Drop No. 8.

Pressure = 12.25 cm. Temperature = 20.0° C.

V = .00313 cm./sec. $a = 3.21 \times 10^{-6}$ cm. k = .357

b	t_g	$\overset{+}{t}_e$	$\overset{-}{t}_e$	τ	s	$N + 10^{22}$	No. Obs.
1 × .00918	2.932	3.683	2.384	.650	5.24	59.5	696
2 × .00918	5.864	6.792	5.030	.881	7.66	63.4	364
3 × .00918	8.796	9.860	7.774	1.043	9.68	67.5	229
4 × .00918	11.728	13.04	10.39	1.326	10.04	54.5	180
5 × .00918	14.660	16.129	12.972	1.578	10.54	49.1	136
6 × .00918	17.592	19.15	16.05	1.55	12.98	60.8	108

Weighted mean $N = 60.1 \times 10^{22}$.

TABLE X.

Drop No. 9.

Pressure = 19.5 cm. Temperature = 20.2° C.

V = .002420 cm./sec. $a = 3.31 \times 10^{-6}$ cm. k = .482.

b	t_g	$\overset{+}{t}_e$	$\overset{-}{t}_e$	τ	s	$N + 10^{22}$	No. Obs.
1 × .00918	3.795	4.708	3.109	.800	5.44	59.6	733
2 × .00918	7.590	8.700	6.500	1.100	7.84	61.8	378
3 × .00918	11.385	12.637	10.093	1.272	10.15	68.9	243

Weighted mean $N = 61.8 \times 10^{22}$.

TABLE XI.

Drop No. 10.

Pressure = 13.34 cm. Temperature = 20.5° C.

V = .003302 cm./sec. a = 3.39 cm. k = .400.

b	t_g	$\overset{+}{t_a}$	$\overset{-}{t_a}$	τ	s	$N + 10^{22}$	No. Obs.
1 × .00918	2.78	3.347	2.282	.532	5.97	58.8	586
2 × .00918	5.56	6.308	4.770	.769	8.20	55.4	301
3 × .00918	8.34	9.303	7.507	.898	10.60	61.7	192

Weighted mean $N = 58.4 \times 10^{22}$.

TABLE XII.

Drop No. 11.

Pressure Temperature

V = .003207 cm./sec. $a = 3.41 \times 10^{-5}$ cm. k = .395.

b	t_g	$\overset{+}{t_a}$	$\overset{-}{t_a}$	τ	s	$N + 10^{22}$	No. Obs.
1 × .00918	2.862	3.496	2.336	.580	5.67	58.2	503
2 × .00918	5.724	6.604	4.962	.821	7.95	57.0	245
3 × .00918	8.586	9.596	7.614	.991	9.83	58.2	159

Weighted mean $N = 57.9 \times 10^{22}$.

TABLE XIII.

Drop No. 12.

Pressure = 24.29 cm. Temperature = 21.1° C.

V = .00251 cm./sec. $a = 3.63 \times 10^{-5}$ cm. k = .572.

b	t_g	$\overset{+}{t_a}$	$\overset{-}{t_a}$	τ	s	$\frac{s}{\sqrt{t_g}}$	No. Obs.
1 × .00922	3.672	4.365	3.078	.6435	6.50	3.39	2056
2 × .00922	7.344	8.349	6.480	.9345	8.93	3.31	1028
3 × .00922	11.016	12.237	9.926	1.156	10.83	3.27	681
4 × .00922	14.688	15.999	13.333	1.333	12.48	3.24	514
5 × .00922	18.360	19.916	16.915	1.501	13.85	3.23	393
6 × .00922	22.032	23.423	20.387	1.513	16.44	3.50	342
7 × .00922	25.704	27.630	23.918	1.856	15.67	3.10	268
8 × .00922	29.376	30.981	27.309	1.831	18.12	3.34	252
9 × .00922	33.048	35.143	30.941	2.10	17.85	3.11	226
10 × .00922	36.720	38.92	34.73	2.09	19.86	3.28	190

Weighted mean $N = 60.0 \times 10^{22}$. Total No. Obs. = 5,950.

Table XIV. contains a summary of the 12 drops with the final weighted mean values of N which were obtained by assigning weights according to the number of observations that were taken. They are arranged in ascending values of l/a in order to show that N has no tendency to gradually increase or decrease, although k changes from .357 to .611. This is a verification that Millikan's expression for k is correct, at least for this region and for air.

TABLE XIV.

Drop No.	$a \times 10^5$	$z \times 10^5$	$1/a$	k	$N + 10^{22}$	No. Obs.
4	3.12	5.75	1.840	.348	58.8	617
8	3.21	5.75	1.790	.357	60.1	1,713
10	3.39	5.30	1.562	.400	58.4	1,079
11	3.41	5.30	1.553	.395	57.9	907
5	3.99	5.19	1.300	.442	58.8	1,854
6	3.69	4.48	1.215	.460	59.5	1,768
9	3.31	3.61	.917	.482	61.8	1,354
7	4.11	3.62	.880	.551	62.2	1,377
12	3.64	2.91	.800	.572	60.0	5,950
3	2.79	2.22	.795	.577	63.8	380
2	3.93	2.83	.710	.604	59.3	557
1	4.04	2.83	.700	.611	61.0	1,281

Weighted mean = 60.0×10^{22}. Total No. Obs. = 18,837.

As indicated heretofore all of the above computations have been made on the assumption that $R = 82.9 \times 10^6$ and $\mu_{23} = .0001826$. Professor Millikan calls my attention to the fact that the correct value of R is 83.15×10^6 and that the most probable value of μ_{23} is .0001824.[1] The introduction of these changes increases the above value of N by .41 per cent. The probable error computed on the ordinary way from column 6 of Table XIV. is $.3 \times 10^{22}$ or .5 per cent. This represents the "probable error" in the z^2 term of equation 10. The *uncertainty* in this term, the mean being obtained from but 12 numbers, is of course considerably larger. Professor Millikan estimates the uncertainty in his determination of the k term of (10) [more accurately the ak term, since this is the quantity actually given by his experiments] for these values of l/a, at about 1 per cent. So that we may consider that the uncertainty in the above determination of N by the Brownian movement method does not exceed 2 per cent. The value actually obtained, viz., 60.3×10^{22} differs by but .5 per cent from the value obtained by Professor Millikan, namely, 60.62×10^{22}.

The results of this investigation show then that the number of molecules in a gram molecule of air is

$$N = 60.3 \times 10^{22} \pm 1.2 \times 10^{22}.$$

In conclusion I wish to express my indebtedness to Mr. Carl F. Eyring, who has assisted me in taking a large portion of the observations and in computing the average values.

Physical Laboratory,
Brigham Young University,
July 17, 1914.

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[1] Ann. d. Phys., 41, p. 759, 1913.

COLOR ANALYSES OF TWO COMPONENT MIXTURES.[1]

BY L. A. JONES.

IT is a well-known fact in physiological optics that if two or more colors are viewed in rapid succession the resultant sensation will be a color which is a mixture of the colors so viewed. For instance, if a disk of which one half is red and the other green be rotated rapidly, the entire disk will appear to be yellow. This mixing is due to what is known as the persistence of vision; that is, the sensation produced by a given stimulus continues to exist for a short period of time after the stimulus has ceased to act.

Now if we have a white screen illuminated by white light from a projection apparatus and in the path of the light is placed a rapidly rotating disk composed of two or more light filters such as colored glass or dyed gelatine film, the screen will appear to be illuminated by light of a color which is a mixture of the colors in the rotating disk. In case the disk is made up of two filters that in color are exactly complementary and if the densities of the filters and the sizes of the sectors are in the proper ratio to each other, the screen will appear white, just as with no disk in the path of the illuminating beam. When this condition is altered in any way, either by changing the colors of the sector so that they are no longer complementary, or by altering the relative sizes or densities of the filters, the screen will appear to be illuminated by light the color of which will depend upon the way in which the disk has been altered. The work of which this paper is a report was carried out in order to determine definitely the hue and purity of all the colors that it is possible to produce in this way with a given pair of color filters. The method used was to produce by means of an adjustable rotating disk made up of the filters to be examined all possible combinations and to analyze the resulting colors with a colorimeter of the monochromatic type.

A diagram of the disk used for producing the mixtures is shown in Fig. 1. Two disks of brass were cut as shown at *a*, Fig. 1. The openings *AA′ BB′* were each 80° in angle, the metal strips between the openings being each 10°. These two disks could be rigidly fastened together by placing the small nuts, *N*, on screws projecting from one of the disks through holes in the other. The filter film was placed between these disks so as to

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[1] Communication No. 11 from the Research Laboratory of the Eastman Kodak Company.

cover the openings and when the screws were tightened was held so rigidly that the disks could be rotated very rapidly without displacing the filters.

A third disk was cut as shown at *B*, Fig. 1, the angles of the openings *CC′* each being 90°. This disk was placed with the two carrying the filters upon the shaft of a small motor and secured in position with a collar and nut.

A scale was laid off on one of the disks as shown in *a*, and one index line, *I*, on the other, *b*. When the index was set at 0 on the scale, the openings *BB′* were completely covered and the ones *AA′* fully open. Now it will be seen that if the openings *AA′* was filled with a filter of one color, and *BB′* with some other, that by moving the index along the scale from 0 to 1.0 all possible combinations of the two filters were obtained. The composition of the mixture produced by the disk was computed as follows:

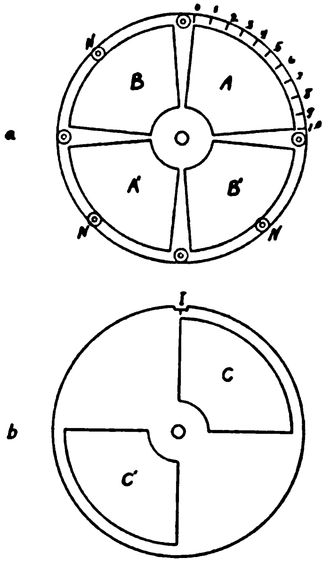

Fig. 1.

Let *S* the setting of the index line on the scale,
Ta the transmission of the filter in openings *AA′*,
Tb the transmission of the filter in *BB′*,
A the ratio of the intensity of the light through the *AA′* filters to the intensity of the mixture produced (in per cent.),
B ratio of the intensity of the light through the *BB′* filters to the intensity of the mixture produced (in per cent.).

$$B = \frac{S \cdot T_b \cdot 100}{S \cdot T_b + (1 - S)Ta} = 100 - A,$$

$$A = \frac{(1 - S)Ta \cdot 100}{S \cdot T_b + (1 - S)Ta} = 100 - B.$$

The mixtures produced were analyzed with a colorimeter of the monochromatic type as designed by P. G. Nutting,[1] and built by Adam Hilger of London. The principle involved in making an analysis of a color with this instrument is that any color can be matched, as far as the visual sensation is concerned, by mixing a pure spectral light of some definite wave length with the proper amount of white light. The only

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[1] P. G. Nutting, Zeit. Instrumentenjunde, Vol. 33, p. 20, Jan., 1913.

exception to this rule is in case the unknown color is a purple, that is, some mixture of red and blue, and in that case if the proper amount of a pure spectral hue be mixed with the unknown a sensation white will be produced.

The spectral hue with which standard white must be mixed to match a given color is known as the dominant hue of the color and the intensity of the standard white expressed as a percentage of the sum of the intensities of the hue and white added together to match the color is known as the per cent. white of the color. Any color then can be accurately analyzed by this method in terms of the wave-length of the dominant hue and the per cent white. In case the color is a purple the terms in which the analysis must be expressed are the wave length of the complementary of the dominant hue and the per cent. hue. The term per cent. hue is defined as the intensity of the pure hue expressed as a percentage of the sum of the intensities of that pure hue and the unknown that must be added together to produce sensation white.

This method of color analysis involves the establishment of a standard of white light. The most logical definition of this is that standard white shall be direct sunlight near noon of a clear day. A large number of measurements extending over a considerable period of time show that the light as above specified is constant in color,—no measurable variations having been observed.

For a complete specification of a color one other factor must be taken into consideration. This third term may be said to be the intensity factor, being for a source the intensity of the light emitted, for a transmitting screen the transmission, and for a reflecting surface the reflecting power. The measurement of this term involves other methods and other apparatus than the colorimeter as used in this work. The nature of a color is by this method of analysis expressed by two terms, the wave length of the dominant hue and the per cent. white, the measurement of which depends in no way upon arbitrary, non-reproducible standards such as are frequently used in colorimeters of other types.

In Fig. 2 is given a diagram showing the arrangement of the essential parts of the instrument. The body of the instrument, *O*, is a rectangular metal box which serves as a triple collimator. On one end of this are attached three metal tubes in each of which is placed a pair of nicol prisms. One prism of each pair is fixed in position and the others can be rotated by means of an attached graduated quadrant which moves past a fixed index line, thus the intensity of the light transmitted by each pair can be changed at will, and by reading the scales the ratio of the incident to the transmitted intensities can be determined for each pair. At the other end of box O are placed the three collimating lenses, L_1, L_2, L_3. P is a dispersing prism of the constant deviation type actuated by a screw which carries the drum, D, graduated so as to read directly

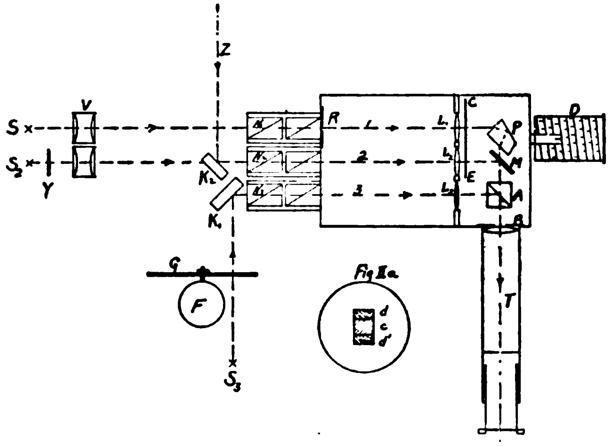

Fig. 2.

in wave-lengths. M is a plane parallel plate of colorless optical glass set so as to reflect a part of the light coming through N_2L_2 along the axis of the observing telescope, T. A is a Lummer-Brodhun cube of special design, the diagonal faces being so ground that the field as seen through the rectangular diaphragm, B, of the observing telescope is as shown in Fig. 2*a*. The portions of the field, *d*, *d'*, are illuminated by the light which comes through N_3L_3 and is totally reflected at the diagonal face of cube A, while the central part of the field, *c*, is illuminated by light which comes through collimators 1 or 2, or by a mixture of the two beams, and is transmitted by the cube A, where the diagonal faces are in optical contact.

This particular type of field is extremely well adapted for use in this instrument. In making a setting the color of field *c* is to be matched with the color of the fields *dd'*. Suppose that the attention is fixed on the dividing line between *c* and *d*, and that the adjustments are made so that there is a just perceptible color difference between fields *c* and *d*. Now if the attention be shifted to the line between *c* and *d'*, the lack of color match will appear much greater than before. This is due to fatigue and the existence of after image in the retina of the observer's eye. By taking advantage of this effect and shifting the attention from one dividing line to the other while making a setting, the color match can be made with greater precision than with other types of fields.

The observing telescope is provided with a pin-hole ocular, the diameter of the opening being about 1.5 mm. The focal lengths of the lenses and the dimensions of the instrument are such that if an image of a light source is formed on the slit, *R*, the central portion of the field *c*, will be illuminated by monochromatic light, the wave-length of which can be adjusted to any desired value by rotation of the drum, *D*.

Now if white light is admitted through N_2, it will be mixed with the light of pure hue at *M* and the field *c* will be illuminated by this mixture. The ratio of the intensities of the two beams thus mixed can be given any desired value by rotating the movable prisms in N_1 and N_2, and the value of the ratio determined by reading the attached scales. The light of unknown color that is to be analyzed is admitted through N_3 and illuminates the field *dd′*. The intensity of this beam can be changed by rotating the nicol in N_3.

At *CE* is a thin metal plate used when making the per cent. white determination, which is so attached to the frame of the instrument that it can be raised out of the field while the color match is being made and lowered into position when it is desired to make the per cent. white reading. This plate is of such shape that when in position it cuts off the lower half of the beam coming through L_1, and the upper half of that coming through L_2. In this way the upper half of the field *c*, Fig. 2*a*, is illuminated by light from N_1L_1, and the lower half by that from N_2L_2. Then by adjusting the movable nicols in N_1 and N_2 a photometric balance between the two halves of field *c* can be obtained and the ratio between the intensities of the pure hue and the white of a given mixture can be determined.

The measurement of the third term necessary for the complete specification of a color involves the use of some form of photometer. In this work the transmissions of the color filters used were measured on a 12-foot bench photometer equipped with a Martens flicker photometer head. The difficulties in measuring the transmission of a colored filter are the same as those which occur in the photometry of colored lights. It has been shown by Ives and others that the flicker method gives the most reliable results in heterochromatic photometry and hence this method was adopted as being best suited for the determination of the transmissions of the filters used in this investigation. The transmissions were determined for the light of the carbon arc which was also used in producing the color mixture.

The arrangement of the colorimeter as used in this work is shown in Fig. 2. *S* is a Nernst glower, one image of which is formed by the condenser *V* upon the slit *R*. A beam of sunlight *Z*, was thrown by means of a heliostat upon a block of magnesium carbonate, K_2, in front of N_2.

This was the source of the standard white light used when possible. On cloudy days a Nernst glower, S_2, screened by a blue filter, Y, to match standard white was used as a source of white light. Several series of measurements were made with both sunlight and the screened glower as the source of standard white and in all cases the differences were so small as to be negligible, being less than the variations liable to occur due to differences in the filter film.

The color to be analyzed was produced by projecting a beam of light from a direct current carbon arc, S_3, through the mixing disk, G, which was rotated at high speed by the motor, F, on the block of magnesium carbonate, K_1, placed in front of N_3. The arc used was an automatic Weuler with 7 mm. carbons taking a current of about 8 amperes The carbon arc is almost always used in practical work as a light source for projection, and as it was desired to imitate practical conditions as nearly as possible the arc was used instead of standard white light which should be used in case the problem is one involving only a high precision analysis of a colored transmitting screen or reflecting surface.

In making the analysis of a given mixture the intensities of the three beams of light 1, 2, and 3 were adjusted by rotating the prisms in N_1, N_2, and N_3, and the wave length of the pure hue was varied by turning D until the field C exactly matched the fields dd' in color and intensity. (It is very important that an intensity match be maintained in order that the color match may be made with precision.)

When a satisfactory match was obtained the wave length of the dominant hue was read from the drum D, and the reading, D_h, of the scale attached to the movable nicol in N was recorded. Then the metal plate CE was dropped into position, the light coming through N_3L_3 cut off, and without changing the setting of the scale on N_2 a photometric balance was made between beams 1 and 2 by rotating the nicol in N_1, and the reading, D_w, of the attached scale recorded. From the readings thus obtained, *i. e.*, D_H and D_w the per cent. white was computed.

TABLE I.

No.	Approx. Color.	Hue ($\mu\mu$).	% W	Tr (%)
22	Orange Red	604.5	3.0	47
25	Red	624.0	3.3	24
44	Blue Green	497	28.0	16
59	Green	547	13.0	22
60	Green	527	27.5	24
94	Red	615	4.0	30
IX	Blue Green	506	61.0	58
2-(IX)	Blue Green	502	53.0	40
4-(IX)	Blue Green	498	46.5	24

The filters used were standard Wratten light filters of dyed gelatine and are designated by catalog number.[1] In Table I. are given the color analyses of the filters used in this investigation.

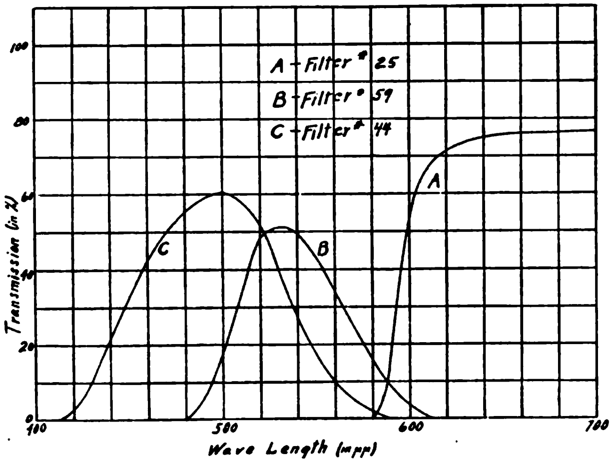

Fig. 3.

In Figs. 3, 4 and 5 are given the spectrophotometric curves of the filters used. The measurements were made with a spectrophotometer of the Hüfner type built by Hilger. The results are plotted with the transmission in per cent. as ordinates, and the wave lengths in — as abscissæ.

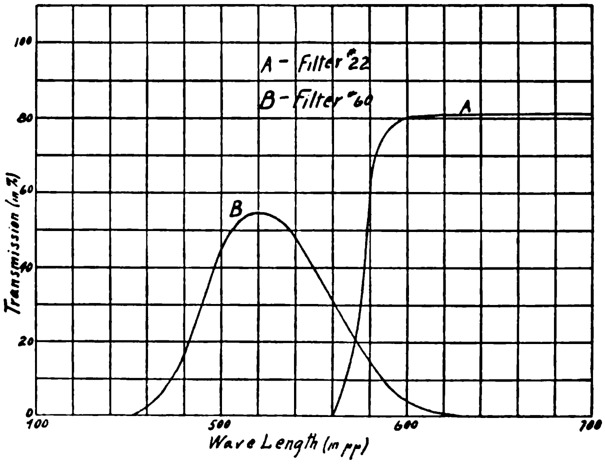

Fig. 4.

The values of the transmission are subject to a probable error of about 2 per cent.

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[1] Atlas of Absorption Spectra, by C. E. Kenneth Mees.

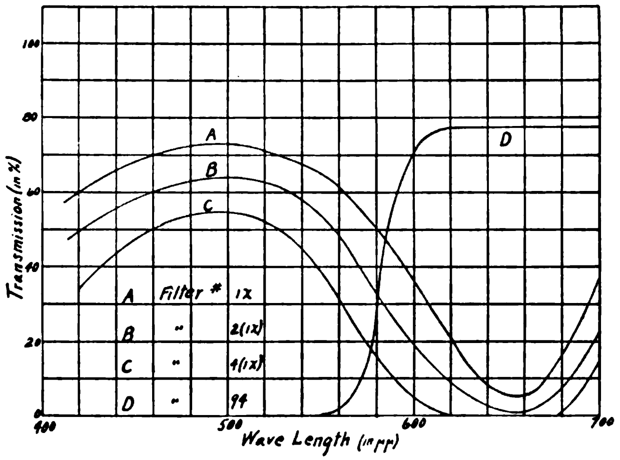

Fig. 5.

In making the color analyses of the mixtures the pair of filters to be used were placed in the disk as previously described and a set of readings taken at successive points from 0 to 1.0 on the scale. This procedure was repeated two to five times on each pair of filters. After computing the readings for each run the mean of all the results for a given scale setting was taken and the results plotted.

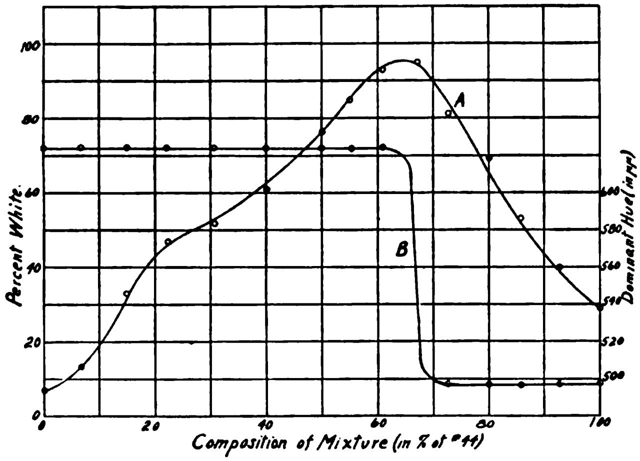

Fig. 6.

In Table II. is given a typical set of measurements including three complete runs made on one pair of filters. This is given to show the agreement of the analyses obtained in the runs made on different days. Runs No. 1 and No. 2 were made with the screened glower as a source of standard white light, No. 3 with sunlight. It will be seen that the agreement obtained is quite good, and that the screened glower answers very well as a source of standard white for work of this kind.

The means of the measurements on this pair of filters, No. 44 and No. 25, are plotted as curves in Fig. 6. The abscissæ are the composition of the mixture expressed in per cent. of filter No. 44. The ordinates on the left are per cent. white referring to curve *A*, those on the right being wavelength of the dominant hue (in $\mu\mu$) referrring to curve *B*. The values of the ordinates of the two curves at per cent. No. 44 0 give the color analysis of the filter No. 25, and at per cent. No. 44 100 the analysis of No. 44. The analysis of any mixture of the two is given by the ordinates of the two curves at the abscissa value representing the given composition.

TABLE II.

Comp. of Mixture.			Dominant Hue ($\mu\mu$).				Per Cent. White.			
S	Per Cent. No. 44.	Per Cent. No. 25.	Run No. 1.	No. 2.	No. 3.	Mean.	Run No. 1.	No. 2.	No. 3.	Mean.
0.00	0	100	624	624	623	623.6	2.4	4.2	3.6	3.4
0.10	6.9	93.1	624	624	623	623.6	12.8	12.0	15.3	13.4
.20	14.3	867	624	623	622	6230	33.8	29.7	31.7	31.7
.30	22.2	77.8	624	624	623	623.6	43.6	47.4	47.9	46.3
.40	30.8	69.2	624	624	623	623.6	51.4	53.8	48.0	51.1
.50	40	60	624	623	623	623.0	61.1	66.9	57.4	61.8
.60	50	50	623	23	22	622.6	76.8	75.2	77.7	76.6
.65	55.3	447	624	22	22	622.6	83.8	81.8	88.2	84.8
.70	60.8	392	630	22	22	6216	92.2	92.7	95.0	93.3
.75	66.7	333	—	—	—	—	96.2	94.2	92.3	94.6
.80	72.7	273	497	498	498	497.6	80.7	80.5	80.0	80.7
.85	79.1	208	497	497	498	497.3	69.8	69.9	68.8	69.5
.90	85.7	14.3	498	497	499	498.0	54.4	54.8	50.6	53.3
.95	92.8	7.2	498	497	498	497.6		—	40.6	40.6
1.00	100	0	498	497	497	497.3	29.1	27.1	28.5	28.2

It will be seen from these curves, that as light though No. 44 was added in increasing amount to the light transmitted by No. 25, that the resultant mixture increased in per cent. white to a max. of about 95 per cent. at per cent. No. 44 65 and then decreased. The dominant hue of the mixture did not change until the per cent. white had reached a maximum and then it shifted suddenly from the dominant hue to filter No. 25 to that of filter No. 44. Thus with this pair of filters it was possible to obtain colors of only *two* dominant hues, but the per cent. white varied from that of the separate filters to 95 per cent. At the point per cent. No. 44 65 the mixture was so near white that the dominant hue was indeterminate, in fact, any color having 95 per cent. white shows a barely perceptible difference from pure white. These filters then are very nearly complementary to each other. That is the light transmitted by them when mixed in proper ratio produces very nearly a sensation white.

The results obtained with the other filters are given in the form of curves, Figs. 7 to 12, plotted as in Fig. 6. Each point plotted represents the mean of several determinations. The variations in the separate readings taken at a given point were of the same order as those shown in Table II.

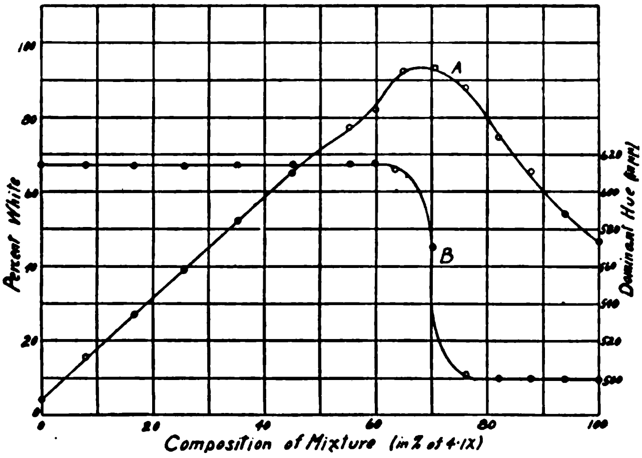

Fig. 7.

In Fig. 7 are given the analyses of the mixtures produced with filters No. 94 and No. IX. Four thicknesses of No. IX were used against one of No. 94. The maximum per cent. white was 94. This combination is very nearly complementary, but not so closely so as the pair used in Fig. 6.

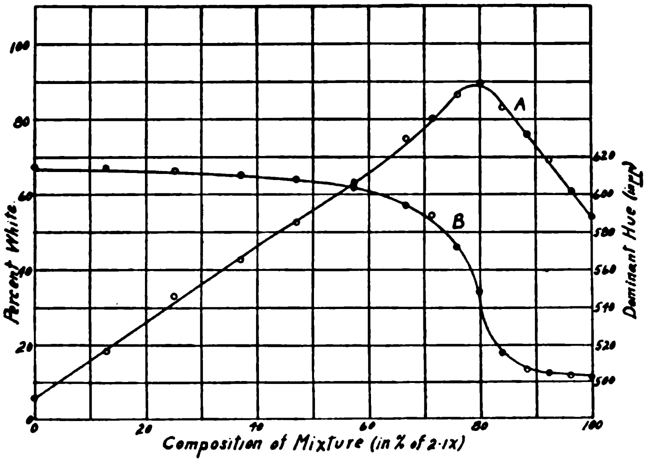

Fig. 8.

Here again the entire range of mixtures produced colors of only two hues, but a wide variation in purity. The change from one hue to the other was very sudden and occurred at the point of maximum per cent. white.

In Fig. 8 the analyses were obtained by using two thicknesses of No. IX. against one of No. 94. Here the maximum per cent. white obtained was 90, and the per cent. white curve, *A*, not so flat on top as in Fig. 6. That is, the range of mixtures that give high per cent. white values was less than with filters that are more nearly complementary. It will be noted also that the change in hue was more gradual, but that here again, the maximum rate of change in hue was at the point of maximum per cent. white. With this pair then colors of all hues from 504 $\mu\mu$ to 615 $\mu\mu$ could be produced, but in the region of hue 550 $\mu\mu$ the colors were very dilute; that is, of high per cent. white.

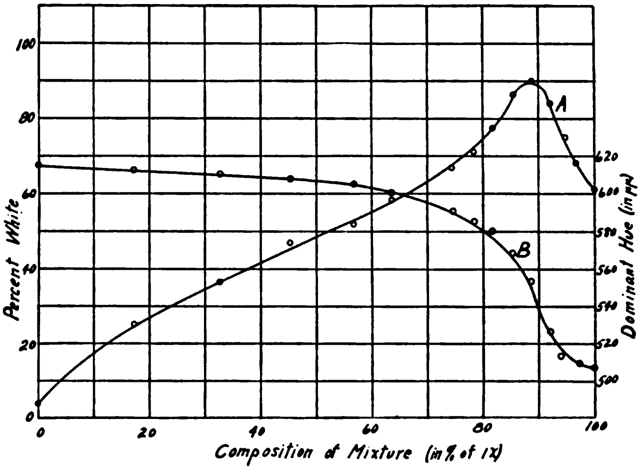

Fig. 9.

In Fig. 9 the analyses are of the mixture obtained with one thickness of No. IX, with one of No. 94. Here the region of high per cent. white was narrower than in any of the preceding cases, the maximum value being 90 per cent. The change in hue was more gradual than before, but again the maximum rate of change in hue was at the position of maximum per cent. white.

It will be noted that in the last three cases discussed, the same red component, No. 94, was used with one, two and four thicknesses of No. IX. As the number of thicknesses used was increased, the total transmission and the per cent. white of the resulting filter decreased, and the wave length of the dominant hue shifted (see Table I.). These changes were accompanied by the changes in the color analyses curves previously referred to in the discussion of Figs. 7, 8, and 9.

In Fig. 10 are given the results obtained with filters No. 22 and No. 60. In this case the filters were far from being complementary, and no mixture produced a color of high per cent. white. The hue curve is almost a straight line, as is also the per cent. white curve. With this pair colors of all hues between those of the component filters were produced, and the per cent. white was always less than that of the green component No. 60.

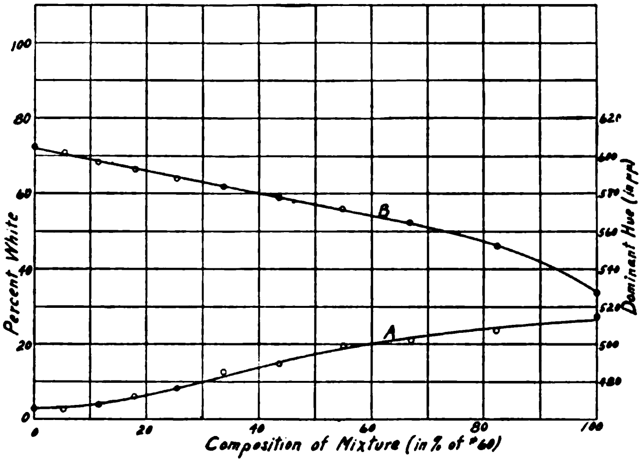

Fig. 10.

In Fig. 11 the curves show the analyses obtained with filters No. 25 and No. 59. Both curves are straight lines to within the possible errors of measurement. The changes in hue and per cent. white were uniform throughout the entire range. The highest per cent. white was that of the

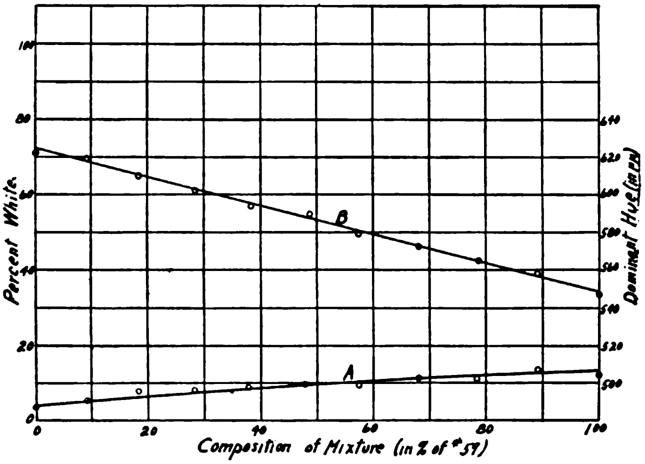

Fig. 11.

green component itself. With this pair colors of all hues between those of the two components were produced.

Conclusions.—The conclusions to be drawn from the results are:

It is not possible with two filters that are complementary or nearly so to produce mixtures that show appreciable color of more than two dominant hues, those hues being the dominant hues of the filters themselves.

With a pair of filters of the tri-color type such as No. 25 and No. 59 it is possible to produce mixtures having low per cent. white values and of all hues lying between the dominant hues of the filters themselves but no mixture can be produced with such a pair that has a high per cent. white value.

This method of analysis affords a very good means of showing how closely any two filters approach the condition of being complementary to each other. The per cent. white curve, in case the filters are exactly complementary, must reach a maximum of 100 per cent. and at the abscissa value corresponding to this maximum the hue curve must change abruptly from the dominant hue of one of the components to that of the other.

The results given are confined to mixtures produced by red and orange red with blue and blue green. A large amount of work remains to be done with other colors, and the author hopes at some time to do something further along this line.

The author wishes at this time to thank Mr. F. A. Elliot for his valuable aid in the computing of data and in the plotting of the results presented in this paper.

THE INFLUENCE OF ANNEALING ON THE CHARACTERISTICS OF LIGHT-SENSITIVE SELENIUM.

By E. O. Dieterich.

AN extension, by Brown and Sieg,[1] of the work of Pfund in analyzing selenium cells by means of the spectrum revealed the fact that, classified according to their wave-length sensibility curves, a great number of different types of selenium cells were possible. Previous investigations by Pfund,[2] Stebbins,[3] Brown and Sieg,[4] and Nicholson[5] pointed to the conclusion that a typical wave-length sensibility curve existed which was essentially the same for all selenium cells, for, in all cases recorded, a maximum in the red end of the spectrum was found. The cells investigated were, in general, made by either Giltay or Ruhmer. The process of making these is a trade secret, but it is quite likely that the same method of crystallization is followed in all the cells of either one of the above types. Pfund, in his earlier experiments, used some cells of his own construction; these, too, were all crystallized in the same manner.

In some later experiments by Brown and Sieg, cells prepared by the author were used. Sensibility curves were obtained which differed widely, some showing a maximum in the red, some having hardly any sensitiveness in the red but a maximum in the blue. An analysis of the data covering the method of construction of these revealed the fact that the crystallization of the selenium took place under different conditions in the different samples. This immediately suggested the possibility that a better knowledge of the conditions governing the construction of light sensitive selenium cells might be of assistance in explaining the action of light sensitive selenium. With this purpose in view, the investigation summarized in this paper was carried out.

This paper will describe the method followed in making the cells, and will show that the resistance and the shape of the wave-length sensibility curves of selenium cells can be controlled by varying the process of annealing. A simple explanation for the different types resulting will also be offered.

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[1] Phys. Rev., Series 2, IV. p., 48, 1914.

[2] Phys. Rev., XXVIII., p. 324, 1909; XXXIV., p. 370, 1912; Phil. Mag., VII., p. 26, 1904.

[3] Astrophys. Jour., XXVII, p. 183.

[4] Phys. Rev., Series 2, II., p. 487, 1913.

[5] Phys. Rev., Series 2, III., p. 1, 1914.

METHOD OF MAKING.

The cells, of which about 40 were constructed for this investigation, were all of the Bidwell type, *i. e.*, two parallel wires were wound spirally around an insulating form and the spaces between the wires filled with selenium.

The selenium used was some in stick form from Merck. No steps were taken to further purify it, since it is, according to other observers, quite pure. Moreover, Pfund[1] has shown that the presence of metallic selenides does not affect the shape of the wave-length sensibility curve. In a few samples made, rather impure selenium was used, and the cells resulting were not as satisfactory as those made with selenium from Merck. The resistance of these was, in general, very high, and they were sluggish in action although they had essentially the same characteristics as those made from the purer element. Marc[2] has observed that impure selenium crystallizes less completely in a given time than pure selenium. To this fact the unsatisfactory action of the samples made with impure selenium is ascribed.

As insulating material soapstone was used. At first, glazed porcelain was tried, but it proved unsatisfactory on account of the difficulty of working into forms. The advantages of the talc over the porcelain are evident. It is just as efficient as an insulator, besides being much softer, and, therefore, capable of being easily machined.

Several kinds of wire were tried as electrodes, copper, nickel, platinum, German silver, and Advance wire. Again although selenides are formed during the crystallization process, due to the high temperature, the shape of the wave-length sensibility curve will not be changed, although the sensitiveness and the resistance of the cell will be affected. These points are to be verified in some later work. Copper, German silver and Advance wires have this disadvantage that, at the temperature of annealing, a film of oxide covering the wire is readily formed. This so materially increases the resistance of the cell as to make it practically useless for investigation except with very sensitive apparatus. Nickel wire is much less easily oxidized and proved as satisfactory as platinum wire, besides being less expensive, and was used in all except the first few cells.

The size of the sensitive surface was about 1 × 3 cm., except in the first few cells which were larger. The distance between the electrodes was a little less than 1 mm.

In applying the selenium to the form the following method was

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[1] Phil. Mag., VII, p. 26, 1904.

[2] Marc, Die Physikalisch-Chemischen Eigenschaften des metallischen Selens, 1907.

adopted as being the most satisfactory. The form was heated, on a hot plate, to a temperature just above the melting point of selenium, 217° C., and then the selenium, in stick form, was rubbed over the heated surface immediately on removing from the hot plate. As is well known, with this treatment, the selenium at once changes, on solidification, to the gray metallic variety and is conducting and light sensitive. By this method smooth, thin films of selenium were obtained. However, in all cases in which the samples were tested out immediately on making, the resistance was found to be very high, of the order of 10^8 ohms, and the sensitiveness, even under intense illumination, in no case greater than 5 to 1. (By sensitiveness is meant the ratio of the resistance of the cell in the dark to that in the light.) Upon making, therefore, the cells were "annealed," the process taking place in an electric oven whose temperature could be quite accurately controlled. It is in this "annealing" process, which consists merely in keeping the cells for some time at a high temperature, or in gradually changing the temperature, that the changes which impart different characteristics to the cells take place. This will be described in greater detail in a later section since it is varied from cell to cell.

In general, all the samples received the same treatment after annealing. They were allowed to come to a temperature of 170° C. while in the oven, then removed and placed in small glass tubes which had been thoroughly cleaned and dried. They were then sealed to prevent the access of moisture and other vapors, and allowed to come to room temperature. Usually they were allowed to rest for 24 hours before being investigated, being kept in a light-tight box in the meanwhile. With these precautions, all the samples were found to be permanent, with respect to light sensitiveness, at least throughout this investigation.

At first, the method generally described for crystallizing the selenium namely, heating it for a few minutes at about 180° C., was followed. The first cell made in this way had a high sensibility but was not permanent. The next few samples were not at all satisfactory, so the method was abandoned. Instead, the cells were annealed at higher temperatures and a longer time was taken for the process. By this method samples were obtained which were quite satisfactory in respect to sensitiveness.

Characteristics of the Cells.

The cells were investigated with respect to resistance, sensitiveness, shape of wave-length sensibility curve and permanence.

Resistance.

The resistance of the cells was measured by means of a Wheatstone bridge. Since the resistance varies with the voltage impressed upon it, the same E.M.F., 16 volts, was used throughout.

Here the observations of Ries[1] in regard to the variation of resistance with annealing were verified. Thus Ries records two samples which were heated at different temperatures, and shows that the higher the temperature to which the cell was heated the lower was its resistance. His method differs from that employed in this investigation, however, in that he subjected the individual cells to a series of temperature changes, alternately heating and cooling each cell, and measuring its resistance and sensitiveness while it was at room temperature. In these experiments each cell was subjected to a high temperature but once, but the results were the same as those obtained by Ries. The following table gives some typical data.

Table I.

No. of Cell.	Temp. of Annealing.	Period of Annealing, Hrs.	Resistance, Ohms.
23	210–200° C.	6	233,000
22	210° C.	4	358,000
28	210° C.	5	490,000
16	180° C.	3½	1,400,000
15	190° C.	2	3,690,000

From the above table the influence of annealing on the resistance of the cells can be clearly seen, namely, that the higher the temperature of annealing and the longer the time, the lower is the resistance.

It was further found that if the cells were heated for a short time only at a high temperature and the annealing carried on to completion at a lower temperature the resistance of the cells was materially reduced. This fact is shown in Table II. Thus, cells No. 18 and 19 were given

Table II.

Cell.	Temp. of Annealing.	Period of Annealing, Hrs.	Resistance, Ohms.
18	210° C.	½	976,000
	180° C.	9	
19	180° C.	9	40,000,000
20	210° C.	½	250,900
	180° C.	14	
21	180° C.	14	9,500,000

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[1] Ries, Das elektrische Verhalten des Kristallinischen Selens gegen Wärme und Licht, 1902.

exactly the same treatment except that cell No. 18 was given a preliminary heating of half an hour at 210° C. The resistance of No. 18 is seen to be less than 1/40 that of No. 19. The same is true of No. 20 and 21.

The resistance of the freshly made cells was, in general, low, but increased gradually, reaching a constant value in a few weeks after making. This gradual, permanent increase is no doubt due to the contraction of the selenium and its consequent tearing away from the electrodes. The resistance of the various samples ranged from 12,000 ohms to 42,000,000 ohms.

Light-Sensitiveness.

With regard to the effect of annealing on the sensitiveness of the cells not much can be said as a result of these experiments. Two cells of remarkably high sensitiveness were obtained, but this was not permanent, the sensitiveness decreasing to about 1/5 of the original value in less than one month. That phase of the subject is under investigation at the present time, nothing definite having been discovered thus far, however. The sensitiveness of the samples, with the exception of the two mentioned above, varied between 5/1 and 20/1, using a 16 c.p. light at a distance of 30 cm. as a source of illumination.

The Wave-Length Sensibility Curves.

In order to determine the wave-length sensibility curves, the same method of procedure as outlined by Brown and Sieg[1] was followed and the same apparatus was used. Three periods of exposure of the cell to light were used; .4 sec., 10 sec., and 30 sec., the time of exposure depending upon the resistance of the cell. The range, in general, was from 460 $\mu\mu$ to 800 $\mu\mu$, since at values lower than 460 $\mu\mu$ the energy of the source was not great enough to measure very accurately. The exposure always took place in the order from short to long wave-lengths, since Brown and Sieg have shown that the order of exposure does not affect the shape of the curve. The slit width and other factors were kept constant throughout the investigation.

It was found that, except in a few cases, the shape of the wave-length sensibility curve did not change, whether the intensity of the incident light were high or low, or whether the time of exposure were long or short. This is shown by the curves for cell No. 10 in Fig. 1. Here the wave-lengths are plotted as abscissæ, and the change in resistance, which is proportional to the deflection of the galvanometer, as ordinates.

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[1] Loc. cit.

It is seen from Fig. 1 that the curves for cells of the type represented by No. 10 do not change in their essential characteristics when the energy of the incident light is varied by as great a factor as 50, nor when the time of exposure changes from .4 sec. to 30 sec. Whenever possible, therefore, on account of the sensitiveness of the apparatus, the exposures were made for .4 sec. Cell 17, Fig. 1, represents the type in which a change in the energy of the incident light changes the shape of the curve. These are the typical curves obtained by other observers for the Giltay type of cells.

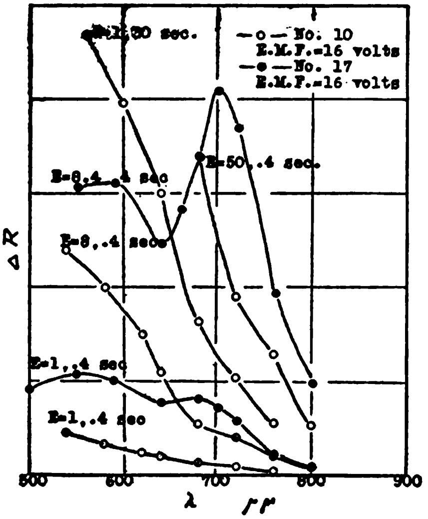

Fig. 1.

A wide variety of types of sensibility curves resulted, maxima being obtained at points not previously recorded. Fig. 2 shows the location of some of the most pronounced maxima developed in this investigation. These curves are not plotted to the same scale and, hence, do not show the relative sensibility of the cells; neither do they represent all the

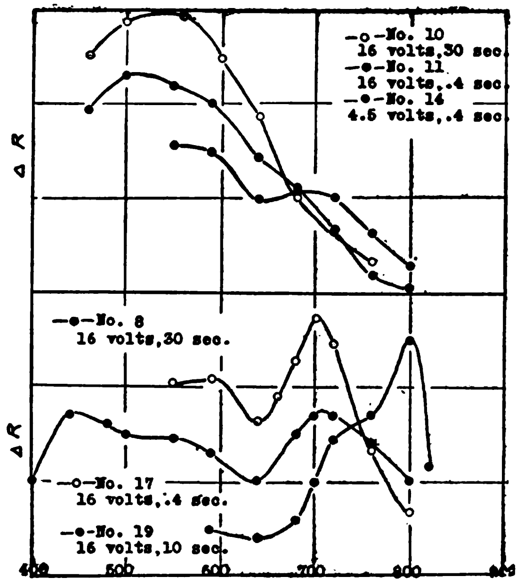

Fig. 2.

types possible to obtain. At the following wave lengths definite maxima were found: 440 μμ, 500 μμ, 550 μμ, 700 μμ, 720 μμ and 800 μμ.

From Fig. 2 it is seen that the cells can be divided, in general, into two groups, viz., those that have their maximum sensibility at wave-lengths greater than 640 μμ, and those in which the maximum occurs at wave-lengths shorter than 640 μμ. In no case was a maximum found at 640 μμ, nor has the author been able to find one recorded at this point.

One type of cell represented by No. 19, Fig. 2, has two maxima, both of about the same magnitude and very sharply defined, one in the violet at 440 μμ, and one at 700 μμ or 720 μμ. This type is obtained when, after applying the selenium to the form in the manner previously described, it is heated for about 10 hours at 180° C.

Fig. 3 shows the effect of a variation in the temperature of annealing on the shape of the wave-length sensibility curve. This set of curves is typical of a large number obtained when the conditions of temperatures were the same as those indicated here.

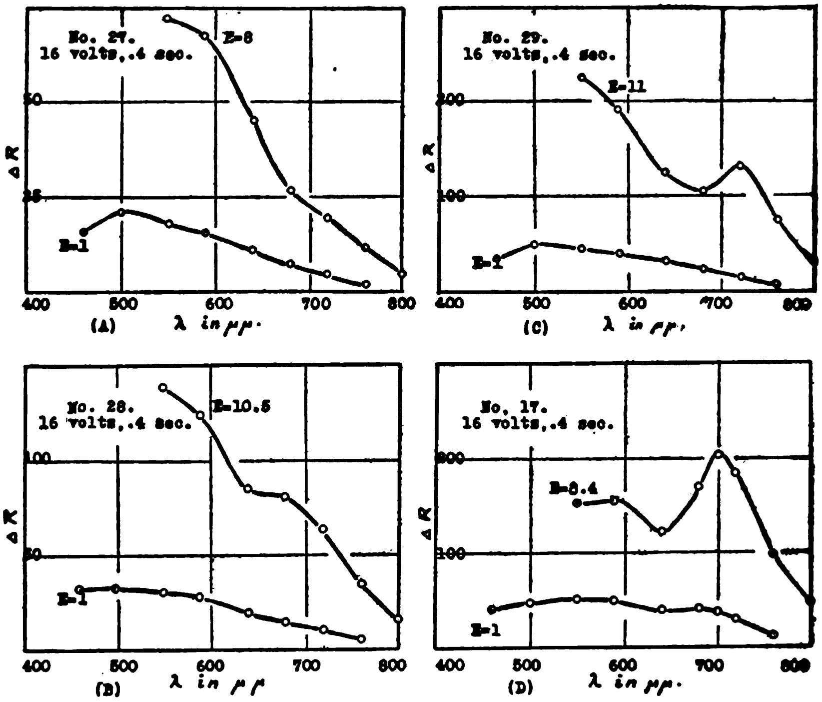

Fig. 3.

(*A*), Fig. 3, represents the type of curve obtained, when the cell, immediately after making, is subjected to a temperature of 210° C. for some time, in this case, 4 hours. This type shows a maximum about 500 $\mu\mu$, and very little sensitiveness above 640 $\mu\mu$.

(*B*), Fig. 3, represents the type resulting when the cell on making is subjected for a short time to a temperature of 210° C., in this case 1 hour, the temperature then allowed to fall to 200° C. and kept constant at this point for 4 hours. Here is seen a change in the shape of the curve and a hint of a maximum in the red.

(*C*), Fig. 3, represents the type obtained when the cell is heated at 210° C. for 30 min., the temperature allowed to fall to 190° C. and kept constant there for 6 hours.

(*D*), Fig. 3, represents the type resulting when the cell was heated for 30 min. at 210° C. and then at an average temperature of 170° C.

The control of the maximum in the red end of the spectrum is clearly shown by the curves in Fig. 3. As the temperature of annealing becomes lower, the maximum in the red becomes, relative to that in the blue, gradually higher, until in the type (*D*) it exceeds that in the blue.

In each case described above the preliminary heating at 210° C. was given in order to lower the resistance of the cell, as was seen to be the case earlier in this paper. This was done in order to increase the accuracy of the measurements, for the apparatus for obtaining the curves is more sensitive and, hence, more accurate, the lower the resistance of the cell measured.

The most obvious explanation of the variation found in the different types of cells, in the light of recent work on selenium, rests on the probable difference in the crystals composing the various types. As we have seen, a rough classification into two general types can be made, those most sensitive to red light and those most sensitive to blue. Recently Professor Brown[1] has succeeded in producing several new forms of selenium crystals of the gray metallic variety by sublimation. A cell made by depositing one variety of these on a form was found to have a maximum at 780 $\mu\mu$. It is not improbable that the maximum sensitiveness of a cell composed of crystals of another variety should lie in the blue end of the spectrum.

In some recent investigations, the results of which have not yet been published, Drs. Brown and Sieg have found the maximum sensitiveness of a single crystal of a certain variety to be in the red end of the spectrum. In another variety two maxima were found, one in the long wave-length and one in the ultra-violet. Moreover, the location of the maximum is found to be dependent upon such factors as the angle of incidence and the intensity of the exciting light, the face of the crystal illuminated, etc.

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[1] PHYS. REV., Series 2, IV, p. 85, 1914.

The explanation of the different types of cells on this basis, then, is simple. A cell, such as is represented by (*A*), Fig. 3, may be assumed to contain crystals which are sensitive to a great extent only to blue light, and few or none at all of those sensitive to red light. In other words, the temperature at which this type is annealed is too high to allow the formation of crystals sensitive to red light. Another type, (*C*), for instance, might be thought of as containing a mixture of the two varieties, the amount of the "red" crystals present not being sufficient to overcome the effect of the "blue" crystals. The type represented by cell No. 19, Fig. 2, can be explained on the same basis, *i. e.*, composed of crystals having maxima both in the red and in the ultra violet, and in which the point of maximum sensibility has been shifted towards the visible spectrum, due to a variation in some one or more of the conditions governing the formation of the crystals. The occurence of maxima separated by only a few wave-lengths can be explained by assuming a difference in the orientation of the crystals in the various cells, so that in different cells different crystal faces are exposed. Or, the light transmitted through the surface layer may suffer changes, by reflection from the crystal surfaces, causing an effect different from that it would have were it direct. This effect might be manifested by a shift in the maximum.

If the above conclusions are correct, the problem of the explanation of the light sensitiveness of selenium reduces to an explanation of the changes taking place in the single crystal under the influence of light and other agencies.

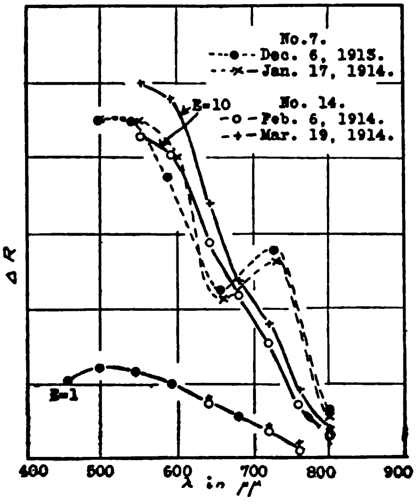

Fig. 4.

Permanence of the Characteristics.

It is evident that no definite statement as to the permanence of the characteristics of the various types can be made at the present time, since the time elapsed is hardly long enough to warrant that. Fig. 4 shows the characteristic curves of two cells taken at different time intervals, under as nearly the same conditions as possible. It is seen that they are the same in the essentials, the slight differences can be attributed to experimental errors.

SUMMARY.

In this paper has been shown:

1. That the resistance of selenium cells depends to a great degree upon the treatment to which they have been subjected while annealing them.

2. That the location of the maximum in the wave-length sensibility curve can be controlled by a variation in the conditions under which the selenium in the cell is crystallized. It was shown that the temperature variations play the most important part.

3. That the various types of cells produced in this investigation can be explained by assuming the presence of various kinds of crystals, or different positions of the same kind of crystals in the cells. The temperature at which one kind is formed may not be favorable for the formation of another kind, hence, the production of the different types of cells.

In conclusion, I wish to acknowledge my indebtedness to the staff of the department of physics for their interest in the problem, especially to Professors Brown and Sieg for the use of their apparatus, and for their many helpful suggestions, and to Messrs. Scott Walker and Paul Helmick who so ably assisted me in much of the experimental work.

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A CONTRAST EQUALITY PHOTOMETER.

BY A. H. PFUND.

SEVERAL years ago the writer described a simple equality-photometer[1] which has been used with success by several investigators. Recently it was found possible to modify the design of the instrument so as to introduce also the principle of contrast and thus to attain (theoretically) double the sensibility of the perfect equality-photometer.

The plan of the photometer-head is shown in Fig. 1 where AA represents a plate of brass set diagonally in a small wooden box, 3 × 3 × 3 cm. An elliptically shaped hole is cut in the center of the brass plate, the axes being 10 and 7 mm. A mirror, consisting of a piece of platinized plate-glass (B), covers one half of the opening, the vertical edge of the glass coinciding with the mirror axis. By viewing this system through a tube T containing a lens which magnifies 3 diameters, a circular field, divided centrally by a perfect edge, is seen. This arrangement constitutes the equality-photometer previously described.

In order to increase the sensibility of the instrument, a piece of smoked or neutral-tint glass was ground down to a very thin optical wedge whose transmission near the thin edge was greater by about 2 per cent. than at the thick edge. After cutting the glass in two so as to procure two identical wedges, the ground surfaces were cemented to pieces of clear glass by means of Canada balsam—thus avoiding the necessity of polishing. These wedges were then mounted at C and C' and were so adjusted that their density gradients, though vertical, were reversed. Upon mounting this photometer-head and establishing a photometric balance an appearance, similar to that shown in Fig. 2, was presented. In this illustration the density gradient was made large purposely so as to bring out the principle involved. It is evident that photometric balance has been established when equality exists at the center of the

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[1] Johns Hopkins University Circular (1906), p. 20.

field and equal contrast in the upper and lower portions. Without going into details it may be shown that if (for example) the eye is capable of detecting a minimum intensity difference of 2 per cent. and if the excess of transmission through the thin part of the wedge be 1 per cent., then the accuracy of setting will be double that obtainable by means of an equality photometer.[1]

Before discussing the results obtained, it may not be out of place to describe the method employed in preparing the mirror *B*. A piece of plate-glass of the approximate dimensions 25 × 15 × 1 mm. is scratched, as shown in Fig. 3, by means of a steel-wheel glass-cutter, and is then broken in two along the dotted line. The portions not touched by the steel-wheel present perfect edges. These plates are subsequently covered cathodically by a hard, highly reflecting layer of platinum and thus two plates, either of which may be used as mirror *B*, are obtained. It is needless to add that subsequent polishing is liable to destroy the perfection of the edge.

A typical series of measurements is recorded below. The comparison of a 50-watt tungsten lamp against a carbon lamp of almost equal candle-power was carried out. It will be pointed out later that a striking feature of this type of photometer is that the eye depends upon difference of contrast and disregards, to a very marked extent, differences of color. In order to accentuate the contrast in color between the tungsten and carbon lamps, a sheet of clear glass of slightly greenish hue was placed between the tungsten lamp and the photometer head. The lamps were about 200 cm. apart and the photometer head, when in position of balance, was nearly half-way between them.

	Distance from Tungsten Lamp to Photometer.	Residuals.	
	102.08 cm.	+.02 cm.	Average deviation from mean setting 9.4 parts in 10,000 or < .01 per cent.
	102.26	+.20	
	102.00	−.04	
	102.13	+.07	
	101.96	−.10	
	102.11	+.05	
	101.91	−.15	
	101.98	−.08	
	101.95	−.11	
	102.20	+.14	
Mean	102.06		

In view of the circumstance that the lamps were permanently separated by a distance of 2 meters the error of setting the movable photometer-head is but one fourth of the error in candle-power measurement—hence,

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[1] After the completion of this work (Jan., 1914) the writer was told by Dr. E. P. Hyde that he had constructed a photometer involving a similar principle. He placed opposed wedges in front of a Lummer-Brodhun contrast photometer and made final settings by obtaining equality of contrast at the edge of one of the trapezoids.—The work was never published.

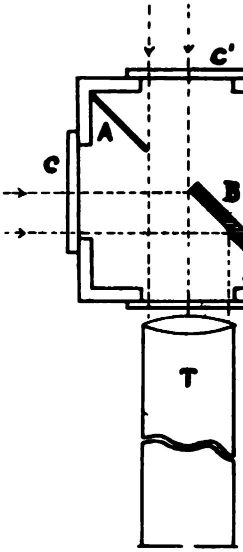

Fig. 1.

Fig. 2.

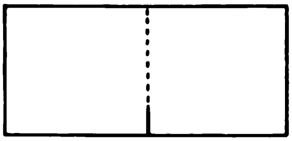

Fig. 3.

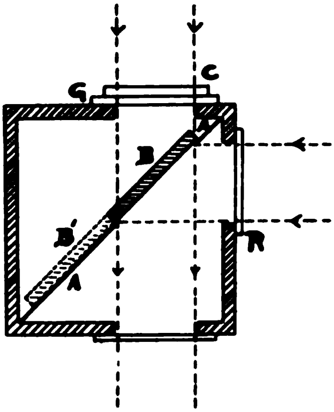

Fig. 4.

A. H. PFUND.

while the error of setting was about 0.1 per cent. the error in c. p. measurement is 0.38 per cent.—The optical wedges used possessed too high a density gradient, hence the ideal conditions, which may be realized with care, were not fulfilled. It is shown, nevertheless, that even though an appreciable color difference exist between the sources, a very fair degree of accuracy is attainable by means of this photometer.

In the course of the work it became evident that the instrument could be used in the comparison of the intensities of colored light-sources. For a successful comparison of such sources it is necessary to increase the density gradient as the difference in color between the light-sources increases for the eye depends largely upon the equality of contrast and not upon a judgment of equality of luminosity. If a red and a green source are to be compared and equality of contrast has been established it is found that the eye has introduced a second (rather diffuse) line into the field of view. This subjective line intersects with the permanent vertical edge at an angle, the point of intersection being at the center of the field of view when a balance is established. By viewing Fig. 1 this subjective line, which divides the field of view into four sectors, may be readily observed. The two criteria of balance, *i. e.*, equality of contrast and central intersection of the two lines, supplement one another and thus increase the accuracy of setting.

This arrangement was tested by inserting a plate of glass covered with green-stained gelatine in front of the tungsten lamp and a similar red-stained plate in front of the carbon lamp. The distance from each source to the usual plaster of paris diffusing screen was very nearly 51 cm. A typical series of settings is given below:

Distance from Lamp to Diffusing Screen.	Residuals.	
Mean 102.06		
51.4 cm.	+0.00	Average deviation from mean setting 4.8 parts in 1,000.
51.5	+0.1	
50.7	−0.7	
51.3	−0.1	
51.5	+0.1	
50.7	−0.7	
51.2	−0.2	
51.4*	0.0	
51.6	+0.2	
51.5	+0.1	
51.2	−0.2	
51.7	+0.3	
51.3	−0.1	
51.7*	+0.3	
51.8	+0.4	
51.1	−0.3	
Mean 51.4		

The stars opposite the 8th and 14th setting indicate that 10 minutes were allowed to elapse before making the next setting in order to determine whether or not fatigue had much influence on the accuracy of setting. It will be noted that even under these extreme conditions of color difference the average deviation from the mean setting is but 4.8 parts in 1,000, whereas the average deviation from the mean candle-power is 1.9 parts in 100 or nearly 2 per cent.

After the completion of this series, Mr. Enoch Karrer, who had had no experience with this instrument, was called in. He succeeded at once in making settings which agreed, within the limits of accuracy, with those previously obtained by the writer. The wedges were then removed and a comparison of the same (red and green) light sources was attempted. The task seemed hopeless as all criteria had disappeared and recourse had to be taken to a very questionable "judgment" of equality.

An attempt was next made to use the optical wedge also in the construction of a flicker-photometer. This instrument, as shown in Fig. 4, consists of the usual brass plate *AA′* with elliptical aperture behind which a platinized mirror is caused to oscillate. The former red and green plates are attached to the photometer-head at *R* and *G*. When the mirror is in the position *B*, the entire field of view appears green and conversely, when the mirror is in the position *B′*, the entire field of view appears red. A single optical wedge with its density gradient vertical was inserted at *C*. It is evident that the criterion for equal intensity is no longer disappearance of flicker over the entire field, but equality of flicker in the upper and lower portions and disappearance of flicker in the center. Surprisingly accurate settings could be made with this device—in fact, when red and green sources were compared, the average deviation from the mean candle-power was but 0.6 per cent.

In order to determine whether or not the flicker could actually be made to disappear over the entire field of view when the wedge was removed, further experiments were carried out on this point. It was noted that, with this arrangement, the flicker could not be made to disappear. As one of the sources was slowly moved up toward the position of equality the flicker first disappeared in (say) the right portion of the field of view but persisted in the left. A still closer approach of the source brought about equality of flicker on the right and left and absence of flicker in the middle—a condition which has been considered as the criterion for photometric balance. Thus it is evident that, even without the use of an optical wedge, the condition of equality of flicker may be realized. The writer assured himself that there was no asymmetry in the instrument capable of accounting for the above effect—and, up to the present, no satisfactory explanation has been found.

The preceding is to be looked upon as a preliminary publication, for much remains to be done in defining the various conditions of maximum sensibility of the several photometric appliances. This work is to be taken up and extended in the near future.

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THE HALL EFFECT AND ALLIED PHENOMENA IN SILICON.

By O. E. Buckley.

THE Hall effect in silicon was first measured by Miss Wick[1] who obtained for the coefficient of the effect, R, the values, -120 for silicon 99 per cent. pure and -28 for silicon 96 per cent. pure. Miss Wick also made measurements of the thermal E.M.F. and the resistance of silicon.

Later Zimmerman[2] found the direction of the thermal E.M.F. of several specimens of silicon against copper to be opposite in direction to that which Miss Wick had determined although some specimens gave a thermal E.M.F. in the same direction as that which she had found. This peculiarity in the behavior of silicon led A. W. Smith[3] to make investigations of the Hall effect in these specimens. For four of his specimens he obtained values of R varying from $+40$ to $+60$. In all of these plates the direction of the thermal E.M.F. was such as to send a current from copper to silicon across the hot junction. In a fifth specimen he found $R = -20$ and in this specimen the thermal E.M.F. was in the opposite direction to that in the plates with the positive Hall effect.

Koenigsberger and Gottstein[4] measured the Hall effect in silicon containing 5 per cent. iron and found by direct measurement $R = +24.5$. They apply two corrections to this value, one for the error due to the Ettinghausen effect and one to change the adiabatic effect to an isothermal effect, the final result being $R = +110$.

Gottstein[5] also measured the Nernst effect in two pieces of silicon and found by direct measurement the values .0132 and .0116 for the coefficient of the effect, Q. He applies corrections as he did for the Hall effect and arrives at the values .0333 and .028. He does not indicate the direction of the effect.

The thermal E.M.F. of silicon against copper has been found by several observers to vary from $+.5$ to $-.5$ millivolt per degree. The

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[1] Phys. Rev., 27, p. 76, 1908.

[2] Trans. Amer. Electro-chem. Soc., 15, p. 395, 1909.

[3] Phys. Rev., 30, p. 1, 1910.

[4] Phys. Zeit., 14, p. 232, 1913.

[5] Ann. der Phys., 43, p. 1079, 1914.

reversal of the thermo-electric behavior has been investigated by Fischer, Lepsius and Baerwind.[1] From their experiments they concluded that the reversal was due to the presence of SiO_2 in the silicon.

Measurements of the resistance of silicon have been made by several observers with widely varying results. Miss Wick[2] found values for the resistance in ohms per c.c. varying from .034 to .080 in the different specimens which she measured. Some of Miss Wick's specimens showed a steady decrease of resistance with rising temperature while other specimens showed minima at about 200° C. Koenigsberger and Schilling[3] with very pure silicon obtained the value .0885 at 15° C. The resistance-temperature curve of this specimen contained two discontinuities.

A list of several physical constants of silicon has been published by Zimmerman.[4] The thermal conductivity and Thomson coefficient have been measured by Koenigsberger and Weiss.[5] The Peltier coefficient has been measured by Gottstein.[6] Koenigsberger[7] has given a discussion of the electrical properties of silicon and similarly behaving substances with reference to the electron theory.

In most of the work on silicon and similar substances measurements of various electrical properties have been made with different specimens. Since different specimens, sometimes even if cut from the same piece, may vary widely in their electrical properties it is important that measurements of different properties be made with the same specimens.

This research was undertaken to determine whether the wide variations and reversal of sign of the Hall effect in various specimens of silicon were accompanied by marked variations in the Nernst effect and resistance. The temperature coefficients of these quantities were also measured to see if any apparent connection existed among them. For this purpose four specimens of silicon of widely varying degrees of purity were used. These specimens were donated by the Carborundum Company, who also kindly furnished analyses. The analyses of the specimens showed impurities as follows:

Unfortunately Plate I. was not very compact. It contained small blow holes, a few of which were over half a millimeter in diameter and over a millimeter long. These holes undoubtedly affected the results with this specimen seriously. Plate III. contained a few small pin holes, not more than four or five to the square centimeter, and of small depth.

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[1] Phys. Zeit., 14, p. 439, 1913.

[2] Phys. Rev., 27, p. 11, 1908.

[3] Ann. der Phys., 32, p. 179, 1910.

[4] Trans. Amer. Electro-chem. Soc., 15, p. 395, 1909.

[5] Ann. der Phys., 35, p. 1, 1911.

[6] Ann. der Phys., 43, p. 1079, 1914.

[7] Jahrbuch der Rad. u. Elek., 11, p. 84, 1914.

Specimen.	Fe, Per Cent.	Al, Per Cent.	Si (by Difference), Per Cent.
I.	0.35	0.19	99.46
II.	3.50	0.23	96.27
III.	3.38	2.55	94.07
IV.	7.27	3.35	89.38

Plates II. and IV. were quite compact. None of the specimens contained any surfaces of separation.

Plates were sawed from irregularly shaped blocks of silicon to the shape shown in Fig. 1 by means of a vulcanite carborundum wheel which was arranged to run in a trough containing water and fine carborundum powder. On account of the limits set by the blocks from which the plates were cut, the dimensions of the plates varied somewhat with the different specimens. The approximate dimensions were, length 4 cm., width 1.8 cm., thickness 0.5 cm., length of side arms from 0.8 cm. to 2.4 cm. In every case the side arms were made as long as possible.

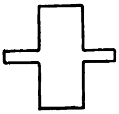

Fig. 1.

The ends of the plates and the ends of the side arms were heavily copper plated. Wires soldered to the copper on the ends of the plate served for leads for the current through the plate. Wires soldered to the copper on the ends of the side arms led to a potentiometer which was used to measure the Hall E.M.F. Near each end of the plate a very narrow copper band was plated onto the silicon completely encircling it. These bands served as contacts to measure the resistance by the fall of potential method. In making measurements of the Hall effect and resistance at various temperatures the plate under test was placed in a double-walled copper vessel held between the poles of a large Weiss electro-magnet. The desired temperature was obtained by circulating water between the walls of the vessel. For the highest temperature steam was used. Readings were taken at four different temperatures between 0° and 100°.

The pole pieces of the magnet were 10 cm. in diameter and were placed 2.2 cm. apart. Over the area of the plate the field was practically uniform. For most of the measurements of the Hall effect a field of about 9,000 gauss was used.

The difference of potential between the side arms and between the resistance contacts was measured by means of a low resistance potentiometer and a sensitive D'Arsonval galvanometer with small period. In making measurements of the Hall effect a current through the plate of about .16 ampere was generally used. A larger current caused objectionable thermal disturbances. The effect of any gradual temperature change was cancelled out by taking a series of readings at equal time intervals, reversing the field after each reading.

The coefficient of the Hall effect, R, is defined by the equation

$$E = RiwH,$$

where E is the Hall E.M.F., i, the current density, w the width of the plate, and H, the intensity of the magnetic field. If I, the whole current in the plate is used in place of i, the current density, the equation may be written

$$R = \frac{Et}{IH},$$

where t is the thickness of the plate.

Plate II. was found to have a negative Hall effect while the other three plates had the positive Hall effect. The values obtained for the coefficient, R, are plotted in Fig. 2. The negative values for Plate II. are plotted as positive.

The resistances of the same plates were measured at various temperatures by comparing the fall of potential through the plate to that through a standard resistance connected in series with it. The fall of potential was measured by connecting the potentiometer to the two copper bands placed near the ends of the silicon plate. Measurements of resistance were made while the plate was held in position for measuring the Hall effect. This arrangement also permitted making a test of the effect of the magnetic field on the resistance of the silicon. No effect could be observed and the change of resistance in a magnetic field of 10,000 gauss was shown to be less than 0.05 per cent.

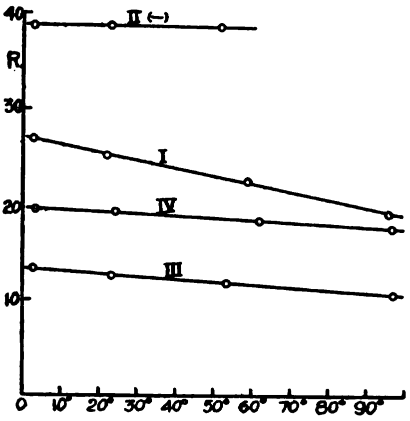

Fig. 2.

Variation of Hall Effect with Temperature.

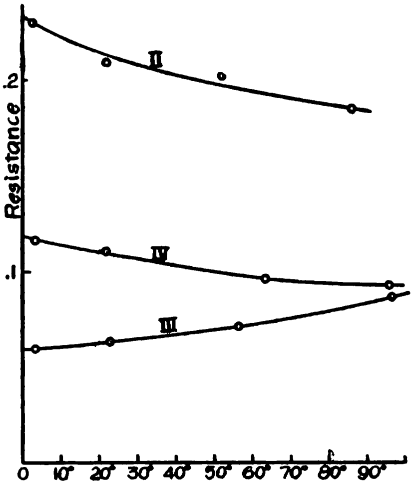

Fig. 3.

Variation of Resistance with Temperature.

In attempting to measure the resistance of Plate I. great difficulty was experienced especially at the higher temperatures. No steady reading could be obtained at the higher temperatures and the apparent resistance at room temperature would be found to have changed after heating and cooling. This was probably due to varying contact resistance at the ends of the plate and consequent deflection of the current flow lines. The results obtained for Plate I. were so erratic that no definite value of the resistance can be given. The same difficulty was experienced to a less degree with Plate II. and the value of its resistance at the highest temperature is doubtful. The other two plates gave results which could be repeated.

The values obtained for the specific resistances of the different specimens are plotted in Fig. 3. No correction has been made for the influence of the side arms on the resistance.

Fig. 4 shows the arrangement of apparatus used in making measurements of the Nernst effect. The copper plating on the ends of the silicon plates was ground to fit the grooves in the ends of the copper bars, *C*. The bars were pressed together against the plate by stiff steel springs, *S*. These bars served both as heat and electrical conductors. For the sake of insuring more uniform contact for heat flow thin tin foil was placed between the copper plating on the ends of the silicon plate and the copper bar.

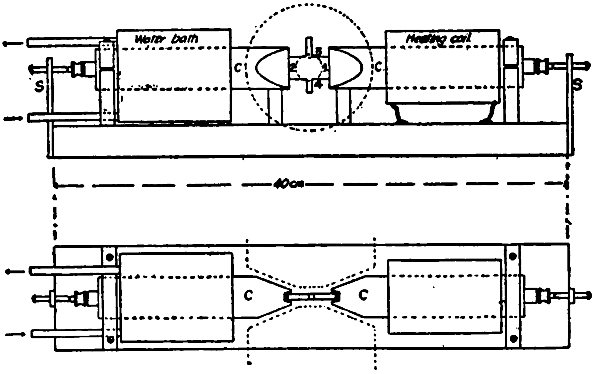

Fig. 4.

One of the copper bars was surrounded by a heating coil of nichrome ribbon insulated from the copper by thin asbestos paper. The heating coil was covered with asbestos paper and a clay cylinder to prevent loss of heat. The other copper bar was provided with a water bath through which water at the desired temperature was circulated. In order to make as uniform a temperature gradient as possible and to prevent thermal disturbances the silicon was covered with two layers of wool felt.

Temperatures were measured by means of copper-advance thermojunctions placed at positions 1, 2, 3 and 4 as indicated in Fig. 4. Each junction was fastened to one side of a small fiber cube and pressed against the silicon by a stiff brass spring. The junctions in positions 1 and 2 were separated from the silicon by thin sheets of mica not over .01 mm. thick. This was found necessary on account of the difficulty in placing the wires exactly parallel to equal temperature lines.

Readings of the Nernst effect were taken at intervals of 30 seconds, the magnetic field being reversed after each reading. By leaving the field on continuously except for the time required for reversal, errors due to the effect of the magnetic field on the E.M.F. of the copper-silicon junctions or to the effect of the magnetic field on the heat current in the plate were avoided. One half the observed change of potential difference between the side arms on reversing the field was taken as the Nernst E.M.F. Correction was made for the non-uniformity of the magnetic field across the plate necessitated by the small area of the pole face.

Measurements of the Nernst E.M.F. were taken with varying magnetic field and for several different temperatures with each plate. In every case the E.M.F. was found to be proportional to the field strength up to the limit of field strength used, about 20,000 gauss.

The coefficient of the Nernst effect, Q, is defined by the equation

$$E = \frac{Qdt}{dx} wH,$$

where dt/dx is the temperature gradient. If we assume that the temperature gradient is uniform and let t_1 and t_2 be the temperatures at junctions 1 and 2 respectively and led d represent the distance apart of those junctions, we may write

$$Q = \frac{Ed}{(t_1 - t_2)wH}.$$

The direction of the Nernst effect was found to be the same in all four of the plates. The values found for the coefficient, Q, are plotted in Fig. 5.

Since the Hall and Nernst effects for silicon are quite large as compared with metals it was expected that the corresponding transverse temperature effects would also be large and possibly measurable by a D'Arsonval galvanometer. However, no positive results for these effects were obtained and all that could be done with this apparatus was to set an upper limit to the values of the coefficients of these effects.

To measure the transverse temperature change the thermojunctions at positions 3 and 4 (Fig. 4) were used. Since they were in contact with the silicon these junctions could not be used in series but each had to be used separately against a similar junction in ice. Two minutes time was allowed after reversing the field, before making a reading, to allow time for the effect, if any, to develop.

The results obtained varied too widely to set a definite value for the effect or to be sure of its direction. For the sake of setting a safe upper limit twice the observed value is given as an upper limit. This is about equivalent to adding to the observed effect three times the probable error in such a way as to increase the value of the effect. The upper limit set in this way for the Ettinghausen coefficient, P, is $P<3\times 10^{-6}$ corresponding to a temperature change of about .02°. Limits of the same order of size were obtained for each of the four plates. Comparison of this value with the values found for other substances shows that it is not surprisingly small and no significance would be attached to it were it not that Gottstein[1] reports a value of P for silicon of 2.65 $\times 10^{-8}$. Such a large effect as that could hardly fail to be detected.

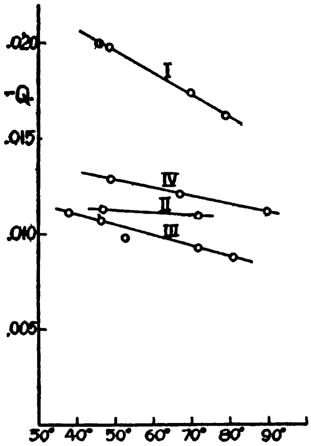

Fig. 5.

Variation of Nernst Effect with Temperature.

Koenigsberger[2] mentions that this effect is so large that it can cause a very large error in the measurement of the Hall effect on account of the transverse change of temperature introducing a thermal E.M.F. We may feel quite sure that this did not cause an error in the measurements reported in this paper, since the use of the side arms would prevent such an effect, and if there were any at all it would be very slow in building up on account of the time required for heat to travel along the side arms to the copper contacts. That there was no such effect was shown by the fact that the galvanometer would come to rest within fifteen seconds after applying the magnetic field.

With a large temperature gradient measurements of the Leduc effect were also attempted. The upper limit set in the same way for the coefficient of the Leduc effect, S, is $S < 5 \times 10^{-8}$ corresponding to a temperature change of about .02°. If this result is compared with the values of S for other substances possessing a large Hall effect it is seen to be very small. Gottstein[3] reports for the Leduc effect in silicon a

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[1] Ann. der Phys., 43, p. 1079, 1914.

[2] Jahrbuch der Rad. u. Elek., 11, p. 84, 1914.

[3] Ann. der Phys., 43, p. 1079, 1914.

value of 2.5×10^{-7} which would result in a temperature difference of about .1° under the conditions of this experiment.

The direction of the thermal E.M.F. of the different specimens against copper was tested by connecting one terminal of the galvanometer in series with a high resistance to the copper plating on the silicon plate. The other terminal was connected to a pointed copper rod which was heated by an insulated heating coil wound around it. The point of the rod was then touched to various points on the plate under test. Plates I., III. and IV. gave a deflection always in the direction indicating a current from copper to silicon at the hot junction although the amount of deflection varied greatly with the different points touched. Plate II. gave mostly the opposite direction although occasional points touched gave the same direction as the other three plates. The occasional points in Plate II. which gave the opposite effect were mostly near one end of the plate and were not distributed in any systematic way as far as could be seen.

The following table gives a summary of the results obtained for purposes of comparison.

Plate.	I.	II.	III.	IV.
Hall effect at 50°	+23	−39	+12	+18
Nernst effect at 50°	−.020	−.0113	−.0105	−.013
Specific resistance at 50°		.20	.07	.10
Temp. coef. of Hall effect	−.0031	0	−.0022	−.0011
Temp. coef. of Nernst effect	−.0057	−.0010	−.0053	−.0032
Temp. coef. of resistance		−.0023	+.0047	−.0022
Direction of thermal E.M.F.	+	−	+	+

According to the conclusions which Koenigsberger has drawn from his work the ratio of the resistance to the coefficient of the Hall effect might be expected to be approximately the same for the different specimens. While this happens to be the case at 50° for the three specimens whose resistance was measured, it appears to be entirely accidental since the temperature coefficients of these quantities are not at all in harmony. There appears to be a much closer relation between the temperature coefficients of the Hall and Nernst effects. If the specimens are arranged in order with reference to the temperature coefficients of the Hall effect they are also in order with reference to the temperature coefficients of the Nernst effect. However there is no apparent connection between the magnitudes of these two effects with the different plates.

Summary.

The Hall effect, Nernst effect and resistance of four specimens of silicon have been measured at several different temperatures.

The reversal of the Hall effect, which is connected with a reversal of its thermal E.M.F., is not accompanied by a reversal or peculiarity in size of the Nernst effect nor by a peculiarity in the resistance.

The temperature coefficients of the Hall and Nernst effect are found to be similar but do not bear any apparent relation to the temperature coefficient of resistance.

The extraordinarily large Ettinghausen and Leduc effects in silicon reported by Gottstein are not confirmed.

The change of resistance of silicon in a magnetic field of 10,000 gauss is less than .05 per cent.

The writer is under obligation to Professor Merritt who proposed the investigation and made many helpful suggestions.

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EXPERIMENTS ON THE PRODUCTION AND MEASUREMENT OF HIGH VACUA.

By J. W. Woodrow.

THIS article describes the actual construction of a sensitive Knudsen absolute manometer and explains its use for the measurement of low pressures. An account is also given of some methods employed for obtaining these pressures.

Introduction.

The theory of an absolute manometer, which is independent of the gas to be measured, has been given by Knudsen.[1] However he merely gives data to show that his theory is confirmed by experiment and does not describe the actual construction of such a gauge. A brief outline of the operation of one type of the Knudsen manometer has been given by Angerer,[2] but some difficulty was experienced by the author of this article in trying to make a sensitive gauge according to his instructions. Therefore, as quite a little time has been spent in this laboratory on the construction of a very sensitive gauge which would be at the same time easily made and simple in operation, it was thought that the results of these experiments were of sufficient value to warrant publication.

Previous experiments had shown that for the measurement of very low pressures the whole gauge must be so constructed that it could be placed in an oven and heated to a temperature of about 200° C. while on the pump. It was further desired to make the volume of the gauge as small as possible and to eliminate all metal which was not absolutely necessary. The gauge, described by Angerer, could not be heated without danger of cracking, a difficulty which has been overcome in the gauge to be described below.

Description of Apparatus.

Several different gauges were constructed varying in sensitivity so as to be used at different pressures. A typical gauge is shown in Figs. 1 and 2 and the electrical circuits are given in Fig. 3. The glass rods *GG* served as supports for the metallic parts of the gauge. All the internal electrical connections and adjustments, with the exception of the final leveling, were made before the outer glass wall *OO* was sealed on at *SS*.

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[1] M. Knudsen, Ann. d. Phys., 32, p. 809, 1910.

[2] E. v. Angerer, Ann. d. Phys., 41, p. 10, 1913.

The suspension *W* was a phosphor-bronze ribbon 50 mm. in length which had been obtained from W. G. Pye & Co. and was listed by them as No. 0000. The movable vane *VV* consisted of a rectangular frame of aluminum 0.076 mm. in thickness, the dimensions of the outer rectangle being 30 × 36 mm. and of the inner 26 × 30 mm. The heating plates *PP* were platinum strips 4 mm. wide, 40 mm. long and 0.025 mm. thick. The deflections of the movable vane were obtained in the usual way by the reflection of a beam of light from the mirror *M*. Fig. 2 is a cross-sectional view through the middle of Fig. 1.

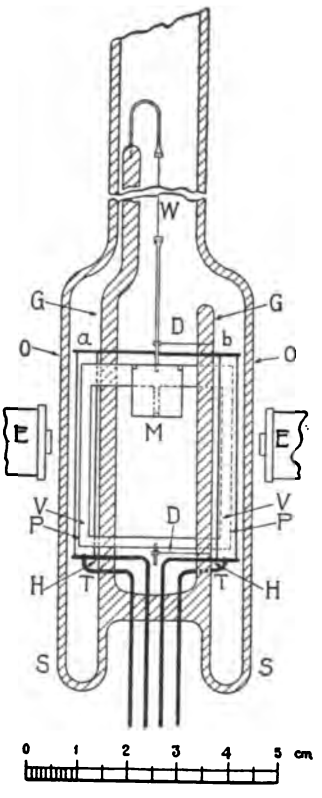

Fig. 1.

All of the platinum connections were made by electric welding, as that was found much more satisfactory than the use of any kind of solder, especially when heated. After a little practice, it was possible to weld the thin platinum heating vanes to the heavy platinum wire so as to make a perfectly continuous contact throughout its width. The phosphor-bronze suspension was connected at both ends by threading through three small holes drilled into the flattened extremities of the platinum and aluminum wires respectively. The small loops *DD* were so placed that they supported the movable vane *V* except when the gauge was leveled for taking readings. This made the gauge readily portable and, by placing in the inverted position when connected to the molecular pump, the danger of the breaking of the suspension by vibration was eliminated. One gauge of medium sensitivity was constructed so as to be sufficiently steady to be used when connected directly to the molecular pump. Large glass tubing was employed in all the connecting portions of the apparatus.

The movable vane used by Angerer consisted of silvered mica, but the present investigation has shown that such a vane is very susceptible to variations in the light falling upon it. An ordinary forty-watt tungsten lamp placed 50 cm. away so as to shine on the vane would cause a deflection of 100 mm. when the pressure was about 10^{-5} mm. of Hg. This phenomenon was shown not to be a direct heat effect by the fact that a non-luminous Bunsen flame had no effect when brought quite close to the apparatus and that with a luminous flame the effect was immediately observed. When the silvered-mica was replaced by aluminum, the effect of light variations was small enough to be neglected. The moving system was electrically connected through the suspension to that terminal of the heating strips which was grounded, the whole being screened from external electrical disturbances by an earthed silver coating on the outside of the glass walls. In an earlier experiment, the silvering was on the inside of the walls, but this was unsatisfactory as it was impossible to free the coating from gases while connected to the molecular pump. However the above method of electrical screening was sufficiently good so long as there was no source of ionization within the apparatus; but if the gauge was connected to an ionization chamber, the electrical disturbances became very troublesome when the ionization was large. No attempt was made in this case to prevent the diffusion of the ions, although this could be easily accomplished by a suitable wire gauge.

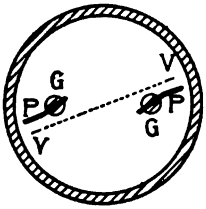

Fig. 2.

A small electromagnet, shown at *E* in Fig. 1, was employed in bringing the moving vane to rest. This was found to be quite necessary in working with the most sensitive gauges, since in a very good vacuum the damping is so small that the vane will not settle down sufficiently for the taking of readings for some time after an accidental disturbance has set it vibrating. It should be noted that the electromagnet must have either an air core or one of good, soft Norway iron, for otherwise the residual magnetism will produce a false zero if the aluminum vane is at all magnetic, as was the case with the samples of metal investigated in this laboratory. Under these conditions it is obvious that the electromagnet should be used only for damping and that the exciting current should be shut off while making observations.

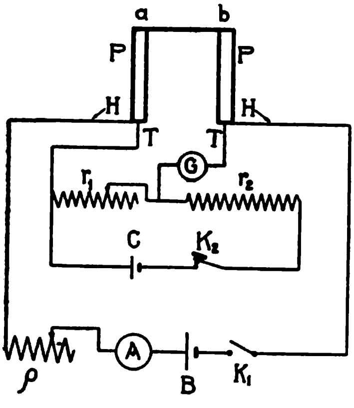

Fig. 3.

Several methods were tried for determining the temperature of the heating strips and that shown in diagram in Fig. 3 was finally settled upon as giving the most satisfactory results. The potentiometer leads *TT* were connected by electric welding to the very extremities of the platinum heating vanes *PP*. The heating current was regulated by the variable resistance ρ and its value was read on the ammeter A. The resistance r_2 was kept constant at 10,000 ohms and r_1 varied to obtain a balance of the sensitive galvanometer G. The potentiometer battery C consisted of a carefully calibrated Weston Standard Cell. This arrangement gave an accurate method of measuring the resistance of the platinum strips PP, plus the heavy platinum wire ab, the total cold resistance being 0.17 ohm. This cold resistance was determined by plotting the curve connecting resistance and heating current under a constant low pressure and extrapolating backward to the intersection with the axis of resistance. If the resistance is measured for small currents, the value at zero current, that is the cold resistance, can be determined very accurately. A typical curve is shown in Fig. 4. The temperature coefficient of resistance of the platinum, which contained a small amount of iridium, was carefully determined and was found to give a linear relation within the range of temperatures employed. The value of the coefficient was 2.35×10^{-3} ohms per degree Centigrade. With this system one can determine the mean temperature of the heating strips with sufficient accuracy, the error for temperature differences of about 50° C. being less than four per cent. It should be mentioned that numerous attempts were made to employ fine thermo-couples, but it was practically impossible to make a good thermal contact with the heaters such that it would remain constant upon heating to 200° C.

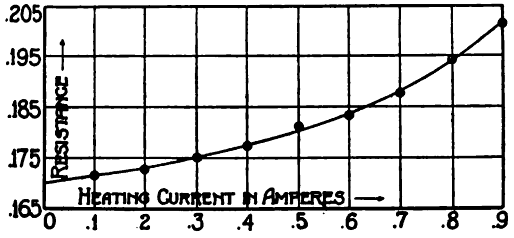

Fig. 4.

CALCULATION OF THE CONSTANTS OF THE GAUGE.

According to the theory given by Knudsen, the gas pressure within the gauge is given by the formula

$$P = \frac{2K}{\sqrt{T_1/T_2} - 1},$$

where K is the uniform pressure in dynes on each cm.^{2} of the movable vane and T_1 and T_2 are the absolute temperatures of the heating strips and of the moving vane respectively. If the difference in temperature is small as compared to the absolute temperature, the formula may be written

$$P = 4K\frac{T_2}{T_1 - T_2}.$$

It is to be noted that this formula is independent of the nature of the gas or vapor to be measured. The constant K was found in terms of deflection, the constants of the suspension being determined in the usual way from the period of the moving system and the calculated moment of inertia of the suspended rectangular frame. For the gauge whose dimensions are given above, the period of a complete oscillation was 10.0 sec. and the calculated moment of inertia of the moving vane was 0.074 gm. cm.^{2} This gives for the pressure

$$P = 2.9 \times 10^{-5}\frac{T_2}{T_1 - T_2}d \text{ dynes},$$

or

$$P = 2.2 \times 10^{-8}\frac{T_2}{T_1 - T_2}d \text{ mm. of Hg.},$$

where d is the deflection in mm. on a scale at a distance of one meter from the mirror. As it is easily possible to obtain a temperature difference of 100° C. and since a deflection of 0.5 mm. can be observed, the gauge will measure a pressure as low as 3×10^{-8} mm. of Hg.

It was thought that at these very low gas pressures the radiation pressure might have an appreciable effect as compared to that of the molecular bombardment. To test for such an effect, the deflection was obtained for several different values of the temperature of the hot platinum strips, when a very constant pressure had been obtained. If the radiation pressure is small enough to be negligible there ought to be a linear relation between the deflection and the temperature difference, while if it is not negligible the deflection should increase more rapidly. The accompanying table shows that the radiation pressure does not have an appreciable effect at a gas pressure of about 10^{-6} mm. of Hg. for temperature differences up to 85° C.

TABLE I.

Temp. Diff. in °C.	Deflection in Mm.	Calculated Pressure $\times 10^{-6}$ Mm. Hg.
8	4	3.4
20	9	3.1
35	17	3.3
65	28	2.7
83	40	3.2

MEASUREMENTS OF LOW PRESSURE.

The lowest pressure obtained with a Gaede molecular pump was measured by a specially constructed gauge, the pressure being 3×10^{-6} mm. of Hg. A Gaede rotary-box pump was used as a fore-pump giving a pressure of 0.01 mm. of Hg., and the molecular pump was run at 8,000 R.P.M. The above vacuum was obtained after pumping for about 3 hours and with the gauge connected as closely as possible to the molecular pump, the total volume consisting only of the gauge and connecting glass-tubing. However in this case the glass walls of the gauge were not heated.

It is interesting to compare the vacuum given by the molecular pump with that obtained when a side tube containing cocoanut charcoal is immersed in liquid air. By the latter method the pressure was reduced after about 36 hours to 2×10^{-7} mm. of Hg. If the liquid air were removed, the pressure would increase to a value somewhat higher than that measured when the gauge was sealed off the molecular pump, but it would then decrease again, slowly approaching the value obtained at sealing off. On the other hand if the liquid air is replaced before this gradual decrease begins the vacuum very quickly reaches the value previously obtained after about 36 hours. This effect is probably due to absorption on the walls of the glass and on the metallic parts of the apparatus which makes the original action of the charcoal and liquid air so slow. If the walls are carefully heated while the gauge is connected to the molecular pump, this phenomenon is not so pronounced.

The lowest pressure which we have measured was obtained by the use of charcoal and liquid air in the usual way supplemented by the evacuating action of a tungsten filament sealed into a side tube and kept running for 130 hours. The pressure had reached the value 5×10^{-8} mm. of Hg. when the investigation was stopped by an accident to the apparatus.

Some interesting data was obtained on the efficacy of rubber valve-grease for holding a good vacuum. A system, which contained three large stopcocks and a large ground-glass joint, attained a vacuum of 10^{-6} mm. of Hg. under the action of charcoal and liquid air. The vacuum remained constant at this value for several days while the charcoal was kept immersed in the liquid air. However when the charcoal tube was disconnected by closing one of the stopcocks, the pressure gradually increased but at a very slow rate.

As the Knudsen absolute manometer measures the pressure independently of the nature of the gas, it affords a convenient method of determining the rate at which charcoal immersed in liquid air will remove different gases. This effect was investigated with oxygen, nitrogen and hydrogen, but it was impossible to get an accurate quantitative comparison as the oxygen was removed too quickly under the action of the liquid air. However some idea of the relative rates may be obtained from the data in Table II. In each case the pressure before inserting the known gas was reduced to the same value, viz. 1.5×10^{-6} mm. of Hg., the charcoal being thoroughly heated while connected to the molecular pump. Hydrogen was investigated first, then oxygen and nitrogen in the order named. The values given in the first column are the times after

Table II.

Time of Action.	Pressure in Mm. of Hg. for		
	Hydrogen.	Oxygen.	Nitrogen.
Zero	485×10^{-6}	500×10^{-6}	450×10^{-6}
5 seconds	———	290×10^{-6}	———
10 seconds	———	170×10^{-6}	———
1 minute	460×10^{-6}	2×10^{-6}	15×10^{-6}
5 minutes	410×10^{-6}	1.5×10^{-6}	———
20 minutes	290×10^{-6}	———	———
1 hour	190×10^{-6}	———	5×10^{-6}
3 hours	135×10^{-6}	1.5×10^{-6}	———
10 hours	135×10^{-6}	1.5×10^{-6}	5×10^{-6}

the charcoal tube has been connected to the gauge by opening the stopcock. For the oxygen, the measurements were taken by leaving the stopcock open only for a few seconds at a time. No special care was taken in the preparation of these gases and consequently they were not very free from other gases and vapors but were sufficiently pure to clearly show the phenomenon.

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New York City.

A COMPARISON OF THE LONGITUDINAL AND TRANSVERSE JOULE MAGNETOSTRICTIVE EFFECTS IN THE SAME SPECIMENS OF STEEL AND NICKEL.

BY S. R. WILLIAMS.

THE interesting discovery made by Joule[1] that a rod of iron would change its length when subjected to a longitudinal magnetic field led to his investigation of the effect of a transverse field upon the length of a piece of iron. In order to get a transverse field imposed upon an elongated specimen of iron, Joule sent an electric current through an insulated wire inside of an iron pipe. The circular magnetic field about the wire in the pipe produced a field normal to the length of the pipe whose change in longitudinal dimension was to be measured. The primary object of this latter investigation by Joule was to discover whether or no volume changes occurred when the iron was magnetized. Joule had no facilities for obtaining high field strengths, hence his results show that in the longitudinal effect only an elongation occurred and in the transverse only a shortening. From this relation he concluded that there was no change in volume.

It is to Bidwell[2] that we owe the extension of our knowledge of the effect of strong longitudinal magnetic fields on iron rods, viz., an elongation for weak fields and a contraction for strong in which for the latter there were certain field strengths which made the specimen shorter than in its virgin state. The question therefore arises as to whether the transverse field produces opposite effects to those of the longitudinal for all field strengths, *i. e.*, will the transverse effect be an elongation, for strong fields in the case of iron?

In 1894, Bidwell[3] contributed two short articles bearing upon the longitudinal and transverse effects which occur in iron rings when they are magnetized. Since the appearance of the papers by Joule and Bidwell, little or no additional work has been done on this subject. Because this subject has an important bearing upon a working hypothesis of magnetization which the author has used with considerable success in

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[1] Joule, Phil. Mag., p. 76, Vol. 30, 1847.

[2] Bidwell, Proc. Roy. Soc., 1885–1890. See Trans. Roy. Soc. also.

[3] Bidwell, Proc. Roy. Soc., 1894.

other magnetic phenomena, it seemed desirable to compare these two effects of magnetization in the same specimens of ferro-magnetic substances. The hypothesis, on which the writer has been working, indicates that qualitatively there should be this reversal for all field strengths in all ferro-magnetic substances, and since the work of Bidwell did not attempt specifically to compare these two effects, but to determine volume changes due to magnetization, it seemed not out of place to present some results which have been obtained upon this subject. Bidwell used toroids in the investigation of the changes in length due to longitudinal and transverse fields. One advantage of the toroid is that, since there are no free poles developed, there is no demagnetizing field, and hence the magnetizing force is the same as the magnetic field which would exist inside of the magnetizing coil without an iron core. There is, however, a practical objection to this form in that every sample of ferro-magnetic substance to be tested requires winding with a magnetizing coil and since this must be done by hand the process is one which consumes a great deal of time. Bidwell's work is limited to two specimens of iron, one in the annealed and the other in the unannealed state. So far as the author knows this has been the extent of comparative work done on these two effects.

In the experiments described below, measurements of changes in dimensions were made upon two specimens of steel and two of nickel, the transverse effect in nickel being measured with considerable difficulty. I wish here to express my thanks to Mr. Paul Weeks, one of our advanced students, for the help and skill in carrying out the measurements.

Methods.

There seem to be two ways of comparing these two magnetostrictive effects. Let A, B, C, D, Fig. 1, represent the cross-section of a cube of steel to which has been applied a magnetic field, H, in the direction indicated. Obviously if the change in length of AC is measured as the field, H, is varied, we have the ordinary longitudinal Joule effect, whereas if DB or the length normal to the page is measured we have the transverse Joule effect. This is the method which Bidwell used in measuring the two effects, *i. e.*, keeping the direction of the magnetic field fixed he measured the variations of two dimensions at right angles to each other, one of which was parallel to the field applied.

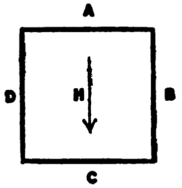

Fig. 1.

The other method is to apply the field, H, as indicated and measure changes in AC, then apply the field parallel to DB and again measure AC.

In this case the dimension whose change in length was measured has been the same and the field has been applied in two directions at right angles to each other, one of which was parallel to the dimension whose variation was measured.

Both methods are open to question, but the latter seemed to have fewer objections than the first, particularly for specimens whose ratio of length to diameter was several hundred. If the first method is pursued the aeolotropic condition of the material will certainly produce a coefficient of change of length different in one direction from that in the other, consequently the comparison of the two changes cannot be made quantitatively. On the other hand when the field is applied in two different directions the aeolotropic nature of rolled and drawn metals prevents the securing of similar fields, as we should have for quantitative comparative tests. The latter method is most readily applied to specimens in long rod or tube form. It seemed, therefore, that conditions most nearly alike in these two effects could be secured when the direction of the field was varied and the direction of the dimension to be measured was kept the same. In other words no absolute comparative tests can be made of these two magnetostrictive effects unless one has a perfectly homogeneous specimen with which to work. So far as the author knows no such samples are obtainable. The author has started work to see what can be done in cast specimens. This study is therefore concerned with a qualitative comparison of the two magnetostrictive effects in the same specimen.

The first specimen of steel was a piece of water pipe, 30.74 cm. long, and whose inner and outer diameters were 10.20 and 11.35 cm. respectively. The wire was wound about it in the form of a toroid, Fig. 2, but not placed directly in contact with the steel so as to restrict the expansion of the cylinder. This was accomplished by winding the insulated wire about a wooden framework which consisted essentially of two wooden rings, *R*, *R*, held apart by small brass rods not shown in the figure. The windings were of No. 13, double cotton covered, copper wire, B. & S. gauge, making three hundred turns in all. The value of the circular field was calculated from the equation,

$$H_c = \frac{2nI}{10r},$$

where n indicates the number of turns, I the current in amperes, and r equals 10.77 cm., the mean radius of the iron cylinder.

The change in length of the cylinder was measured by means of the displacement of interference bands produced by the light reflected from the two glass surfaces, *g*, *g*, shown in Fig. 2. Light from a sodium flame, *S*, was reflected by the mirror, *M*, down to the optical surfaces of *g*, *g* and thence reflected back by nearly the same path to an observing micrometer microscope. This optical device for measuring the change in length could be used without disturbing the system both in the longitudinal and transverse effects. The method of observing the displacement of the bands was to locate the position of one band with a fixed cross-wire in the micrometer eye-piece and then set the movable cross-wire about where the band would come when the field was applied. After a trial or two the distance between the movable and fixed cross-wires would represent with considerable accuracy the displacement of the band. Knowing the width of the band it was possible to determine the amount of the change in length of the steel cylinder. This method of measuring the shift did not require the magnetizing current to be left on for any length of time and consequently heat effects were minimized.

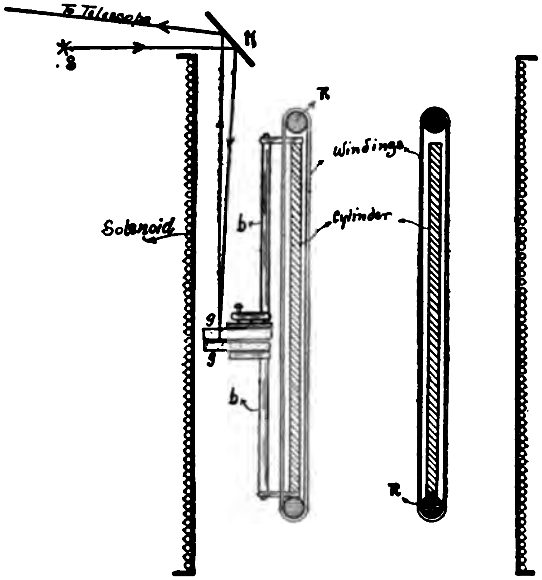

Fig. 2.

The change in length was measured as the field was increased from zero upward, with several makes and breaks of the circuit for each field strength measured. Successive sets of readings were taken in this way and the mean results are plotted in Curve 2, Fig. 3. Demagnetization by reversals occurred after each series of readings.

The longitudinal effect was obtained by placing a solenoid of a large cross-section around the toroid as shown in Fig. 2. The axis of the toroid coincided with that of the solenoid. The ends of the cylinder were 11.5 cm. back from the ends of the solenoid. The current both for the toroid and the solenoid was supplied by a storage battery. The measurement of the change in length due to the longitudinal field was carried out in exactly the same way as for the transverse field. The results are shown in Curve 1, Fig. 3, after making corrections for the end effects of the solenoid and the demagnetizing effect of the cylinder itself.

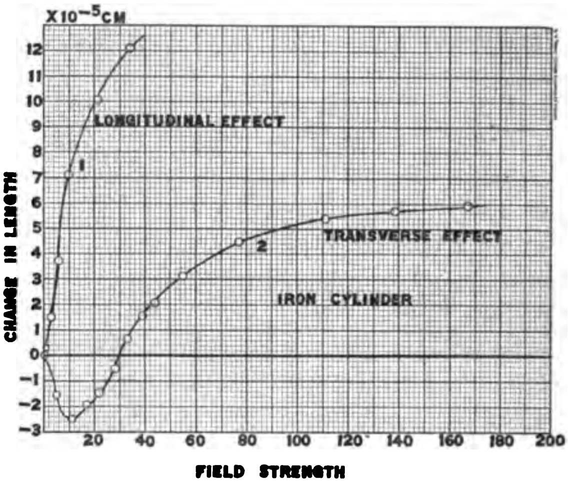

Fig. 3.

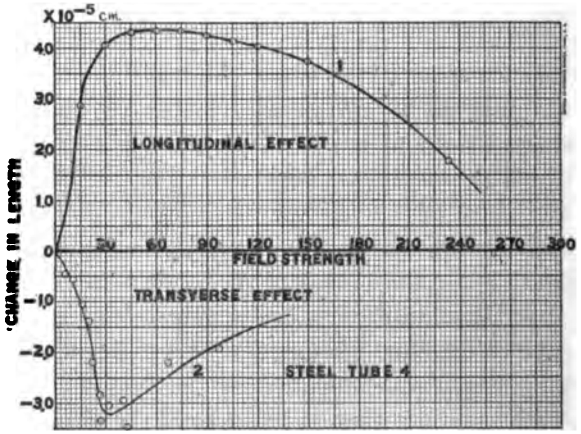

Fig. 4.

The solenoid had a length of 53.7 cm. and an average diameter of 28.74 cm. There were six layers with a total number of 1,466 turns.

The next specimen tested was a steel tube 80 cm. long and 0.297 cm.

in mean radius. This is known as tube 4 in previous investigations.[1] The device for measuring the changes in length due to the longitudinal and transverse fields was the same as used in former work. The transverse field was obtained by passing a current through only a few turns of insulated wire inside of the tube and consequently not very large fields could be obtained. The results are shown in Curves 1 and 2, Fig. 4.

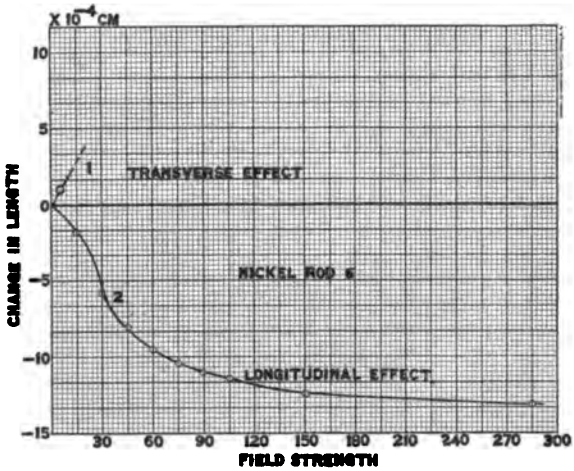

Fig. 5.

For the two specimens of nickel which were tried, much more difficulty was encountered in measuring the change in length for the transverse field than was experienced with the steel samples. This was due to the sensitiveness of nickel for heat effects. Curves are shown for only one specimen as the two were similar. For the few field strengths which could be measured only an increase in length was found which is the reverse of that for the longitudinal field (Fig. 5).

Discussion of Results.

Roughly, the curves seem to indicate this reversal of effect when the longitudinal and transverse fields are applied. The results obtained from the long specimen (80 cm.) seem the most trustworthy. The large demagnetizing factor for the short steel tube decreased the effective value of *H*, the field of the solenoid, to so large an extent that large field values could not be obtained for comparison. The values for the demagnetizing factors used were obtained from equations given by Ewing[2] and applied to a hollow cylinder, just as one would to a solid cylinder. This was done because Grotrian[3] has shown that for fields which do not attain

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[1] Phys. Rev., p. 261, Fig. 7, Vol. 34, 1912.

[2] Ewing, Magn. Induc. 3d ed., p. 25.

[3] Grotrian, Annal. der Physik, Vol. 52, p. 735, Vol. 54, p. 452.

values giving a saturated condition the demagnetizing factors of solid and hollow cylinders are practically the same. As to the reversal of effects, suppose one had a homogeneous specimen of a ferro-magnetic substance and assumed that the turning of the elementary magnets was the cause of the changes in length; then if the elementary magnet had the form of an oblate spheroid it would be easy to see why the application of a transverse field produced the reversed effect to that brought about by the longitudinal field. I have indicated this possibility in a former paper.[1] If the curves shown there for the single elementary magnet held for the group, then qualitatively the longitudinal and transverse effects should be the reverse of each other; but as pointed out, the aeolotropic condition of the specimen precludes any exact relation occurring in ferro-magnetic substances in drawn and rolled samples.

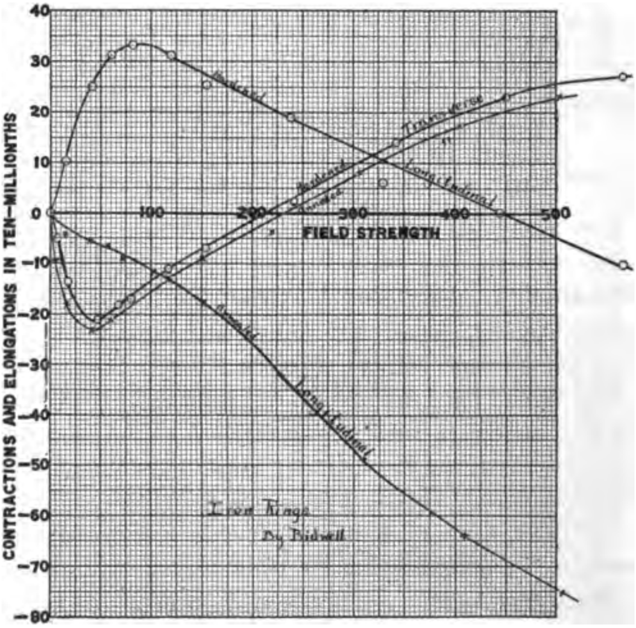

Fig. 6.

Bidwell[2] states that he had a specimen of iron which showed a contraction for all field strengths, and cites it as the only specimen which has yet been found showing this phenomenon. The author has found a specimen of steel which to all appearances showed the same phenomenon. Not until a much larger multiplying power of the device for indicating the change in length was used did it show a very small

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[1] PHYS. REV., p. 40, Vol. 34, 1912.

[2] Bidwell, Proc. Roy. Soc., Vol. 56, 1894.

increase in length for weak fields. If this was the case with Bidwell's specimen then his work would be still more confirmatory of the reversibility of the two effects. His results are shown graphically in Fig. 6. Bidwell's specimens were very small while the rods and tubes used in my work were 80 cm. long and consequently gave an advantage for multiplying the effect.

As one studies the various theories of magnetism he is impressed with how little they have to say regarding magnetostrictive effects and yet if one should classify the various magnetic phenomena, the various magnetostrictive effects would make the major part of the phenomena. One is reminded of the words of Joule,[1] that "when we are able to answer some of the questions regarding magnetostrictive effects in a satisfactory manner we shall probably have a much more complete acquaintance with the real nature of magnetism than we at present possess."

SUMMARY.

1. The results show a qualitative reversal of the two effects as the planetesimal hypothesis would indicate.

2. An exact quantitative reversal is not to be expected for several reasons: (*a*) non-homogeneity of the specimens, (*b*) in the transverse field there is not a uniform magnetic field applied, varying inversely as the distance out from the center of the specimen when the specimen is a hollow cylinder and the current is conducted through an insulated wire inside of the cylinder. In case the specimen is a solid cylinder then the field varies inversely as the square[2] of the distance from the center because the current flows in the specimen itself as a conductor, (*c*) the elementary magnets affected by the longitudinal field will be different from those influenced by the transverse field.

3. This comparison of the two Joule magnetostrictive effects should, according to previous investigators, give some clue as to change of volume. Knott[3] was undoubtedly the first to show that magnetization is generally attended by considerable change in volume, but later investigators did not find a relation existing between different specimens for volume changes which they had found for changes in length. For instance, since iron elongates and nickel contracts for initial field strengths, one might expect a difference in behavior of volume changes. Such is not the case as pointed out by Nagaoka and Honda.[4] Both metals increase in volume by magnetization. From the standpoint of

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[1] Joule, Phil. Mag., p. 76, Vol. 30, 1847.

[2] Northrop, PHYS. REV., Vol. 24, p. 479, 1907.

[3] Bidwell, Proc. Roy. Soc., vol. 56, 1894, p. 99.

[4] Nagaoka and Honda, Phil. Mag., Vol. 4, p. 71, 1902.

the planetesimal hypothesis, change in volume arises from the changed piling of the elementary magnets due to their orientation and there should be no particular relation between volume changes and the other magnetostrictive effects just as Nagaoka and Honda have shown.

4. This work points out why Berget[1] in working with a cylindrical bar measuring 5.2 cm. long and 1.95 cm. in diameter found that the length of the bar was still increasing at a field strength of 540 units. He neglected to take account of the demagnetizing factor which I have done in the case of specimen 1. The desirability of using long thin specimens in measuring the various magnetic effects is here again pointed out.

5. A complete theory of magnetism must explain the various magnetostrictive effects as well as the other magnetic phenomena.

PHYSICAL LABORATORY,
OBERLIN COLLEGE, OBERLIN, OHIO,
June, 1914.

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[1] Berget, Comp. Rend., p. 722, Nov. 7, 1892.

WAVE-LENGTH-SENSIBILITY CURVES OF CERTAIN CRYSTALS OF METALLIC SELENIUM; AND A PARTIAL EXPLANATION OF THE COMPLEXITY OF LIGHT-ACTION IN SELENIUM CELLS.

By L. P. Sieg and F. C. Brown.

In a recent paper[1] it was shown that there was no apparent characteristic wave-length-sensibility curve for light-sensitive selenium, except when that selenium was made light-sensitive under very definite physical conditions. Some of these conditions have recently been worked out and published by Dieterich[2]. The conclusion seemed advisable if not necessary that the selenium in the selenium cell must be made up of a varying crystal structure, and that in some way the varying characteristics could be explained in terms of the individual crystal units composing the selenium. The recent production by one of us of several crystals of large size[3] has made it possible to obtain further evidence as to the manner in which the individual crystals might explain the variations in mass selenium. In the present work we have studied the wave-length-sensibility curves of a number of crystal forms and have already obtained curves which can explain most of the variations found in selenium. As predicted in our former paper we have found these crystals to have directive axes in the property of light-action as well as in the property of light transmission and crystal structure.

The method for obtaining the wave-length-sensibility curves was in the main the same as that described in our former papers. However for adjusting to equal energy throughout the spectrum the procedure was altered to permit of more rapid working. An aluminium sector disc was calibrated in use with a given Nernst glower, in such a manner that a particular setting of the sector for a corresponding wave-length always allowed the same energy to fall on the thermopile or the crystal as desired. For comparative work with different crystals this method was very satisfactory indeed. From time to time the energy values were checked just prior to each light exposure on the crystal, and as these results

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[1] Phys. Rev., N.S., Vol. 4, p. 48, 1914.

[2] Phys. Rev., N.S., Vol. 4, p. 467, 1914.

[3] Phys. Rev., N.S., Vol. 4, p. 85, 1914.

checked and because crystals of like character always gave the same sensibility curves under the same conditions, it was certain that no important errors accrued from the method used.

Some of the crystals were not sensitive enough to be used with short periods of illumination. For this reason all crystals were compared for illumination of 30 seconds' duration. Just what differences exist because of varying periods should be investigated in connection with the rates of change as designated in previous papers.

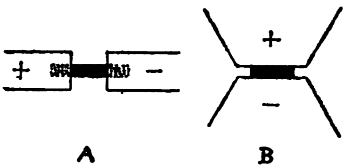

Fig. 1.

The light-sensibility of the crystal was obtained when it was placed between electrodes of the same material as diagrammed in Fig. 1. Both silver and brass electrodes were used at different times, but thus far there does not appear to be any importance to be attached to the material or the structure of the electrodes. In one apparatus, designated *A*, the crystal was pressed against a glass plate by two silver electrodes acting on the ends of the crystal. The illumination could thus be applied to either the front or the back side of crystal, as well as to the edges of the crystal. In the apparatus, *B*, the entire crystal was clamped between brass electrodes. It is to be noted that in the apparatus *A* the light acts directly on the part of the selenium that is not under pressure by the electrodes and that conversely the crystal when placed in apparatus *B* was under pressure at the place of illumination. Further on will be mentioned slight modifications of these methods of clamping. It will appear that the advantages of any one method will depend largely on the shape of the crystal to be used and the crystal face to be illuminated.

Comparative Results with Individual Crystals and with Selenium Cells.

At different times we have shown that light-sensitive selenium is not a simple substance but a complex mechanism and we have taken the view that the complexities arise from more or less elementary constituents. This requires that the atom of selenium can not be the sole unit in the mechanism acted upon. The information that we have thus far is very strong evidence that the variation of the properties of light-sensitive selenium is inherent in the crystals that compose it, that these crystals by virtue of their form, position and location determine the properties of the selenium mass. The reader will we believe become at once sympathetic to this view by a casual observance of the wave-length-sensibility curves in Figs. 2*a* and 2*b*. Fig. 2*a* is taken from our former paper[1] and represents the characteristic curves of various varieties of light-sensitive selenium as found in selenium cells. Fig. 2*b* represents the characteristic sensibility curves of individual crystals, which differ in crystal form or differ as to the direction of the crystallographic axis with regard to the direction of illumination. We do not claim to have exhausted the list of crystals or possible curves that might be obtained with individual crystals. In fact some crystals already discovered have not been investigated because of their insufficiently large size. However the resemblance between the two sets is striking indeed. Both show some curves with a maximum in the ultra-violet, and both show the same range of the maximum in the extreme red. The only lack of agreement is in the fact that we have not yet discovered that crystal which has a distinct maximum in the region of .55 μ, such as found in the typical Dieterich cell. And along with this fact it must be noted that we have examined no crystal which in any position showed a diminishing sensibility at the blue end of the spectrum. In making these comparisons we have made no allowance for the enormous difference in the reflecting power of the selenium in selenium cell from that of the crystal. The former absorbs about 98 per cent. of the light in all parts of the spectrum, while the

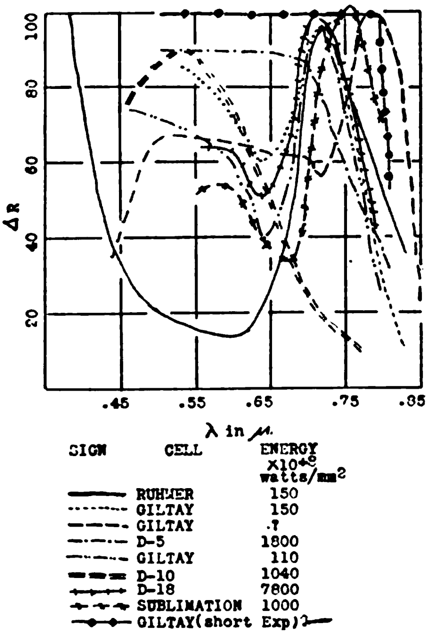

Fig. 2*a*.

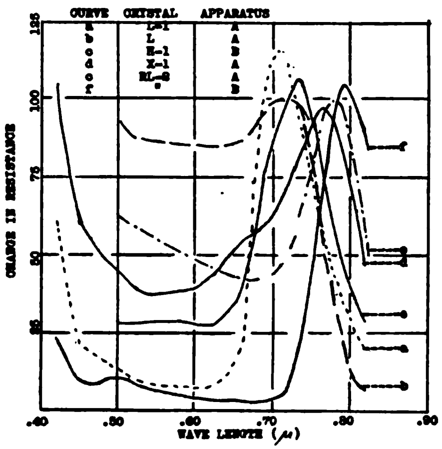

Fig. 2*b*.

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[1] Loc. cit.

latter probably does not absorb more than 60 to 70 per cent. of the incident light. What may be the consequences of this difference is not certain. It will of course be advisable to measure the absorption constant of the crystals.[1] In as much as the selenium in mass does not show a very great selective absorption and since the shifting of the red maximum is almost identical with the selenium crystals and the selenium mass, it is not expected that the crystals will show much of a variation in reflecting power in this region.

In what ways the crystals give rise to varying sensibility curves will be taken up in the remainder of the paper.

EACH CRYSTALLINE FORM HAS A DISTINCTIVE CHARACTERISTIC CURVE.

Before asserting that each crystalline form of selenium has characteristic photo-electrical properties, it is advisable to know that the character of the curve is not altered by other physical conditions which might vary in the experiments. For example it was determined that the direction of current flow did not alter the sensibility curves when the illumination was on the same part of the crystal. Since it had been shown by one of us[2] that the resistance might vary by a factor of several hundred depending on the pressure on the crystal, there was considerable doubt as to whether the pressure might control the character of the sensibility curves. In fact we mentioned in one of our former papers that a part of the variations in the selenium cells might be accounted for by variations in the pressure on the selenium as it was placed on the cell form. We therefore placed an acicular hexagonal crystal between the electrodes of apparatus *B* and mapped out the sensibility curves when the pressures were of such a magnitude that the conductivity increased by a factor of 50 to 1. The intensity of illumination and other physical conditions were unchanged. The results of the investigation are shown by the two upper curves in Fig. 3. There is no evidence of any change in the position of the maximum or in the ratio of the values of the maximum to the minimum as a result of the change in pressure.

Since Pfund and ourselves had observed an ironing out of the maximum in certain selenium cells by diminished illumination it was thought advisable to see if the sensibility curves underwent any change when the intensity of illumination was varied. We used the same crystal and apparatus just mentioned above. These comparison curves are also

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[1] Recently a preliminary test was made to determine the reflecting power of these crystals throughout the spectrum. This was found to be practically constant, and probably between 30 and 40 per cent, thus agreeing with the values of Foersterling and Freédericksz (Amer. d. Phys. 43, 1227, 1914) for metallic and amorphous selenium mirrors.

[2] PHYS. REV., N.S., Vol. 4, p. 93; 1914.

shown in Fig. 3. Again there is no apparent change in the curves due to varying intensity. This result is particularly important at this point because it has not been determined if there is a variation of the reflection coefficient in different crystals or along different axes of the same crystal. If there is such a selective variation we have no means of knowing just what is the relative amounts of energy absorbed when we can measure only the incident energy. Unless the selective variation is of some magnitude, we can be reasonably certain that the amount of light absorbed by the crystal does not concern the character of the sensibility curve. Of course electrical theory requires that the reflection constant should vary with the conductivity of the material. But since the resistance of these crystals is so large, it is believed that the reflection due to the free or conducting electrons is small compared to the reflection by the charges in the bound atomic structure. The values of the intensity in Fig. 3 are for the incident energy. The values of the absorbed energy must be many times smaller, but even if the absorbed energy does vary by a factor of fifty the ratios of the absorbed energy surely can not be enough different to make any material error in the conclusions that we have drawn, concerning the independence of light-action and the pressure effect.

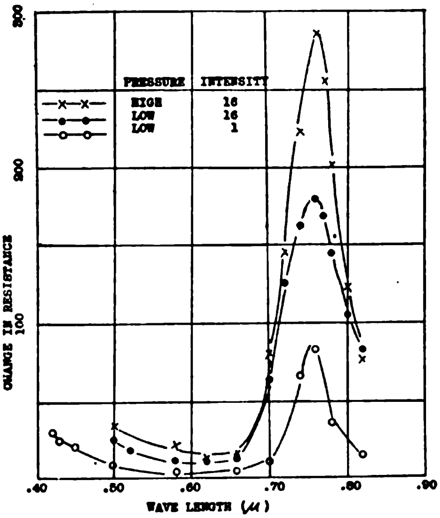

Fig. 3.

It has been shown[1] for selenium cells and also for single crystals that the resistance varies greatly with the electromotive force across the selenium. It is sufficient to mention here that we could not observe any change in the sensibility-curves due to a variation of the resistance by use of varying potential differences between the electrodes.

In the formation of the crystals there were sometimes variations of 20° or 30° in the oven temperature. As nearly as could be discerned such temperature treatment did not alter the sensibility curve of a given crystal form.

Having shown that the preceding conditions did not alter the character

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[1] Paper by Brown and Stebbins, Phys. Rev., 26, p. 273, 1908 and Brown, Phys. Rev., 33, p. 1, 1911, and other paper not yet published.

of the sensibility curves, we proceeded to investigate the variation of the sensibility curves for the different crystal forms. For this we selected three specimens which were of sufficient size for the purpose. One (*L*–1) was a lamellar crystal[1] with axes perpendicular to the edges. Its size was about 4 × 2 × .2 mm. A second was of the acicular hexagonal form (*H*–2), of size about 10 × .4 × .4 mm., and the third was a combination of crystals that had grown out from a central hexagonal spine. The angles and the form of this are not yet known, but the crystal is probably of the second crystal system. Crystals of the above types are shown in the above articles referred to.

The lamellar crystals were placed in apparatus *A* and illuminated perpendicular to either of the large flat surfaces. In either case the sensibility was that shown by the upper curve in Fig. 4.

The acicular crystals were placed in apparatus *B*. They were illuminated perpendicular to the axis of the crystal. It was apparently immaterial as to which of the six faces were illuminated. The curve for this type showed a sharp maximum in the same place as that for the lamellar crystal (see *H*–2, Fig. 4), but it was very much sharper and its magnitude was relatively much greater than for the lamellar crystal.

The third crystal showed a maximum at wave-length .7 μ, but it was not as sharp as that of the acicular crystal (see *X*–1, Fig. 4). It should be mentioned that all the curves in this paper are drawn each to an arbitrary scale, so that the relative sensibility of each crystal can not be ascertained from the curves. In the crystals just referred to however the sensibility was of the same order of magnitude in each.

THE SAME CRYSTAL GIVES DIFFERENT CURVES WHEN ILLUMINATED ALONG DIFFERENT AXES.

Previously we have shown the crystals to be doubly refracting.[2] Electrical theory requires that any material having directive axes in its optical properties in this way, shall also show directive properties in any phenomena involving electro-optical interactions. Simply stated if a crystal shows electrical phenomena as a result of light acting upon the crystal as a crystal and not as an element, the electrical phenomena should vary when the illumination is along different axes. On the other hand if the light action should be on the atoms without regard to the crystal mechanism it is not conceivable how there could be any difference in the sensibility curves arising from illuminating different sides of the crystal providing the reflecting power is constant.

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[1] For further information about these crystals see papers by Brown, PHYS. REV., loc. cit., and by Brown and Sieg, Phil. Mag. (6), Vol. 28, p. 497. 1914.

[2] Brown, loc. cit.

Several crystals were investigated as to the conductivity change when illuminated along different axes. The best specimen was a lamellar crystal 4 × 2 × 0.2 mm. The large surfaces were rectangular and approximately parallel. We were not able to determine the angles of the edge faces.

When the illumination was directed on either of the large faces as shown by the arrow, marked 1 (Fig. 5), the sensibility began to increase rapidly at wave-length .66 μ and held a broad maximum with the mean position at .74 μ. At λ = .80 μ there was a very large sensibility. By illuminating either of the short edges of the crystal the sensibility had just started to rise at λ = .74 μ, the position where with side illumination the maximum occurred. In this instance the maximum was very sharp and located at .79 μ. In the infra red beyond wave-length .82 μ this edge of the crystal was more sensitive relatively to the minimum than was the flat surface at its maximum. When either of the longer edges was illuminated as shown by arrow 3, the maximum was at .76 μ. Thus a single crystal may have its maximum shifted at least five wave-lengths by changing the side of the crystal that is illuminated. We carefully observed that the direction of current flow made no difference in these experiments.

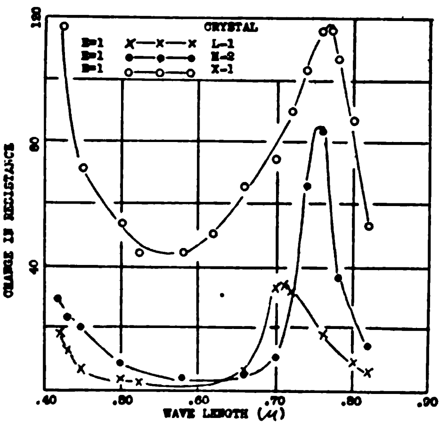

Fig. 4.

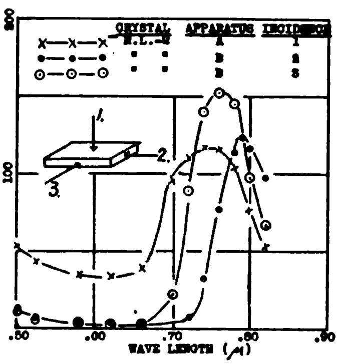

Fig. 5.

It is altogether improbable that the variation of the selective absorption plays an important rôle in giving these different sensibility curves. If so the results are consitent with the idea that the light acts upon the crystal structure rather than upon the atoms. However this mode of explanation of the shifting of the maximum can be sidestepped perhaps if we wish, by attributing the shift toward the longer wave-lengths to transmitted action as will be explained in the last section of this paper.

THE ACTION OF POLARIZED LIGHT.

We have made some observations on the action of polarized light, the significance of which is satisfying only in a general way. The experiments were called forth after we had investigated the effect of varying the angle of incidence on a lamellar crystal. By such variations of the angle of incidence, sensibility curves were obtained for different angles as indicated in Fig. 6. For an angle of about 60° there was a decided change in the character of the curve but for angles larger or smaller the curve form was almost unchanged. It was thought that this might be about the angle of maximum polarization, and that consequently only one component of the light might enter the crystal at this angle.

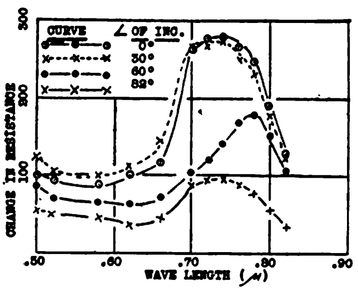

Fig. 6.

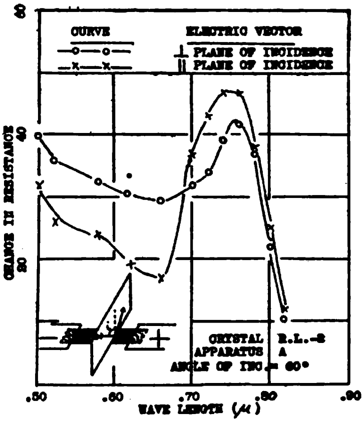

Fig. 7.

We therefore tried first light polarized in the plane of incidence and then at right angles to this plane. For all angles of incidence except 60° there was no change in the character of the sensibility curve, but at about this angle the effect of light polarized in opposite planes was quite different. When the electric vector (see Fig. 7) of the light was perpendicular to the plane of incidence the sensibility curve was much like that shown for the same angle shown in Fig. 6. But when the electric vector was parallel to the planes of incidence the maximum was relatively much broader and higher than in the previous case.

A part of the difference between the location of the maximum in Figs. 6 and 7 may have arisen from the fact that we had no accurate means of setting the angles.

We are concluding from our experiments first that our Hilger monochromator partially polarizes the light that passes through it and secondly that under certain conditions a given intensity of illumination may produce different results depending on its state of polarization.

The Direct and the Transmitted Action of Light.

Recently we have shown[1] that the seat of light action in selenium crystals is distributed throughout the crystal, and that light may alter the conductivity of a crystal 10 mm. or more away from the point of impingement. At that time we had not succeeded in determining whether this action could be transmitted from one crystal to another. Now we have gone further and found not only that the action of light can be transmitted from one crystal to another, but we have also mapped out the sensibility curve for such indirect action and compared it with the direct action, where the light falls on the part of the selenium between the electrodes.

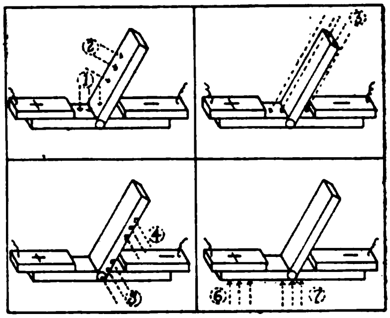

Fig. 8.

We chose for this work a lamellar crystal to the middle portion of which was grown another crystal. This crystal and the apparatus *A* are shown in a very highly conventionalized way in Fig. 8. The dotted arrows show the portions that were illuminated. Wherever any or most of the light fell upon the part of the crystal carrying the current it was called direct action as indicated by the change of conductivity of the crystal. Contrarily if the light impinged on any part of the crystal that did not carry the current, the light effect was designated as transmitted light action.

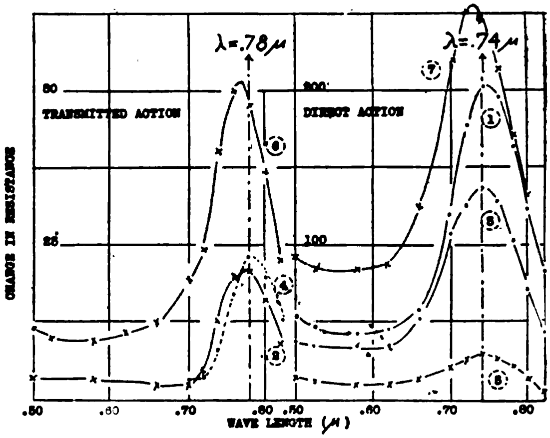

Fig. 9.

In Fig. 9 are shown the sensibility curves, on one side for the trans-

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[1] Phil. Mag., Ser. 6, Vol. 28, p. 497, 1914 and Phys. Rev. N. S., Vol. 4, p. 85.

mitted action of light and on the right side for the direct action of light. It may be seen that in positions (1), (3), (5) and (7) Fig. 8 the light acts directly on the part of the selenium that directly takes part in the conductivity. The corresponding curves in Fig. 9 show the maximum in all cases to be at about .74 μ. But when the conductivity is altered by transmitted action as by the impingement of the light at positions (2), (4) and (6) the maximum is shifted to = .78 μ. Also it should be noted that in the transmitted action the ratio of the maximum to the minimum is of the order of 5 to 1 while in the direct action this ratio is only about half as great. Thus the position and the relative magnitude of the maximum depend on whether the light impinges directly on the part of the crystal that conducts or whether the change of conduction takes place at a distance from the point of impingement. Of course it is impossible to say what constitutes direct action or to separate entirely the two actions in the so-called direct action. This latter difficulty arises partly from the fact that a part of the light is always absorbed near the surface and does not penetrate all the conducting selenium even in the most favorable condition. Thus if we could obtain and isolate the direct action completely we might find the maximum at considerably shorter wave-lengths than we have indicated.

In this connection an interesting experiment that suggests itself is to see if the thickness of the selenium in the selenium cell determines the position of maximum sensibility. Thus the selenium blocks studied by White[1] should show a maximum far out in the red, because most of the change of conductivity by light in these blocks must be an indirect or transmitted action.

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[1] Phil. Mag., Ser. VI, Vol. 27, p. 370, 1914.

ON THE SILVERING OF QUARTZ FIBERS BY THE CATHODE SPRAY.[1]

By Horatio B. Williams.

QUARTZ fibers coated with a layer of some electrical conductor are often used in the construction of delicate electrometers and similar instruments and it is convenient to be able to prepare them with certainty and ease. The method usually employed to secure a conducting layer depends on the deposition of silver by one of the processes of chemical reduction. For some purposes this method is adequate, but the reduction is difficult to control, the deposit is usually rough and crystalline and the conductivity attained is out of all proportion small for the weight of material deposited. In torsional instruments a thick coating of metal is disadvantageous by reason of the diminution of elasticity entailed. Bestelmeyer[2] reached a satisfactory solution of the problem for such instruments as electrometers through platinizing the fibers by disintegration of a platinum cathode in vacuo. Fibers coated by this method have a very high resistance and are suitable only for electrostatic instruments. Within the last few years the string galvanometer of Einthoven has come into very general use in physiological laboratories. It seems to be less well known among physicists though it has many excellent qualities which would seem to recommend it to a place in the physical laboratory. In this instrument the silvered thread which constitutes the moving system is required to have relatively low resistance—35,000 to a few hundred ohms. It is usually desirable that the fiber be as light as possible for a given conductivity and for some purposes one may wish to limit the air damping which is always a relatively large part of the total damping unless the resistance of the circuit is low. Chemical silvering entails a large air damping due to the roughness of the deposit, and the inertia of a rough thread must be manifestly greater for a given conductivity than it would be if the coating were smooth and coherent. Cannegieter[3] prepared satisfactory fibers for use in the string galvanom-

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[1] See preliminary report, Phys. Rev., Vol. 2, Series 2, p. 402, 1913.

[2] A. Bestelmeyer, Leitendmachen von Quarzfäden, Zeitschr. f. Instrumentenk, Vol. 25, p. 339, 1905.

[3] H. G. Cannegieter, Leitendmachen dünner Quarzfäden durch Kathodenzerstaübung mit nachfolgender galvanischer versilberung, Zeitschr. f. Biologische Technik u. Methodik, Vol. 2, p. 21, 1910.

eter by platinizing or silvering them by cathode disintegration and subsequently increasing their conductivity by electroplating with silver. His process involves securing the fiber on the anode of the discharge tube and after the spraying process, transferring it to a glass frame and immersing in a silvering solution with subsequent rinsing. The risk of loss by breakage is naturally increased by so great a number of manipulations. By the process outlined below the necessary conductivity is secured by cathode disintegration alone.

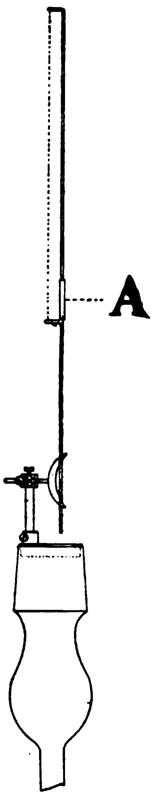

Fig. 1.

Methods of preparing the naked quartz fibers have been described by Boys,[1] Nichols,[2] and Wertheim-Salomonson.[3] I have found a modification of Wertheim-Salomonson's method (which was based on that of Nichols), more satisfactory in the case of extremely fine fibers than the original cross-bow method of Boys. The fibers are collected on frames made of fused silica rod[4] 3 mm. in diameter of the form indicated in Fig. 1. Fused shellac in a thin layer is used to cement the fibers to the frames. A short piece of glass tubing, *A*, Fig. 1, is slipped over the frame. This prevents deposition of silver on the part of the rod covered and in subsequently measuring the resistance of the thread the tube is slid along the rod, insulating the ends of the fiber from each other. The frame is held in the discharge tube by a brass support, also shown in the figure, which permits of adjustment in every direction. Fig. 2 is a schema of the assembled apparatus. The discharge tube is shown at *A*; *B* indicates a liquid air trap and *C* an arrangement for electrolytic generation of hydrogen. The cathode, similar in form to that of Cannegieter, is made of a piece of sheet silver 20 cm. long, 9 cm. wide and .25 mm. thick, bent to a U-shaped section.

The silvering is effected by passing a direct current of about .012 ampere through the tube in an atmosphere of dry hydrogen at a pressure of approximately .2 mm. The current and fall of potential across the tube are kept nearly constant and the pressure varied as may be necessary.

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[1] C. V. Boys, On the Production, Properties and Some Suggested Uses of the Finest Threads, Phil. Mag., Vol. 23, 5th Series, p. 489, 1887. Also C. V. Boys, Quartz Fibers, The Electrician, Vol. 38, p. 205, 1896.

[2] Edward L. Nichols, The Galvanometer, Electric Power, Vol. 6, p. 178, 1894.

[3] J. K. A. Wertheim-Salomonson, Anfertigung und Gebrauch dünner Quarzfäden, Zeitschr. f. Biol. Tech. u. Method., Vol. 1, p. 35, 1908.

[4] This rod, which is also used to make the fibers, can be obtained from The Thermal Syndicate, Ltd., 50 East 41st St., New York.

The usual drop across the tube is about 800 volts. Current is supplied by small generators. The discharge tube is made considerably longer than the cathode in order to keep the ground joint well away from the discharge so as to avoid contamination of the tube by the mixture used to lubricate the joint. Extension of the tube well above the cathode delays silvering of the top which would prevent inspection of the interior after closure. The large volume of the tube is advantageous since the effect of self-evacuation is less troublesome as was pointed out by Stark.[1] In a preliminary report to the Physical Society it was mentioned that large quantities of hydrogen seemed to be occluded. At that time the supporting frames were held in a perforated cork stopper. On substituting the brass holder the rapid disappearance of gas ceased.[2] The vacuum now changes very slowly and admission of small amounts of gas at intervals of 10 minutes or more suffice to maintain the necessary pressure.

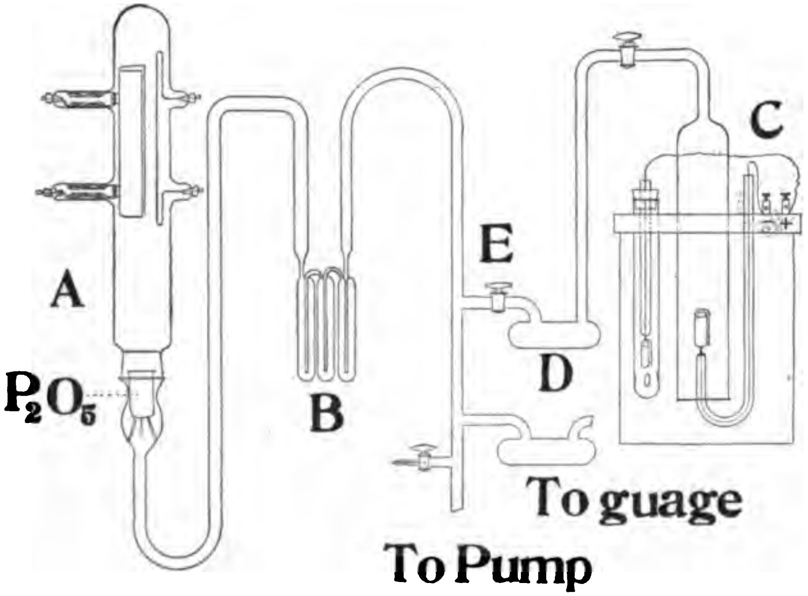

Fig. 2.

It has been shown by Schmidt,[3] that the canal rays exert apparent

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[1] J. Stark, Der Kathodenfall des Glimmstromes als Funktion von Temperatur, Stromstärke und Gasdruck, Ann. der Physik, Vol. 12, p. 9, 1903.

[2] Since the above was written this fluctuation of pressure has been shown to be due to a reaction in which hydrogen phosphide is formed from the phosphorus pentoxide used for drying. The reaction seems to be conditioned on the presence of traces of water vapor in the gas and can be produced without the cork stopper by removing the liquid air from the trap.

[3] G. C. Schmidt, Ueber die chemischen Wirkungen der Canalstrahlen, Ann. der Physik. 4th Series, Vol. 9, p. 711, 1902.

oxidizing or reducing influences according to the nature of the gas in the tube. Hydrogen was chosen to secure the reducing effect, though I had presumed an inert gas like nitrogen might be equally good. Recent work by Kohlschütter and Noll,[1] makes it appear likely that hydrogen is preferable for the reason that in it the metal deposits in a more coherent layer. In earlier experiments using as the source of current an induction coil with rectifying valve tubes, uniform deposits were obtained whose conductivity was very slight, though they attained a thickness several times that of the quartz core. This occurred even with gold electrodes in a hydrogen atmosphere. At this time no liquid air trap was used and mercury vapor was undoubtedly present, also probably more water vapor than with the use of the trap. The experiments of Kohlschütter and Noll suggest that the presence of these vapors may have been partly responsible for the result, but the source of current was an important factor, since with a direct current from generators good deposits are often obtained without using the liquid air trap.

Tyndall and Hughes[2] have shown that the rate of disintegration of a cathode for a constant current and cathode fall is constant over a considerable range of pressures. In attempting to secure a definite rate of increase in the conductivity of a fiber so as to be able to judge the time necessary to obtain a desired result, it would be better to measure the cathode fall than to take the drop across the entire tube. However, where the length of the positive glow is comparatively small, the difference between cathode fall and total fall is not great and the uniformity of results secured by maintaining constant current and approximately constant fall of potential across the entire tube has been satisfactory. The time required to produce a given conductivity in a fiber of given dimensions may be best illustrated by a few actual experimental results.

Diameter of Quartz Fiber.	Time of Discharge.	Resistance of 14 Cm. Length.
.0025 mm.	1 hour,	3,020 ohms.
.0025 mm.	1 hour, 30 minutes,	2,400 ohms.
.002 mm.	2 hours,	2,700 ohms.
.0014 mm.	2 hours,	15,000 ohms.
.0013 mm.	3 hours, 13 minutes.	11,200 ohms.

Under the microscope the deposit appears smooth and uniform. No increase in diameter can be measured after silvering, at least not by the

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[1] V. Kohlschütter and A. Noll, Ueber feine Metallzerteilungen, Zeitschr. f. Elektrochem., Vol. 18, p. 419, 1912.

[2] A. M. Tyndall and H. G. Hughes, Kathode Disintegration in a Vacuum Tube, Phil. Mag., Vol. 27, p. 415, 1914. For a very complete bibliography of the subject of cathode disintegration from 1852 to 1912, see V. Kohlschütter, Die Zerstaübung durch Kanalstrahlen, Jahrbuch der Radioaktivität und Elketronik, Vol. 9, p. 355, 1912.

microscopic methods at my disposal. Chemically silvered fibers show a distinct increase in diameter for deposits which give them resistances of the order of those in the above table. The silver deposited in the cathode tube must be much more compact and coherent. The resistance per unit length is very uniform and considering the silver as a uniform thin sheet applied to a circular cylinder, one may calculate the thickness from the conductivity. For such a fiber as the last in the table the calculated thickness is 48 × 10^{-6} mm. This is probably less than the actual thickness, since it is doubtful if the coating is ever as perfectly coherent as the silver in a solid wire.

The durability of the coatings has been good considering their great tenuity. All the fibers have increased in resistance with time. The behavior of several fibers in this respect is tabulated below.

Number of Fiber.	Data.	Resistance, Ohms.
6	Dec. 9, 1913,	2,400
	Dec. 25, 1913,	3,000
	June 27, 1914,	3,390
	August 3, 1914,	3,420
7	Dec. 9, 1913,	3,020
	Dec. 25, 1913,	3,450
	June 22, 1914,	3,750
	August 3, 1914,	3,820
12	June 27, 1914,	15,300
	July 8, 1914,	16,000
	August 3, 1914,	17,140

Using the fibers at maximum tension in the string galvanometer seems to hasten deterioration and on increasing the tension nearly to the breaking point I have seen the silver coating become electrically discontinuous without rupturing the quartz.

The fibers are fastened by soldering to brass tips for insertion in the galvanometer. Ordinary tin-lead solder without flux is satisfactory for chemically silvered fibers, but the heat necessary to melt it seemed at times to injure the cathode silvered threads. I have used Wood's metal for about a year with good results. This alloy does not adhere well to brass, but if the brass is first "tinned" with ordinary solder and a little Wood's metal allowed to melt on the tinned surface before it cools, the soft alloy can be made to adhere subsequently without difficulty. Frequently when fibers cease to conduct it is found that the soldered joint is the point of discontinuity and resoldering is usually possible.

Department of Physiology,
Columbia University, New York,
August, 1914.

NOTE ON THE DIRECT DETERMINATION OF *h*.

By O. W. Richardson.

In a paper published in the Physical Review, Vol. 4, p. 73 (July, 1914), under the title "A direct determination of *h*" Professor Millikan submits a resumé of previous experimental work which I venture to think may be misapprehended by those who are insufficiently familiar with the history of this subject. In discussing Einstein's photoelectric equation he points out that it involves the following consequences:

"(1) That there is a linear relation between the frequency of the impressed light and the maximum energy of emission of the electrons ejected by it.

"(2) That the slope of the line representing the linear relation between P.D. and ν is h/e, *i. e.*, that this slope times e is Planck's "*h*."

"(3) That the intercept of the P.D. line on the ν axis gives the frequency ν_0 at which the metal in question first begins to be photoelectrically active."

He then goes on to state that "The second and third of these assertions have not heretofore been made the subject of accurate test nor can they be so made without simultaneous measurement *in vacuo* of both contact potentials and photo-potentials in the case of metals which are sensitive throughout a long range of frequencies. The spectral lines used in the test must further be strictly monochromatic and of frequency determinable to a high degree of precision."

As regards the first of the three assertions he quotes an opinion of Pohl and Pringsheim as to the inadequacy, for deciding between various alternatives, of the experiments of Hughes and of Richardson and Compton. I shall not enter specifically into the merits of this particular question since the objections raised by Pohl and Pringsheim have already been answered by Professor Hughes.[1]

I should like, however, to be allowed to call attention to the following facts:

1. The three assertions enumerated above form three of the principal

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[1] Phil. Mag., Vol. 27, p. 473 (1914).

conclusions drawn by Richardson and Compton[1] from the experiments they describe in the paper to which Professor Millikan refers.

2. In those experiments the contact potentials and photo-potentials were determined *in vacuo* simultaneously. It is true that the method used for getting the contact potentials was a novel one and depends upon the determination of the applied potential difference at which the photo-electric current just becomes saturated. This method had, however, previously been shown to be reliable by the experiments of Compton[2] and its validity has since been confirmed by Page.[3]

3. Although the experiments did not cover so wide a range of frequency in the exciting light, the metals employed include sodium, the one examined by Professor Millikan.

4. So far as monochromatism and definiteness of frequency are concerned there is no doubt that the apparatus used by Richardson and Compton was quite adequate to the purposes of the experiments..

In drawing attention to the facts mentioned above I am far from desiring to disparage Professor Millikan's achievement, the importance of which can hardly be exaggerated. There is no doubt that the work of Kadesch[4] exhibits an improvement in accuracy over the earlier experiments of Hughes and of Richardson and Compton and that Professor Millikan has succeeded in effecting a still greater advance in the same direction. The results of his investigation I am particularly glad to welcome as they are so entirely in agreement with the views which the experiments made in collaboration with Dr. Compton led me to advocate.

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[1] Phil. Mag., Vol. 24, p. 575 (1912), Abstract in Phys. Rev., Vol. 34, p. 393 (1912).

[2] Phil. Mag., Vol. 23, p. 579 (1912).

[3] Amer. Journal of Science, Vol. 36, p. 501 (1913).

[4] Phys. Rev., Vol. 3, p. 367 (1914).

TEMPERATURE DISTRIBUTION IN AN INCANDESCENT LAMP FILAMENT IN THE NEIGHBORHOOD OF A COOLING JUNCTION.

By A. G. Worthing.

Introduction.

THE filaments of incandescent lamps are subject to end-losses which result from the cooling effects of supports. These losses are sometimes of considerable magnitude and must be determined if it is desirable to study certain of the properties of the filament. The variations existing near such a cooling junction may, indeed, afford the opportunity for the convenient measurement of certain other properties, such as the thermal conductivity or the Thomson electromotive force, subjects on which the writer hopes to present papers soon. A knowledge of the temperature distribution in such cases is of considerable value.

Theoretical Considerations.

The theory which is involved here assumes a homogeneous, cylindrical filament of uniform cross-section and surface condition, in an evacuated chamber, the filament being heated by an electric current which is uniformly distributed over any cross-section. A small amount of speculation leads one to expect, as one proceeds from the cooling junction, a rapid rise in temperature which approaches a maximum value according to some exponential law. The isothermal surfaces within might be expected to be somewhat similar to portions of paraboloids of revolution belonging to a family whose axis is the axis of the filament. The fundamental differential equation as used here is an expression in cylindrical coordinates stating that, for the condition of a steady state, the net rate of conduction of heat into a small element of such a filament plus the rate of development of heat electrically within it equates to zero. Consider a hollow cylindrical element of such a filament of length Δl, of radius r and of radial thickness Δr, whose axis coincides with the axis of the filament. There follows:

$$(1)\quad 2\pi r\left[k\frac{\partial^2 T}{\partial l^2}+\frac{dk}{dT}\left(\frac{\partial T}{\partial l}\right)^2\right]\Delta l\Delta r+2\pi r\left[k\frac{\partial^2 T}{\partial r^2}+\frac{k}{r}\frac{\partial T}{\partial r}+\frac{dk}{dT}\left(\frac{\partial T}{\partial r}\right)^2\right]\Delta l\Delta r+2\pi r i^2\rho\Delta l\Delta r=0.$$

The meanings of the above symbols together with others to be used later are given in Table I. The first, second and third terms represent respectively the net rate of heat conduction into the element of volume through the ends, the net rate of heat conduction into the element of volume radially, and the rate of heat production in the element of volume electrically. (1) leads directly to

$$\frac{\partial^2 T}{\partial l^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{k}\frac{dk}{dT}\left[\left(\frac{\partial T}{\partial l}\right)^2 + \left(\frac{\partial T}{\partial r}\right)^2\right] + \frac{i^2\rho}{k} = 0. \tag{2}$$

TABLE I.

T	Temperature.
l	Distance parallel to filament axis measured from the cooling support.
r	Radial distance.
ρ	Resistivity.
k	Thermal conductivity.
i	Current density.
r_0	Radius of the filament.
T_0	Temperature at the filament surface.
T_{0m}	Maximum surface temperature.
k_m	Thermal conductivity at the maximum filament temperature.
k', T', T_0', T_{0m}'	Quantities similar to k, T, T_0, T_{0m} in a system where $(dk'/dT') = 0$.
c, a, μ, l_0, ν, A	Constants whose meanings will be defined later.

(2) may be changed to a more suitable form by assuming a scale of temperature T', in which

$$\frac{dk'}{dT'} = 0; \tag{3}$$

we then have

$$\frac{\partial^2 T'}{\partial l^2} + \frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r} + \frac{i^2\rho}{k'} = 0. \tag{4}$$

In case

$$\frac{i^2\rho}{k'} = a^2 T' \tag{5}$$

we finally have

$$\frac{\partial^2 T'}{\partial l^2} + \frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r} + a^2 T' = 0. \tag{6}$$

By means of the well-known device of assuming

$$T' = RL, \tag{7}$$

where R and L are respectively functions only of r and of l, two independent equations may be obtained from (6):

$$\frac{d^2 L}{dl^2} - \mu^2 L = 0, \tag{8}$$

$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}+(\mu^2+a^2)R=0, \tag{9}$$

where μ^2 is a constant.

There are three boundary conditions which must be fulfilled.

$$\left[\frac{T'}{T_{0m}'}=\frac{J_0(ar)}{J_0(ar_0)}\right]_{l=\infty}, \tag{10}$$

where J_0 represents a Bessel function of zero order.

$$\left[\frac{T'}{T_{0m}'}=f(l)\right]_{r=r_0}, \tag{11}$$

$$\left[\frac{T'}{T_{0m}'}=F(r)\right]_{l=0}. \tag{12}$$

The first of these only is theoretical and merely states the particular solution of (6) for the condition of no conduction of heat along the filament, a condition which is fulfilled at a considerable distance from a cooling junction. The second boundary condition gives the surface temperature distribution. It is necessarily one that must be experimentally determined. The third boundary condition naturally specifies the temperature distribution across some surface near the cooling junction. In practice this distribution must be assumed.

In case $f(l)$ is expressed in terms of a hyperbolic sine or a hyperbolic cosine series, a solution may be obtained which represents accurately the distribution of temperature by means of an infinite series whose terms are of the type

$$A\sinh(\mu l)\,J_0(\sqrt{\mu^2-a^2}\,\sqrt{-1}\,r).$$

In case $f(l)$ is expressed in terms of an exponential series, as has been done by the writer for some of his determinations, an expression may be obtained for the temperature distribution which although it theoretically fails under certain conditions, represents the temperature distribution to a high degree of accuracy in all practical cases which the writer has considered. Assume thus the following solution of (8)

$$f(l)=1-A_1e^{-\mu(l+l_0)}+A_2e^{-2\mu(l+l_0)}-\cdots, \tag{13}$$

where l_0 is a constant depending on the materials and dimensions at the cooling support and on the conditions of operation. For $F(r)$ in practice one can not, as already stated, make any experimental determination. It is evident from simple considerations and from the later determinations of temperature difference between filament axis and filament surface that such an assumption as

$$F(r)=\text{const.} \tag{14}$$

is warranted. A double series suffices to give the resultant temperature distribution.

$$\frac{T'}{T_{0m}'} = S_1 + S_2 = \frac{J_0(ar)}{J_0(ar_0)} - A_1 e^{-\mu(l+l_0)} \frac{J_0(\sqrt{\mu^2 + a^2}\, r)}{J_0(\sqrt{\mu^2 + a^2}\, r_0)}$$

$$(15) \qquad + A_2 e^{-2\mu(l+l_0)} \frac{J_0(\sqrt{(2\mu)^2 + a^2}\, r)}{J_0(\sqrt{(2\mu)^2 + a^2}\, r_0)} - \cdots + B_1 e^{-\nu(l+l_0)} J_0(\sqrt{\nu_1^2 + a^2}\, r)$$

$$+ B_2 e^{-\nu_2(l+l_0)} J_0(\sqrt{\nu_2^2 + a^2}\, r) - \cdots,$$

where $B_1, B_2 \cdots \nu_1, \nu_2 \cdots$ represent undetermined constants. If $\nu_1, \nu_2 \cdots$ be considered particular values of μ, it is readily seen that each term of (15) is a solution of (8) and of (9) and therefore of (6). The second of the two series S_2 in (15) represents a correction term which enters, due to the failure of the first series S_1 by itself to fulfill the boundary condition (14). In determining the constants of S_2 it is to be considered as a separate series. Its boundary conditions as such, since the first two of the three main boundary conditions are fulfilled by S_1, are

(16) $[S_2 = 0]_{l=\infty},$

(17) $[S_2 = 0]_{r=r_0},$

(18) $[S_2 = -S_1]_{l=0}.$

The method of evaluating the constants here may be found in appropriate texts dealing with Bessel's functions.[1] It is of course necessary to first evaluate S_1 for the position $l = 0$. Evidently the sum of S_1 and S_2 fulfills the boundaries (10), (11) and (12).

The most satisfactory comprehension of a temperature distribution results from a consideration of $1/T_{0m}'\,\partial T'/\partial l$ and $1/T_{0m}'\partial T'/\partial r$ and through them of the axial and the radial components of the temperature gradient $\partial T/\partial l$ and $\partial T/\partial r$ and of the angle which the temperature gradient makes with the axis of the filament. Differentiation of (15) gives

$$(19) \qquad \frac{1}{T_{0m}'}\frac{\partial T'}{\partial l} = A_1 \mu e^{-\mu(l+l_0)} \frac{J_0(\sqrt{\mu^2 + a^2}\, r)}{J_0(\sqrt{\mu^2 + a^2}\, r_0)} - 2A_2 \mu e^{-2\mu(l+l_0)} \frac{J_0(\sqrt{(2\mu)^2 + a^2}\, r)}{J_0(\sqrt{(2\mu)_2 + a^2}\, r_0)} + \cdots$$

and

$$(20) \qquad \frac{1}{T_{0m}'}\frac{\partial T'}{\partial r} = \frac{\partial/\partial r\ J_0(ar)}{J_0(ar_0)} - A_1 e^{-\mu(l+l_0)} \frac{\partial/\partial r\ J_0(\sqrt{\mu^2 + a^2}\, r)}{J_0(\sqrt{\mu^2 + a^2}\, r_0)} + \cdots$$

For computations using (19) and (20) often the quantities represented

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[1] Byerly's Fourier's Series and Spherical Harmonics, p. 228.

by Bessel's functions may be taken as unity, while for expressions such as $\partial/\partial r\ J_0\ (cr)$ may be taken the approximation,

$$\frac{\partial}{\partial r}J_0(cr) \doteq -cJ_1(cr) \doteq -\frac{c^2r}{2}\,. \tag{21}$$

The method of procedure for obtaining the corresponding values for $\partial T/\partial l$ and $\partial T/\partial r$ will be evident when the relation between T and T' is once determined.

THE SURFACE TEMPERATURE DISTRIBUTION.

The method of study was based on the Holborn-Kurlbaum optical pyrometer principle and is very similar to that employed by Hyde, Cady and Worthing[1] in a study of energy losses in electric incandescent lamps and by the writer in a study of the variations from Lambert's cosine law.[2] All of the precautions which have been enumerated by Worthing and Forsythe[3] were made use of. The arrangement of apparatus is shown in Fig. 1. The filaments studied were of tungsten mounted as U-shaped loops in evacuated bulbs. The lamps, which were well aged in order to remove gases, were located at A. The necessary measurements consist in balancing in brightness the pyrometer filament D against the background filament A in the neighborhood of the cooling junction as that filament, heated to incandescence by a constant current, is raised or lowered. Next with exactly the same arrangement of apparatus, excepting that a black body is substituted at A, and is operated at various temperatures, a calibration of the pyrometer filament current as a function of the black body temperature of the background is obtained. Then with the aid of a black body temperature—true temperature calibration of the background filament, taking account of the glass of the bulb of the lamp placed at A, one easily obtains the surface temperature distribution along the filament placed at A. The writer, however, used

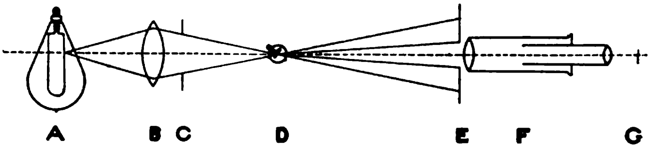

Fig. 1.

Diagram showing arrangement of apparatus. *A*, background; *B*, objective lens; *C*, entrance cone diaphragm; *D*, pyrometer filament; *E*, eyepiece diaphragm; *F*, eyepiece; *G*, monochromatic glass filter.

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[1] Trans. of Ill. Eng. Soc. (U.S.), 6, p. 238, 1911. Illum. Eng. (Lond.), 4, p. 389, 1911.

[2] Astrophys. Jour., 36, p. 345, 1912.

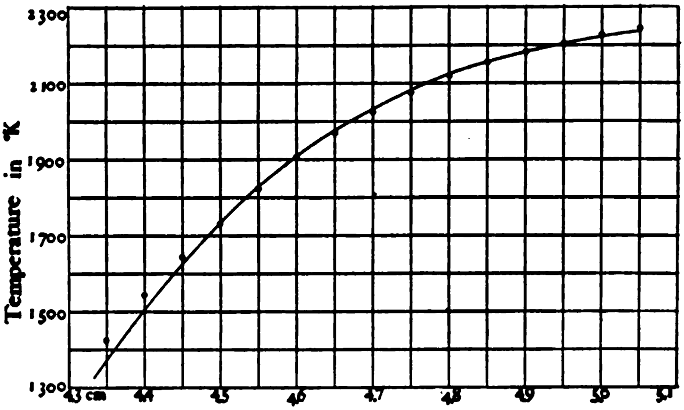
[3] PHYS. REV. II, 4, p. 163 (1914).

a somewhat more roundabout method in this work, due to his interest in certain other considerations. The effects of the Thomson electromotive force were eliminated by averaging the results as to temperature distribution obtained with direct currents flowing through the filament first in one direction then in the opposite direction. The black-body temperature—true temperature calibration used was that obtained by Mendenhall and Forsythe.[1]

As has already been stated an exponential series relation between surface temperature and distance was found,[2]

$$(22)\qquad \frac{T_0}{T_{0m}} = [1 - e^{-\mu(l+l_0)}]^n = 1 - ne^{-\mu(l+l_0)} + \frac{n(n-1)}{2}e^{-2\mu(l+l_0)} - \cdots.$$

How nearly this represents the distribution experimentally obtained in a particular case may be seen from Fig. 2. The average of a great number of such determinations has shown all of the deviations to be noticed, with one exception, to be accidental. Deviations in a direction to make the computed value of T_0/T_{0m}, too small in the immediate neighborhood of the cooling junction always occur. They may result from a noticeable progressive change in the surface condition as one proceeds toward the junction. These deviations are neglected in this paper. Measurements on several different sized filaments operated at different maximum temperatures, indicate that n is constant throughout and that μ varies so that



Distance along the filament in cm. from an arbitrarily chosen zero.

Fig. 2.

$$(23)\qquad \frac{i}{\mu} = \text{constant (approx.)}$$

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[1] Astrophys. Jour., 37, p. 380, 1913.

[2] See following paper.

The deviation from constancy occurs only with a change in the maximum operating temperature, the ratio decreasing slightly with decreasing maximum temperatures. For the case of a change in μ due only to a change in filament size, (23) may be shown, on assuming the same general surface distribution in the two cases, to be theoretically true. Of two filaments of radii r_1 and r_2 operated at the same maximum temperature, consider two corresponding elements of filament of equal length and the same mean temperature. We have for both elements the identity,

(24) Electrical input = radiation output + conduction output.

Consequently

$$(25)\qquad \pi r_1^2 \Delta l\, i_1^2 \rho = 2\pi r_1 \Delta l f(T) + \pi r_1^2\, \Delta l \left[k \left(\frac{\partial^2 T}{\partial l^2} \right)_1 + \frac{dk}{dT} \left(\frac{\partial T}{dl} \right)_1^2 \right],$$

$$(26)\qquad \pi r_2^2\, \Delta l\, i_2^2 \rho = 2\pi r_2 \Delta l f(T) + \pi r_2^2 \Delta l \left[k \left(\frac{\partial^2 T}{\partial l^2} \right)_2 + \frac{dk}{dT} \left(\frac{\partial T}{\partial l} \right)_2^2 \right],$$

where $f(T)$ represents the radiation intensity.

The well-known relation

$$(27)\qquad \left[\frac{i_1}{i_2} = \left(\frac{r_2}{r_1} \right)^{\frac{3}{2}} \right]_{T_1 = T_2},$$

When combined with (25) and (26) gives

$$(28)\qquad \frac{r_2}{r_1} = \frac{\left(\frac{\partial^2 T}{\partial l^2} \right)_1 + \frac{1}{k} \frac{dk}{dT} \left(\frac{\partial T}{\partial l} \right)_1^2}{\left(\frac{\partial^2 T}{\partial l^2} \right)_2 + \frac{1}{k} \frac{dk}{dT} \left(\frac{\partial T}{\partial l} \right)_2^2}.$$

If for filament 1 we have

$$(29)\qquad T_1 = f_1(l),$$

and for filament 2 the assumed same general surface distribution such that

$$(30)\qquad \left[T_2 = f_2(l) = f_1 \left(\frac{\left(\frac{\partial T}{\partial l} \right)_2}{\left(\frac{\partial T}{\partial l} \right)_1} l \right) \right]_{T_1 = T_2},$$

there follow

$$(31)\qquad \frac{\left(\frac{\partial^2 T}{\partial l^2} \right)_1}{\left(\frac{\partial^2 T}{\partial l^2} \right)_2} = \frac{\left(\frac{\partial T}{\partial l} \right)_1^2}{\left(\frac{\partial T}{\partial l} \right)_2^2}$$

and

$$(32) \qquad \left(\frac{\partial T}{\partial l}\right)_1 i_2 = \left(\frac{\partial T}{\partial l}\right)_2 i_1.$$

When $f(l)$ has the form given in (22), (23) results.

This relation is important in that it enables one to go directly from a surface distribution for one filament operating at a definite maximum temperature to that of another filament operating at the same maximum temperature.

THE T' SCALE OF TEMPERATURE.

Results on the thermal conductivity of tungsten[1] indicate that

$$(33) \qquad k = k_m + C_1(T - T_m).$$

The new temperature scale T' having the necessary condition (3) of constant thermal conductivity was further arbitrarily fixed by making

$$(34) \qquad k' = k_m$$

and

$$(35) \qquad [T' = 0]_{T=0}.$$

Consequently

$$(36) \qquad T' = \left(1 - C_1\frac{T_m}{k_m}\right) T + \frac{C_1}{2k_m} T^2.$$

The substitution of (22) in (36) gives the previously assumed relation (13) in which A_1, A_2, etc., are functions only of n and of the coefficients of (36).

APPLICATION OF FOREGOING TO A SPECIAL TUNGSTEN FILAMENT.

The value of certain constants relating to the particular filament used and the coefficients of (22) and (33) are incorporated in Table II.

TABLE II.

Certain Important Constants.

Constant.	Value.	Constant.	Value.
r_0	.01045 cm.	μ	$3.80 \dfrac{1}{\text{cm}}$
i	$12600 \dfrac{\text{amp}}{\text{cm}^2}$	n	1.87
ρ_m	75.8×10^{-6} ohms×cm.	k_m	$1.385 \dfrac{\text{watts}}{\text{cm} \times \text{deg}}$
T_{0m}	2315°K.	C_1	$5.10^{-4} \dfrac{\text{watts}}{\text{cm} \times \text{deg}^2}$
l_0	.3 cm (roughly)	a	$2.60 \dfrac{1}{\text{cm}}$

The value given for a represents an average value for the interval 1500° K. to 2450° K., throughout which range there is a decrease of approximately

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[1] See later paper.

10 per cent. in going from the lower to the higher temperature. The assumption of constancy is here allowable in case one is concerned chiefly with the magnitude and the direction of the temperature gradient. In case one is interested in the differences in temperature between the surface and the axis at given cross sections, only values of the right order of magnitude will be obtained. The values thus found, however, will be directly related to the true differences approximately as the square of the a used in the computations is related to the squares of the true values of a for the regions under consideration.

Another method of obtaining the temperature differences consists in computing the distributions for various values of a and then applying to the regions in question the appropriate distributions. This consideration is based on computations which show that, for positions where the axial components of the temperature gradients are large in comparison with the radial components, small changes in the temperature distribution at one cross-section appreciably affect only a very limited region thereabout.

The equation for the special case resulting from (15) is

$$\begin{aligned}\frac{T'}{T_{0m}'} = S_1 + S_2 = \frac{J_0(2.60r)}{J_0(2.60r_0)} - 3.21e^{-3.8(l+l_0)}\frac{J_0(4.61r)}{J_0(4.61r_0)} \\ +3.91e^{-7.6(l+l_0)}\frac{J_0(8.04r)}{J_0(8.04r_0)} - \cdots - 6.4\times 10^{-5}e^{-230(l+l_0)}J_0(230r) + \cdots.^{1}\end{aligned} \tag{37}$$

As has already been noted, this solution fails under certain conditions,—*e. g.*, when $J_0(\sqrt{\mu^2_n + a^2}\, r_0)$ is zero. In ordinary cases the series in (13) may consist of a finite number of terms so terminated that this condition is impossible. The first term only of S_2 in (37) produces any noticeable effect on the temperature distribution.

The most interesting results for a particular application based on a constant value for a are indicated in Table III.

On account of a slightly different relation obtained for k as a function of T in the preliminary work published in an abstract,[1] values there given will not be found to be entirely consistent with those here given.

The formation of an expression for $\partial T'/\partial r$ from the S_1 terms of (37) shows that it, so far as first order terms are concerned, is proportional to the radial distance. Evidently also at any cross-section

$$\frac{\partial T}{\partial r} = - C_2 r, \tag{38}$$

where C_2 is a constant. Integration gives

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[1] PHYS. REV., II., 3, p. 67 (1914).

TABLE III.

Temperature Distribution Results in a Filament of Tungsten (r_0 = 0.01045 cm.) Heated to a Maximum Temperature of 2315° *K.*

l	T_0	Relative Emission Intensity.	Temperature Gradient at Surface.			Temperature Difference Between Axis and Surface.
			Radial Component.	Axial Component.	Angle Made with Filament Axis.	
0.0 cm	1149°K	0.023	$0\ \frac{\text{deg}}{\text{cm}}$	$3750\ \frac{\text{deg}}{\text{cm}}$	0.0°	0.00°
0.2	1711	.200	15	2130	0.3	.08
0.5	2114	.619	31	756	2.3	.16
0.7	2225	.808	39	364	6.1	.20
1.2	2301	.967	46	54.7	40	.24
1.7	2314	.9952	47	8.3	80	.25
∞	2315	1.0000	47	0.0	90	.25

$$T - T_0 = \frac{C_2}{2}(r_0^2 - r^2). \tag{39}$$

Therefore

$$[T - T_0]_{r=0} = \frac{C_2}{2} r_0^2 = -\frac{r_0}{2}\left(\frac{\partial T}{\partial r}\right)_{r=r_0}. \tag{40}$$

Equation (40) was used in computing the last column of Table III. It readily follows with slight approximation that the isothermal surfaces are portions of paraboloids of revolution which become more and more convex toward the cooling junction as one recedes along the filament from it. Further (39) and (40) show that the difference in temperature between a point on the axis and another point on the same cross-section varies as the square of the radial distance.

When, however, corresponding points on two filaments of the same material but of different radii and operated at the same maximum temperature, are compared, (23) together with the derivative $\partial T'/\partial r$, which may be obtained with the aid of (20), indicate that the differences in temperature, particularly when not too far from the maximum temperature, between corresponding axial points and other points on the same cross-section vary as the radii of the filaments. This fact would appear as evident from the equation given by Angell[1] for the portion of the filament where the temperature gradient is wholly axial.

A close approximation to the temperature distribution in a tungsten filament near a cooling junction for a certain definite heating current, when once the distribution is known for some other heating current may likewise be obtained quite simply. A consideration of (19) in which the coefficients A_1, A_2, etc., are practically constant shows, for corresponding

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[1] Phys. Rev., 33, p. 421 (1911).

points chosen from the standpoint of T_0'/T_{0m}', that the expression $1/T_{0m}' \, \partial T'/\partial l$ varies as μ, which in turn according to (23) varies as i. The axial components of the temperature gradient may evidently be everywhere determined for the unknown surface distribution. The variation in $1/T_{0m}' \, \partial T'/\partial r$ may be obtained from similar considerations By (5) we see that a varies as i. From (20) and (21) we see that each term of $1/T_{0m}' \, \partial T'/\partial r$ and that therefore the quantity as a whole, since both a and μ vary as i, varies as i^2. The deviations mentioned in both (5) and (23) are such as to counteract each other. An accurate knowledge of the temperature distribution for one tungsten filament, together with the above considerations will enable one to determine quite accurately the distribution in any case for a filament of the same material.

SUMMARY.

A solution has been obtained which represents with considerable accuracy the temperature distribution in a cylindrical tungsten filament of uniform cross-section and surface condition, heated electrically in an evacuated chamber.

This solution has been applied to a special case in which a tungsten filament (radius = 0.01045 cm.) is heated by a current to a maximum central temperature of 2315° K. (See Table III.)

The method of obtaining from the known distribution the temperature distribution in another filament of the same material, but of different radius heated under the same conditions to the same maximum temperature, is indicated.

The method of obtaining from the known distribution the temperature distribution in the same filament heated under the same conditions to a different maximum temperature is also indicated.

The surface temperature distribution for a tungsten filament for a given case and the approximate laws of variation with changes in filament size and maximum operating temperature have been experimentally determined. See equations (22) and (23).

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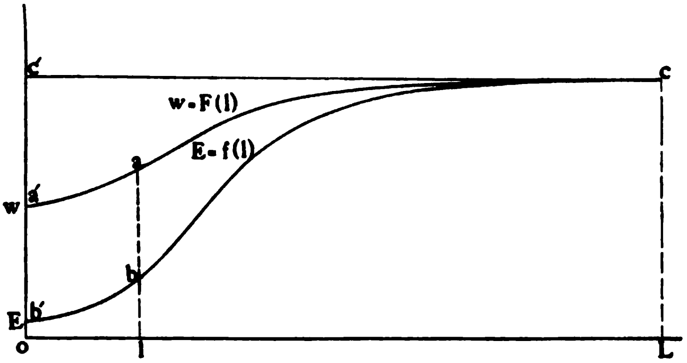
THE THERMAL CONDUCTIVITIES OF TUNGSTEN, TANTALUM AND CARBON AT INCANDESCENT TEMPERATURES BY AN OPTICAL PYROMETER METHOD.

By A. G. Worthing.

Introduction.

THE present paper is the direct outcome of a study of the energy losses in incandescent lamps by Hyde, Cady and Worthing.[1] With a single exception the question of thermal conductivity at incandescent temperatures seems not to have been attempted heretofore. Angell,[2] using a method suggested by Mendenhall, investigated aluminum and nickel up to temperatures of 600° C. and 1200° C. respectively. The temperature measurements were made by means of thermocouples located on the inner and the outer surfaces of hollow cylindrical rods which were electrically heated. Extremely large heating currents and fairly large specimens of material were, of course, necessitated. The question of incandescence did not enter in the method. In the present paper a method of measuring thermal conductivity depending on the incandescence of small filaments mounted in evacuated chambers, such as filaments in ordinary lamp bulbs, is described and made use of.



Distance along filament.

Fig. 1.

Diagram to illustrate method for deteming thermal conductivities.

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[1] Trans. Illum. Eng. Soc. (U.S.), 6, p. 238 (1911); Illum. Eng. (Lond.), 4, p. 389 (1911).

[2] Phys. Rev., 33, p. 421 (1911).

THEORY.

Consider the energy supply and radiation intensity distributions per unit of filament surface for a small filament of uniform surface and cross section, mounted in an evacuated bulb and heated to a steady state of incandescence by an electric current. These quantities, w and E respectively, as functions of the distance along the filament are diagrammatically represented by the curves $a'ac$ and $b'bc$ respectively of Fig. 1. The meanings of the symbols here used together with others are given in Table I. Evidently at the center of the filament loop where there is no temperature gradient along the filament

$$[w = E = w_m = E_m]_{l=L}. \tag{1}$$

TABLE I.

Symbols and Quantities Used.

Symbol.	
w	Rate of heat production electrically at a given filament cross-section per unit of radiating surface.
E	Radiation intensity, or rate of radiation of energy per unit of radiating surface.
l	Distance measured along the filament.
L	One half of the filament length.
dH/dt	Rate of conduction of heat along the filament.
r	Radius of filament.
k	Thermal conductivity.
T	Temperature.
i	Current density.
ρ	Resistivity.
$\varkappa$	Electrical conductivity.
w_m, E_m, T_m	Values of w, E and T at center of filament loop.
μ, σ, β, γ, A, and B	Constants defined by equations (8), (9) and (10).

It is easily seen that the areas $lacLl$, $lbcLl$ and $bacb$ respectively represent quantities proportional to the rate of heat production in the filament length lL, the rate of radiation of energy from the filament length lL, and the rate of heat conduction along the filament at l. For the last named quantity, *i. e.*, $\frac{dH}{dt}$, we may write

$$\frac{dH}{dt} = 2\pi r E_m \int_l^L \left(\frac{w}{w_m} - \frac{E}{E_m}\right) dl. \tag{2}$$

This together with the general equation

$$\frac{dH}{dt} = \pi r^2 k \frac{dT}{dl} \tag{3}$$

enables one to write for the thermal conductivity at l

$$k = \frac{2E_m}{r\dfrac{dT}{dl}} \int_l^L \left(\frac{w}{w_m} - \frac{E}{E_m} \right) dl. \tag{4}$$

This assumes that the radial component of the temperature gradient at l is negligible in comparison with the axial component. Determinations of k for different positions l and hence for different temperatures T lead to the determination of k as a function of T.

Apparatus and Method.

The arrangement of apparatus and the general method of procedure were the same as those described in the study of the temperature distribution in an incandescent lamp filament near a cooling junction.[1] The filaments studied were U-shaped lamp filaments mounted with welded junctions in large well-exhausted lamp bulbs. The lamps which were initially well aged were used as backgrounds. The pyrometer filaments were of tungsten, sufficiently small, usually about 0.025 mm. in diameter, so that even when seen projected against the background at a place where the axial temperature gradient was a maximum, there was no apparent difference in the brightness of the background at the two edges of the projected pyrometer filament.

The precautions noted for optical pyrometric methods of this type by Worthing and Forsythe[2] have been carefully observed in the work on tungsten. The work on tantalum and carbon is less accurate; and while at the time of the experimental work on them all of these precautions were not known, it is believed that the conditions were not such as to cause very great errors.

The experimental work consisted in determining the following three relations,

$$E = \varphi(T), \tag{5}$$

$$E = f(l), \tag{6}$$

$$w = F(l). \tag{7}$$

Equation (5) was obtained by balancing the pyrometer filament first against a central portion of the loop of the filament under investigation when operated successively at various currents, and then against a black body similarly placed and operated at various temperatures. In the former case the brightness of the background image was so reduced by sectored disks that the pyrometer filament currents were of the same order of magnitude in both instances. With the aid of ordinary electrical

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[1] Phys. Rev., Vol. IV, No. 6.

[2] Phys. Rev., II, 4, p. 163 (1914).

and length measurements on the background lamp filament, one may then express E as a function of the black body temperature. The temperature relations determined by Mendenhall and Forsythe[1] were used to convert these into the desired functions of the true temperatures.

Equations (6) and (7) were determined in much the same manner that T as a function of l was determined in the paper on temperature distribution already mentioned. In fact (5) enables one to obtain (6) from the temperature distribution there given. The simple determination of the relation between ρ and E with the aid of (6) leads to (7), since obviously w is proportional to ρ everywhere in a uniform filament heated to a steady state by an electric current. The relations (6) and (7) are those represented in Fig. 1 respectively by the curves $b'bc$ and $a'ac$.

The effects of a Thomson electromotive force located in the filaments near the junction were eliminated by averaging the results obtained with the heating current flowing first in one direction through the filament and then in the reversed direction. In determining (5) account was taken of the fact that the average rate of energy supply electrically per unit surface of filament does not equal (less than, as represented in Fig. 1) the rate of energy radiation per unit surface of filament at a point remote from the filament junction. The correction factor is the ratio (area $a'c'ca'$)/(area $Oa'cLO$).[2]

Once given (5), (6) and (7) a method of applying (4) in order to determine k is obvious. Considerable difficulty was experienced in obtaining satisfactory relations for (6), due for the most part to the inability to realize the fundamental assumptions of uniform filament surface and cross-section. The fact that the glass of the bulb enclosing

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[1] Astrophys. Jour., 37, p. 380 (1913).

[2] The area $a'c'ca'$ represents an input loss. For many purposes this loss may be considered as producing an apparent decrease in the effective length of the filament ΔL_1 given by (area $a'c'ca'$)/(length oc'). Similarly from the standpoint of the energy radiated, there is an apparent decrease in the effective length ΔL_2, given by (area $b'c'cb'$)/(length oc'). In a like manner one may obtain from the standpoint of the luminous flux from the filament, the apparent decrease ΔL_3. For uniform tungsten filaments, in loops sufficiently long so that there is no sensible heat conduction along the filament at the centers of the loops, there have been found in practice an approximately constant ratio between the junction temperatures, when fused with copper, and the maximum filament temperatures. For such cases general formulæ may be derived for ΔL_1, ΔL_2 and ΔL_3. Thus

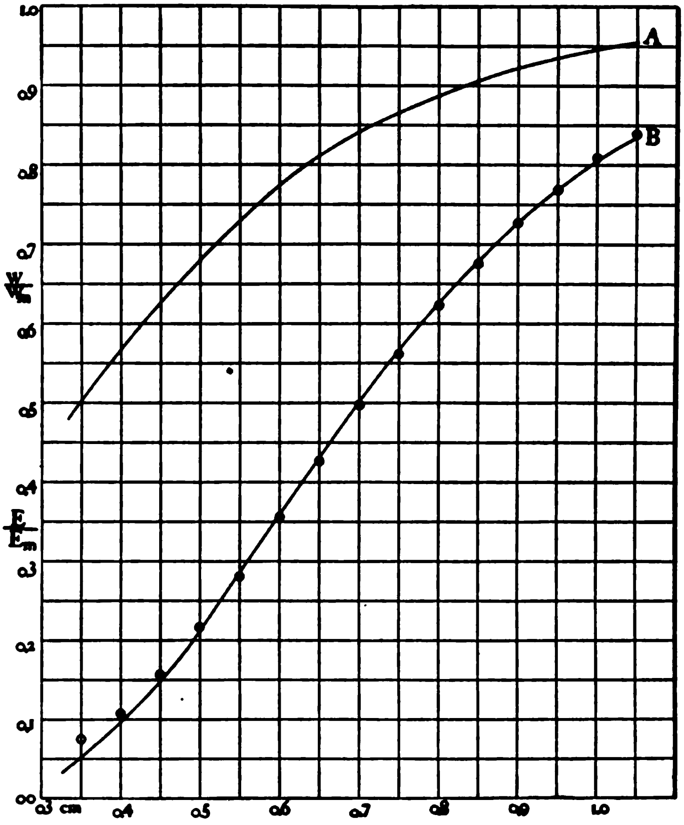
$$\Delta L_1 \cdot i = 2700 \frac{\text{amps}}{\text{cm}}$$

where i is the current density L_1 and L_2 are approximately thereafter determined by

$$\Delta L_1 : \Delta L_2 : \Delta L_3 = 3 : 7 : 10.$$

For the lamp mentioned in the main article, operating at 2315° K., the apparent decreases for each junction are respectively 0.21 cm., 0.50 cm. and 0.71 cm.

the background was not optically good was also a source of error. However, as nearly as possible the law was empirically determined for the finite range $0.2 < (E/E_m) < 0.85$ and then assumed to hold also for the range $0.85 < (E/E_m) < 1.00$. Slight errors in pyrometer balancing in this region were always productive of comparatively great errors in the constants of the law sought. Measurements made there, however, always on the whole justified the assumption. Below $E/E_m = 0.2$ deviations from the empirical law found for the middle region always occurred in a definite direction, hence the limits chosen. No values obtained for k were based on the application of the empirical equation to regions where $E/E_m < 0.2$. How well the empirical law fills the requirements may be seen in Fig. 2, in which there have been platted the data



Distance along filament in cm.

Fig. 2.

The relative input (A) and the relative radiation intensity (B) distributions for a tungsten filament ($r = 0.01045$ cm.) heated to a maximum temperature of 2315° K. The circles represent experimental values of radiation intensities obtained in a particular set of measurements. The curves represent distributions given by equations (9) and (10).

for a single experimental determination of the E and w distributions for the same lamp operated at the same maximum temperature as the one for which the temperature distribution has been given in a paper already referred to. The deviations, with the exception of the ones for small values of E/E_m, are accidental and in the average of many curves are eliminated.

RESULTS.

For tungsten the empirical relations found which determine the character of the functions in (5), (6) and (7) are of the types

$$E = \sigma T^{\beta}, \tag{8}$$

$$\frac{E}{E_m} = [1 - e^{-\mu(l+l_0)}]^{\gamma}, \tag{9}$$

$$\log \frac{w}{w_m} = \log \frac{E}{E_m}\left(A + B \log \frac{E}{E_m}\right). \tag{10}$$

Equations (8) and (10) seem to be quite exact. The values of various constants which enter in connection with the data platted in Fig. 2 in the application of (4) to the determination of k are given in Table II.

TABLE II.

Constants for a Particular Application of (4) to a Tungsten Filament.

Quantity.	Value.	Quantity.	Value.
r	0.01045 cm.	β	5.35
i	$12600 \frac{\text{amp}}{\text{cm}^2}$	σ	$6.2 \times 10^{-17} \frac{\text{watts}}{\text{cm}^2 \times \text{deg.}^{5.35}}$
E_m	$62.1 \frac{\text{watts}}{\text{cm}^2}$	γ	10.
T_m	2315° K	A	0.257
μ	$3.80 \frac{1}{\text{cm}}$	B	0.015

The value obtained for β, while probably fairly exact, is to be regarded as preliminary. It is hoped that a future paper from this laboratory will consider it along with other similar determinations in detail. In Fig. 3 there are given the results obtained by the application of (4) to the particular filament when operated at four different maximum temperatures. For each such maximum temperature four determinations have been made of k at various temperatures corresponding respectively to values for E/E_m of 0.8, 0.6, 0.4 and 0.3. That the assumption made in the derivation of (4), of a negligible difference between the temperature gradient and its axial component is justified for these positions, may be seen from an inspection of Table III. of the paper on temperature distribution. In the case of $l = 0.7$ cm., or $E/E_m = 0.808$, at the

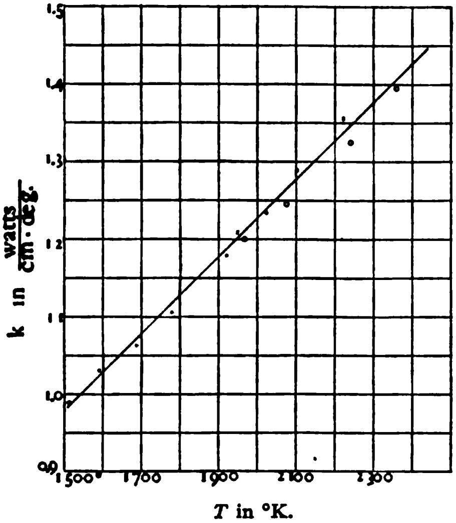


Fig. 3.

Thermal conductivity of tungsten as a function of temperature.

o	14280 $\frac{\text{amp}}{\text{cm}^2}$	2468° K
x	12600	2315
x	10520	2111
.	8420	1890

surface of the filament the temperature gradient is only 0.6 per cent. greater than the axial component. The average over the filament cross section would naturally give a still smaller variation. In case the empirical distributions (9) and (10) are correct, no errors are introduced in the values of k thus determined, due to the fact that as E/E_m approaches unity, the axial component of the temperature gradient is no longer approximately equal to the whole gradient. The general agreement of the results is quite satisfying.

Values of k at various temperatures for tungsten, tantalum and untreated carbon are included in Table III. It is of interest to note that the order in which the materials are given represents their order as to thermal conductivity, tungsten having the highest values; that for carbon there is only a very small temperature coefficient, and that for both tungsten and tantalum there are fairly large positive temperature coefficients. With the exception of carbon no published values with which to compare results have been found. Of these only the rough determination of Hansen[1] on carbon electrodes (these carbon electrodes probably contain carbon in much the same form as do the untreated carbon filaments used by the writer) are directly comparable, since the

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[1] Trans. of Amer. Electrochem. Soc., 16, p. 351 (1909).

TABLE III.

Thermal Conductivities k, and Values of $\frac{k}{xT}$ *for Tungsten, Tantalum and Carbon in the Range 1500° K. to 2500° K.*

Temperature, ° K.	Tungsten		Tantalum.		Carbon (Untreated).	
	$k, \frac{\text{watts}}{\text{cm}\times\text{deg}}$.	$10^{-8}\times\frac{k}{xT}$, C.G.S. Units.	$k, \frac{\text{watts}}{\text{cm}\times\text{deg}}$.	$10^{-8}\times\frac{k}{xT}$, C.G.S. Units	$k, \frac{\text{watts}}{\text{cm}\times\text{deg}}$.	$10^{-8}\times\frac{k}{xT}$, C.G.S. Units.
1500	0.98	0.283		1.62		
1700	1.08	.322	0.73	3.3	0.084	11.9
1900	1.18	.362	.78	3.4	.086	10.1
2100	1.28	.405	.83	3.6	.088	8.8
2300	1.38	.450				
2500	1.48	.502				

others have experimented on the crystalline forms graphite and diamond. Hansen's results give as an average about 0.06 watts/cm. × deg. for the neighborhood 0° C. to 400° C., which is qualitatively in good agreement with the writer's values. He also mentions a value of about 0.008 watts/cm. × deg. for carbon at 3000° C. This does not check at all with the writer's values.

Because of the theoretical interest, determinations have been made of the function k/xT. Its values for all three substances for certain temperatures have been included in Table III. It is of interest to note that k/xT is not approximately constant for any one of them, there being a large negative temperature coefficient for carbon and large positive coefficients for both tungsten and carbon. Variations of a similar character for nickel and aluminum at high temperatures are indicated by the work of Angell,[1] although attention has not been called to them in his paper. Aluminum and nickel respectively have a large positive and a large negative temperature coefficient for k/xT. These deviations have an important theoretical bearing. The classic work of Jaeger and Diessehorst[2] on this function between 18° C. and 100° C. has usually been accepted as indicating its probable constancy at least for pure metals at higher temperatures. This conclusion has fitted in well with electronic theories. The work of Lees[3] and later particularly that of Meissner,[4] experimenting at temperatures as low as 20° K., indicated deviations from constancy such as would be consistent with a zero value at absolute zero. Such deviations were found to disappear, however,

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[1] PHYS. REV., 33, p. 421 (1911).

[2] Wissensch. Abh. d. Phys. Techn. Reichsanstalt, 3, p. 269 (1900).

[3] Phil. Trans. (A), 208, p. 381 (1908).

[4] Verh. d. Deut. Phys. Gessell., 16, p. 262 (1914)

at the temperatures used by Jaeger and Diesselhorst. Modifications of the electron theories have been made in accord with these deviations. Further modifications are evidently necessary. Some data obtained by the method here used seem to indicate that the thermal conductivity of tungsten may be a function of the current density in the filament. The author hopes to consider this point in more detail.

SUMMARY.

1. A method based on optical pyrometry for obtaining the thermal conductivity of certain solid substances at incandescent temperatures has been described.

2. The thermal conductivities of tungsten, tantalum and carbon in the region 1500° K. to 2500° K. have been obtained. The results on carbon check well with results by Hansen.

3. Values for the function k/xT (k thermal conductivity, x electrical conductivity, T absolute temperature) have been determined for the same substances and for the same temperature ranges. Large temperature coefficients of the function were found, in contradiction to what would be expected from electronic theories.

4. As a preliminary determination of the relation between the radiation intensity E and the absolute temperature T for tungsten, the equation

$$E = \sigma T^{\beta},$$

where $\beta = 5.35$ and $\sigma = 6.2 \times 10^{-17}$ watts/cm.^{2} $\times$ deg.$^{5.35}$, has been found to hold.

The writer is indebted to Messrs. K. Linder, G. Cadisch, A. Shapero and A. Scheel for efficient aid rendered at various times during the course of the experimental work.

Nela Research Laboratory,
National Lamp Works of General Electric Company,
Nela Park, Cleveland, Ohio,
July, 1914.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SEVENTY-THIRD MEETING.

A REGULAR meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York, on Saturday, October 31, 1914. President Merritt presided at the meeting. The morning session began at 10 o'clock and the following papers were presented:

Some Applications of Simpson's Rule in Integrating Periodic Functions. **FREDERICK BEDELL AND RALPH BOWN.**

Five Lithium Lines and their Magnetic Separation. **NORTON A. KENT.**

A Comparison of Stellar Radiometers and Radiometric Measurements on 110 Stars. (By title.) **W. W. COBLENTZ.**

An Achromatoscope. (By title.) **S. R. WILLIAMS.**

Electrostatic Measurement of Single Electrode Potentials. **ARTHUR W. EWELL.**

The Relation Between the Wave-length and Absorption of X-Rays. **WILLIAM DUANE.**

An Addition to a Theory of Ionization by Impact. **H. H. FARWELL.**

The Electron Emission from Tungsten Filaments, containing Thorium. **IRVING LANGMUIR.**

Application of a Theory of Ionization By Impact to the Experiments of Franck and Hertz. **BERGEN DAVIS.**

Theory of The Flicker Photometer. **HERBERT E. IVES AND E. F. KINGSBURY.**

Brightness Flicker and Color Flicker in the Flicker Photometer. **HERBERT E. IVES AND E. F. KINGSBURY.**

Adjourned for lunch at 12.45.

Afternoon Session, 2.15 P. M.

A Direct Method of Determining The Radiant Luminous Efficiency of Light Sources. **E. KARRER.**

The Emissivity of Metals and Oxides, II; Measurements with the Micropyrometer. **G. K. BURGESS AND R. G. WALTENBERG.**

The Total Emissivity of Platinum and the Relation Between Total Emissivity and Resistivity. **PAUL D. FOOTE.**

A Simple Method for Demonstrating and Measuring Anomalous Dispersion. (By title.) S. R. Williams.

Measurements with the Coolidge Tube. Transformer Characteristics. J. S. Shearer.

Measurements with the Coolidge Tube; Brightness of Fluoroscopic Screens. J. S. Shearer.

Physical Photometry with a Thermopile Artificial Eye. Herbert E. Ives and E. F. Kingsbury.

Note on the Construction of a Short Vacuum Gauge. E. Karrer.

The Electric Resolution of the Series Lines in the Mercury Spectrum. Georg Wendt and Reinhard A. Wetzel.

The Longitudinal Stark Effect upon Aluminum Doublets. Reinhard A. Wetzel.

New Vapor Lamps and a Preliminary Study of their Spectra in the Schumann Region. Frederick A. Saunders.

Graphic Record of the Period and Damping Factor of a Needle Galvanometer. Horatio B. Williams.

A Model Illustrating the Relativity Concepts of Einstein. Reinhard A. Wetzel.

Note on the Melting Point of Tungsten. Irving Langmuir.

Adjourned at 4.30.

Alfred D. Cole,
Secretary.

A COMPARISON OF STELLAR RADIOMETERS AND RADIOMETRIC MEASUREMENTS ON 110 STARS.[1]

By W. W. Coblentz.[1]

In this paper experiments are described showing that there is but little difference in the radiation sensitivity of stellar thermocouples constructed of bismuth-platinum, and thermocouples of bismuth—bismuth + tin alloy which have a 50 per cent. higher thermoelectric power.

Improvements are described in the method of maintaining a vacuum, by means of metallic calcium, whereby it will be possible to go to the remotest regions for making radiation measurements without carrying an expensive vacuum pump.

This radiometric outfit was used in connection with the 3-foot Crossley reflector at the Lick Observatory, Mt. Hamilton, Calif. Measurements were made on the radiation of 112 celestial objects. This includes measurements on the bright and dark bands of Jupiter (also a pair of his satellites) the rings of Saturn, and a planetary nebula.

Quantitative measurements were made on stars down to the 5.3 magnitude, and high-grade qualitative measurements were made on stars down to the 6.7 magnitude.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 31, 1914.

It was found that red stars emit from two to three times as much total radiation as blue stars of the same photometric magnitude.

Measurements were made on the transmission of the radiations from stars and planets through an absorption cell of water. By this means it was shown that, of the total radiation emitted, the blue stars have about two times as much radiation as the yellow stars—and about three times as much radiation as the red stars, in the spectral region to which the eye is sensitive.

A stellar thermocouple and a bolometer were compared, and the former was found to be the more sensitive. The conclusion arrived at is that, from the appearance of the data at hand, greater improvements are to be expected in stellar thermocouples than in stellar bolometers.

The object of the investigation was to obtain some estimate of the sensitivity required in order to be able to observe spectral energy curves of stars. The radiation sensitivity of the present apparatus was such that, when combined with a 3-foot reflecting telescope, a deflection of 1 mm. would have resulted when exposed to a candle placed at a distance of 53 miles. In order, however, to do much successful work on stellar spectral energy curves, a sensitivity 100 times this value is desirable. In other words, assuming that the rays are not absorbed in passing through the intervening space, the radiometric equipment (radiometer and mirror) must be sufficiently sensitive to detect the radiation from a candle removed to a distance of 500 miles. This can be accomplished by using a 7-foot mirror and by increasing the sensitivity of the present radiometer (thermocouple and galvanometer) 20 times. This increase in sensitivity is possible.

Measurements were made to determine the amount of stellar radiation falling upon 1 cm.^{2} of the earth's surface. The value found is so small that it would require the radiations from *Polaris* falling upon 1 cm.^{2}, to be absorbed and conserved continuously for a period of one million years in order to raise the temperature of one gram of water 1° C. If the total radiation from all the stars falling upon 1 cm.^{2} were thus collected and conserved it would require from 100 to 200 years to raise the temperature of one gram of water 1° C. In marked contrast with this value, the solar rays, which reach the earth's surface, can produce the same effect in about one minute.

Bureau of Standards, Washington.

THE EMISSIVITY OF METALS AND OXIDES, II: MEASUREMENTS WITH THE MICROPYROMETER.[1]

By G. K. Burgess and R. G. Waltenberg.

The micropyrometer may be used conveniently for the approximate determination of the monochromatic emissivities of metals, oxides, etc., in microscopic quantities at high temperatures. It is possible to determine to one per cent. the emissivity of a mass of 0.01 mg. having an area of

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 31, 1914.

0.25 mm.^{2} and a thickness of 0.005 mm. The determination of the temperature-coefficient of emissivity is readily made, as well as the detection of variation of emissivity with change of state, as at the melting point.

Assuming the emissivity for solid platinum is 0.33 for red light of wavelength $\lambda = 0.65\mu$ and 0.38 for $\lambda = 0.55\mu$, the following among other results have been obtained.

Emissivities of Metals and Oxides with Micropyrometer.

Metals.		Cu.	Ag.	Au.	Pd.	Pt.	Ir.	Rh.	Ni.	Co.	Fe.	Mn.	Ti.
$e_\lambda = 0.65\mu$	solid.....	.10	.04	.14	.33	.33	.30	.29	.36	.36	.37	.59	.63
	liquid....	.15	.07	.22	.37	.38	....	.30	.37	.37	.37	.59	.65
$e_\lambda = 0.55\mu$	solid.....	.38	.35	.38	.38	.38	....	....	.44	....	....	....	.75
	liquid....	.36	.35	.38	....	....	....	....	.46	....	....	....	.75

		Zr.	Th.	Y.	Er.	Be.	Cb.	V.	Cr.	Mo.	W.	U.	
$e_\lambda = 0.65\mu$	solid.....	.32	.36	.35	.55	.61	.49	.35	.39	.43	.39	.54	....
	liquid....	.30	.40	.35	.38	.61	.40	.32	.39	.40	....	.34	....
$e_\lambda = 0.55\mu$	solid.....	....	.36	....	....	.61	.61	.29	.53	....	....	.77	....
	liquid....	....	....	....	.30	.81	....	....	....	....	....	....	....

Oxides Near F. P. s.		NiO.	Co_3O_4	Fe_3O_4	Mn_3O_4	TiO_2	ThO_2	Y_2O_3	BeO	CbO_x	V_2O_3	Cr_2O_3	U_3O_8
$e_\lambda = 0.65\mu$	solid.....	.89	.77	.63	....	.52	.57	.61	.37	.71	.69	.60	.30
	liquid....	.68	.63	.53	.47	.51	.69	....	....	....	....	....	.31

For none of the metals examined was there a marked temperature coefficient in the range 900 to 2000° C. and for the most of them this coefficient is negligible, the emissivity usually agreeing also with the value at 20° C. For the white metals the emissivity usually shows very slight or no change at the melting point, but for gold, silver, copper, and uranium there is a marked discontinuity with red light. For palladium, there are anomalies in the behavior of the emissivity at the melting point; and for platinum the fact that there is a change in emissivity (for $\lambda = 0.65\mu$) on melting would influence the constancy of the Violle standard of light.

Bureau of Standards, Washington.

Electrostatic Measurement of Single Electrode Potentials.[1]

By A. W. Ewell.

Determinations of the absolute values of single electrode potentials have often been sought from electrocapillary phenomena "dropping electrodes," endosmotic phenomena, etc. Considerable uncertainty attaches to the values obtained. Recent work by Billitzer gives values for electrode potential approximately 0.7 volt lower than the absolute values commonly given in tables.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 31, 1914.

The writer has been seeking for several years to obtain absolute single electrode potentials by an electrostatic method. The final apparatus, in brief, consisted of a glass or quartz flask covered on the outside with metal which is connected to a Dolezalek electrometer. The flask contained the electrolyte and from the change of potential of the metal outside was determined the potential acquired by the electrolyte when the earthed electrode was dipped into the electrolyte.

Observations with zinc and zinc sulphate, copper and copper sulphate, silver and silver nitrate, with glass and quartz flasks, and silver and aluminum films on the outside, all gave values for the absolute single potentials which are consistent among themselves and are about 0.2 volt higher than Billitzer's values, *i. e.*, approximately 0.5 volt lower than the values found in recent tables. Errors from leakage and from conduction through quartz or glass were easily detected and corrected. Volta effects were proved to be zero by the substitution of mercury for the electrolyte.

WORCESTER POLYTECHNIC INSTITUTE.

Some Applications of Simpson's Rule in Integrating Periodic Functions.[1]

By Frederick Bedell and Ralph Bown.

If a quantity, x, varies as the time-integral of a second quantity, y (illustrated by the case of magnetic flux and electromotive force), that is, $x \propto \int y dt$, successive values of x may be determined from successive values of y, by plotting y as a curve and determining the areas between any initial ordinate and successive ordinates y_1, y_2, y_3, etc.

If these successive ordinates are taken at equal time-intervals and the curve for y is assumed to be made up of arcs of parabolas, the whole area is made up of a number of parabolic trapezettes, the areas of which can be accurately and easily determined by Simpson's Rule. This becomes most simple when y is periodically positive and negative, as in case of alternating electro-motive forces and fluxes, the negative half-waves being a repetition of the positive. In this case, applying Simpson's Rule, we have values of x in two series, thus: *First series,* $x_0 = 0$, $x_2 = y_0 + 4y_1 + y_2$, $x_4 = x_2 + y_2 + 4y_3 + y_4$, $x_6 = x_4 + y_4 + 4y_5 + y_6$, etc., to which must be added a constant C_1; *second series,* $x_1 = 0$, $x_3 = y_1 + 4y_2 + y_3$, $x_5 = x_3 + y_3 + 4y_4 + y_5$, $x_7 = x_5 + y_5 + 4y_6 + y_7$, etc., to which must be added a constant C_2. The constant to be added in each series is the mean value of two values of x, as found in that series, the time interval between which is one-half period. Thus, if values of y are given every 10° (where 360° is one period) and x is determined for this same interval, $C_1 = (y_0 + y_{18}) \div 2$, and $C_2 = (y_1 + y_{19}) \div 2$.

To test the method, y was assumed to be a sine function with values taken every 30° from a table; values of x determined by Simpson's Rule, as described above, were found accurate in this case to a twenty fifth of one per cent.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 31, 1914.

As another test, values of y were taken every 10° from the equation

$$y = 1000 \sin x + 333 \sin 3x(x - 10^\circ) + 100 \sin 5(x - 15^\circ) + 50 \sin 13x.$$

Corresponding values of x, determined by the above rule, varied not more than a sixth of one per cent. from the values determined by tables. For curves even more irregular, it is believed the method will give results sufficiently accurate for most purposes and usually more accurate than can be otherwise obtained. For a fuller discussion, references and a comparison with other methods, see a paper about to be published by the authors entitled "Derivation of Wave-form of Flux from Wave-form of Electromotive Force," Electric Journal, 1915.

CORNELL UNIVERSITY.

THE ELECTRIC RESOLUTION OF THE SERIES LINES IN THE MERCURY SPECTRUM.[1]

BY GEORG WENDT AND REINHARD A. WETZEL.

THE transverse Stark effect was obtained in canal rays with electric fields measuring from 50,000 to 90,000 volts per centimeter.

In the sharp subordinate series of mercury triplets the following lines were investigated: λ2,925; 2,894; 2,752; 2,576; 2,464Å. The photograms obtained show no appreciable splitting or displacement of the lines with these fields under the dispersion at our disposal.

In the diffuse subordinate series of mercury triplets we investigated the following: λ3,132; 3,024; 2,967; 2,803; 2,699; 2,652; 2,534; 2,482Å. With the exception of λ2,967 all show splitting and displacements due to the action of the electric field. Quantitatively the effect is not the same upon each member of the triplet. It is largest upon the first, smaller upon the second, and smallest upon the third component of the triplet. The effect also increases rapidly with the increasing term numbers of the series.

Results.—(1) A confirmation of Stark's series laws for the electric effect—viz., there is a difference between sharp and diffuse subordinate series, there is an increase of the effect with increase of term number; (2) A confirmation of the fact that for the different elements the nature and magnitude of the electric effect upon series lines of the same term number is not a function of their atomic weights but first and foremost a characteristic of the chemical element (its valence electrons). The photograms show this fact even more beautifully than those obtained by Stark and Kirschbaum.

The above investigation was made at the Physical Institute of the Technical High School at Aachen, Germany, during July, 1914. Thanks are due Prof. Stark for the privileges of the laboratory. His rich experience and thoughtful suggestion also served us during the entire work.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 31, 1914.

THE TRANSVERSE STARK EFFECT UPON ALUMINUM DOUBLETS.[1]

BY REINHARD A. WETZEL.

Sharp Subordinate Series.—Electric fields of from 50,000 to 90,000 volts per centimeter show no appreciable effect upon the doublet λ2,660 — 2,652Å. under the dispersion used.

Diffuse Subordinate Series.—Under the above conditions the following doublets were investigated:

λ3,093 — 3,082; 2,575 — 2,568; 2,373 — 2,367Å.

Upon the first two pairs no appreciable effect is noticeable on the photographic plates. Of the last pair the line λ2,373Å. shows a small displacement toward the red, both in the vectors oscillating parallel and in those oscillating perpendicular to the field.

Acknowledgement of indebtedness is due to Prof. Stark for suggesting the research and the opportunities that he so generously placed at our disposal in his laboratory at Aachen.

THE TOTAL EMISSIVITY OF PLATINUM AND THE RELATION BETWEEN TOTAL EMISSIVITY AND RESISTIVITY.[1]

BY PAUL D. FOOTE.

THE theory of radiation from metals developed by Aschkinass has been extended and the following general equation has been derived for the total emissivity of a metal:

$$E = 0.5736\sqrt{rT} - 0.1769rT,$$

where E is the total emissivity at the absolute temperature T and r the volume resistivity of the metal at this temperature. Experimental observations upon platinum obtained by the use of radiation pyrometers confirmed the above theoretical relation. A table of corrections is given for converting temperatures observed with a radiation pyrometer sighted upon platinum, to true temperatures. The following table represents the total emissivity of pure platinum as a function of the temperature.

° C.	0	200	400	600	800	1000	1200	1400	1700
E	.030	.051	.070	.089	.108	.124	.140	.155	.175

BUREAU OF STANDARDS, WASHINGTON.

AN ACHROMATOSCOPE.[1]

BY S. R. WILLIAMS.

AN optical device for attaching to any spectroscope has been devised whereby the channeled spectra of light reflected from two thin films may be compared. When the thin films are wedge shaped the thickness of one film may be varied until the optical path in the two films are the same for any

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 31, 1914.

given wave-length. If complete achromatization occurs the interference bands will be in coincidence throughout the spectrum when the order of interference, N, for a certain wave-length in one film is the same for the same wave-length in the other film. In other words, we have the relation holding that,

$$\frac{\partial N_1}{-\partial\lambda} = \frac{\partial N_2}{-\partial\lambda}.$$

The thin films may be easily prepared and instead of laboriously measuring the indices of refraction in order to determine the degree of achromatization of crown and flint glasses, thin sections of the two kinds of glasses are slipped into position and on adjusting the interference bands in the field of view of the spectroscope, the degree of achromatization may be obtained by inspection.

When sunlight is used as a source of light the Fraunhofer lines are simultaneously produced with the interference bands and in photographing afford a convenient scale of wave-lengths.

Physical Laboratory, Oberlin College,
July, 1914.

NEW BOOKS.

Die Lichtelektrischen Erscheinungen. By R. POHL AND P. PRINGSHEIM. Braunschweig: Vieweg and Sohn, 1914. Pp. 1 + 114. Price Mk 3.

This monograph, on photoelectric phenomena, deals especially with the subject as it has been developed through the researches of the authors, and is therefore more limited in its scope than the recently published monographs by Ries, and Stanley Allen.

Special prominence is given to those phases of the subject dealing with the selective and normal photo-effects of metals.

The work includes chapters on the photo-effect of non-metallically conducting bodies, the effect of surface layers on the photo-effect, and the ionization of gases by light. The closing chapter gives a resumé of the work done by Lindemann, Debyje Sommerfeld and others on the interpretation of photoelectric action in the light of the quantum theory.

The bibliography which is appended is most exhaustive and is brought down to September, 1913. It is regrettable that the work was written before the publication of papers by Wiedemann and Hallwachs, Friedenhagen, Küstner and Hughes on the part played by occluded gases in photoelectric phenomena.

J. C. M.

La Telegraphie et La Telephonie Simultanees et La Telephonie Multiple. By K. BERGER. Paris: Gauthier-Villars, 1913. Pp. 1 + 134.

The book contains three chapters: (1) Telephony over telegraph lines; (2) Telegraphy over telephone lines; and (3) The simultaneous transmission of several telephonic messages over the same line. The treatment is rather technical in character, but the physicist will find much of interest, especially the historical data and the clear presentation of the underlying scientific principles with which each chapter is introduced. The book is a French translation by Le Normand of the German work.

K. E. G.

Taschenbuch für Mathematiker und Physiker. By FELIX AUERBACH AND RUDOLF ROTHE. Leipzig: Druck, 1913. Pp. x + 463. Price, Mk. 6.

Into this small book is compressed a large amount of valuable information, both in mathematics and physics. Beginning with an account of the life of Friedrich Kohlrausch, there follow a number of numerical tables of various functions, and a very compendious presentation of various departments of mathematics including a short account of integral equations. In physics there are also useful tables, with much general information. Particularly interesting is an account of the quantum theory by Sommerfeld. A list of recent books is included.

A. G. W.

Collected Papers in Physics and Engineering. By James Thomson. Cambridge: University Press, 1912. Pp. civ + 484.

This volume adds to the debt of gratitude which students of physics owe to Sir Joseph Larmor for the editorial labor and skill he has devoted to the collection, in convenient and accessible form, of the scientific papers and correspondence of the great physicists of the Victorian era. In the present instance he has had the assistance of James Thomson, M.A., the son of Professor Thomson; and the very interesting Biographical Memoir, with which the volume opens, "has been constructed largely out of narratives and recollections obtained by his daughter Mary Hancock Thomson."

James Thomson was two years older than his brother William, and the two were from childhood devotedly attached to each other. It is inevitable that this close association with his more brilliant junior must have had a considerable effect upon his own scientific work; yet these papers show very clearly that James possessed a mind of great independence and originality, and the correspondence between the brothers (a few examples of which are contained in this volume) make it plain that the influence and help were not all on one side. The difference in the characters of the two men are distinctly shown in their published work. After a perusal of the present volume, one is entirely convinced of the accuracy of the statement in the biographical sketch that, as boys and throughout their lives, "James was the more careful and exact, and William the more quick and ready."

James Thomson was, by profession, an engineer and was for many years professor of engineering, first in Queen's College, Belfast, and afterward in the University of Glasgow. He made many investigations and inventions in the subject of hydraulics, accounts of which are contained in the first section of the present volume. He invented a turbine water-wheel, a centrifugal pump and a jet pump which were much used in their time; and he made great improvements in the measurement of running water by weir-boards, especially by his introduction of the triangular notch. One short paper in this section contains his explanation of the "tears of strong wine" which is familiar to all readers of Maxwell's Theory of Heat.

The second division of papers on "Congelation and Liquefaction" begins with the celebrated calculation, from Carnot's theory, of the lowering of the freezing point of water by pressure. As was still customary at the time (1849), notwithstanding the previous work of Mayer, Joule and Helmholtz, the argument assumes, like that of Carnot himself, the perfect conservation of heat. As is well known, the calculation was numerically verified soon after by Lord Kelvin; it is quite plain from his language that the fact that "an entirely novel physical phenomenon was *predicted* in anticipation of any direct experiment" had a great effect upon his mind, and must have contributed to his reluctance to accept Joule's theory until he had, with much difficulty, reconciled it with Carnot's.

The remaining papers of this section (together with some correspondence with Faraday which is included) is mainly concerned with regelation and the plasticity of ice in glaciers; there was much difference of opinion on these subjects, but Thomson's views were well considered and sound, and he maintained them with courage and tenacity even against the great authority of Faraday.

During his residence in Belfast, Thomson was a colleague of Andrews and was deeply interested in his experiments upon the critical state and the continuity of the liquid and gaseous states. His contributions to this subject form the third section of papers. As is well known, he first suggested some of the ideas which were afterward worked out more completely by van der Waals. He also investigated the triple-point and made the first thermodynamic model in three dimensions.

The fourth section on "Dynamics and Elasticity" is perhaps of less general interest than the preceding ones. The paper on "Inertia, Chronometry, Absolute Motion" however might well arrest attention as a fore-runner of the modern doctrine of relativity—especially the statement on p. 380 of the difficulty of defining simultaneity.

The book concludes with three papers on geological subjects, some miscellaneous papers, and an appendix containing three papers of general, rather than scientific, interest.

H. A. B.

Molecular Physics. By JAMES ARNOLD CROWTHER, M.A. Philadelphia: P. Blakiston's Son & Co., 1914. Pp. viii + 167. Price, $1.25.

This little book has been prepared to meet the need of those who desire to know what has been learned through recent researches of the constitution and properties of atoms and molecules.

It contains chapters which deal with the physics of the electron, the properties of alpha rays, positive ray analysis, as developed by Sir J. J. Thomson, the chemistry of the atom in the light of the electron theory, the Zeeman effect, spectral series, the origin of spectra, the kinetic theory of heat, the electronic theory of thermal and electrical conduction, radioactivity and radioactive transformations. It also contains three appendices of mathematical notes illustrating the matter of the text and closes with a table of atomic data, and a bibliography. The book has been written with care and will prove extremely useful.

J. C. M.

La Technique de La Radiotelegraphie. By DR. ING. H. REIN. Paris: Gauthier-Villars, 1913. Pp. x + 257.

This book is a translation into French of the second edition of Rein's Radiotelegraphisches Praktikum. It was originally written as a laboratory guide for students of wireless telegraphy, but in its present form it contains much which would not be found in a mere laboratory manual. After a short introduction, discussing the characteristic features of various systems of wireless telegraphy the main part of the book is devoted to a description of high frequency instruments and the best methods for their calibration. Measurements of capacities, self and mutual inductances, wave-length and damping under such conditions as are found in actual practice are given in detail and in each case a large number of different methods is given. Then follows a chapter on generators and receivers of various types. The book closes with a discussion of the guiding principles for the erection of wireless stations.

The illustrations are practically restricted to apparatus used in Germany and the bibliography contains only a few references to publications in any other but the German language. The treatment of the subject is excellent and the book should be found in every laboratory of wireless telegraphy. We are indebted to the translator, G. Viard, to have made this book available to students who are unable to read it in the original.

K. E. G.

Practical Physics. By N. H. Black and H. N. Davis. New York: The Macmillan Company, 1913. Pp. ix + 487. Price, $1.25 net.

This is an elementary physics text-book which follows for the most part established lines. It is a somewhat longer treatise than other elementary text-books due largely to more descriptions of commercial devices.

E. S. B.

Second Course in Algebra. By William Benjamin Fite, New York: D. C. Heath and Company, 1914. Pp. v + 247.

This is an attractive and clearly written text for students who have had a year of algebra.

F. B.

Elektrische Spektralanalyse chemischer Atome. By Dr. J. Stark. Leipzig: S. Hirzel, 1914. Pp. viii + 138. Price, Mk. 5.

This monograph contains an account of the phenomenon of electric resolution of spectral lines recently discovered by Stark, and includes a detailed description of the different experimental methods of observing the effect.

The chapter dealing with the dependence on field intensity of the strength and the distance apart of the electrically resolved constituents of a line as well as that on the analysis of lines of the same series and of different elements is specially well prepared.

The monograph contains a chapter on phenomena allied to the Stark effect, and also one which deals entirely with theoretical considerations regarding it. The bibliography appended is excellent.

J. C. M.

Experimental Studies in Electricity and Magnetism. By FRANCIS E. NIPHER. Philadelphia: P. Blakiston's Son & Co., 1914. Pp. 1 + 73. Price, $1.25.

This book contains in connected form the results of Professor Nipher's researches, which have been published from time to time since 1910 by the Academy of Sciences of St. Louis. The author undertakes to obtain evidence from the ordinary electric discharge in air which can be used to decide between the claims of the "one-fluid" and the "two-fluid" theories of electricity. His observations are extremely curious and interesting. They well deserve the consideration of anyone who is concerned with the phenomena of the electric discharge. The author concludes that they can be explained on the "one-fluid" theory. He agrees with Aepinus that this theory necessitates the mutual repulsion of material particles, and believes that he observes this in the explosion of a lead wire into which a discharge passes from a neighboring positive terminal. The creeping of a wire in the direction opposite to that of the corpuscular (*i. e.*, negative) discharge passing along it is remarkable.

A short section at the end of the book deals with the author's observations of the connection between magnetic storms and certain conditions of weather.

W. F. M.

Gas Analysis. By L. M. DENNIS. New York: The Macmillan Company, 1913. Pp. xvi + 434. Price, $2.10.

This book is written for technical chemists and contains descriptions of apparatus and methods for rapid gas analysis. Elementary physical laws are briefly mentioned at the proper places. Some contact with the physicist is made in such chapters as: "The determination of the specific gravity of a gas; the purification of mercury; the determination of the heating value of fuel; the examination of atmospheric air." K. E. G.

Geometry of Four Dimensions. By HENRY PARKER MANNING, New York: The Macmillan Company, 1914. Pp. ix + 348. Price, $2.00.

Aristotle made the statement that "beyond these (three dimensions) there is no other magnitude because these three are all." After an interesting historical introduction, reviewing the development of the geometry of more than three dimensions from its negation by Aristotle until its recognition as a branch of modern mathematics, the author discusses the subject in chapters as follows: The Foundations of Four-Dimensional Geometry; Perpendicularity and Simple Angles; Angles of Two Planes and Angles of Higher Order; Symmetry, Order, and Motion; Hyperpyramids, Hypercones, and the Hypersphere; Euclidean Geometry; Figures with Parallel Elements; Measurement of Volume and Hypervolume in Hyperspace; The Regular Polyhedroids.

F. B.

Mathematical Theory of Heat Conduction. By L. R. INGERSOLL AND O. J. ZOBEL. Boston: Ginn and Company, 1913. Pp. vi + 171. Price, $1.60.

The authors have succeeded in producing a very attractive short treatise on this very important subject to the physicist. By a judicious selection of problems they have managed to include the most important methods and at the same time to treat a great variety of practical questions of interest to the engineer or the geologist. The figures are clear and luminous, and the graphical representation of the approximations by Fourier's series is to be commended. An idea of the physical good sense displayed in the choice of subject-matter may be obtained from the following, chosen at random; the theory of "cold waves," flow in engine-cylinder walls, the fireproof wall, annealing of castings, the formation of ice, all worked out with numerical values.

The mathematician will regret that in the treatment of Fourier's theorem, page 60, Fourier's error in giving the wrong order of integration is perpetuated, but this will not worry the physicist much, and to him the book is unhesitatingly to be commended. A. G. W.

College Physiography. By Ralph Stockman Tarr. Published under the Editorial Direction of Lawrence Martin. New York: The Macmillan Company, 1914. Pp. xxii + 837. Price, $3.50. (*Received.*)

The Electrical Conductivity and Ionization Constants of Organic Compounds. By Heyward Scudder. New York: D. Van Nostrand Company, 1914. Pp. 1 + 568. Price $3.00.

Besides an explanatory introduction, lists of abbreviations, etc., this book contains three tables. The first consisting of 280 pages, is an alphabetical list of all the organic compounds whose electro-chemical properties have been hitherto studied; under each compound are given values of its specific conductivity and ionization constant as determined by different observers at various temperatures and under a variety of other conditions. In each case cross-references (by number) are given to the third table, or "Author List," which the author believes to be an almost or quite complete bibliography of the subject from 1889 to 1910. The second table is an index of the chemical formulæ of the compounds mentioned, and there is at the end of the book a Subject Index giving references to the bibliography.

The work appears to be done in a very thorough and painstaking manner and must have involved great labor. It will doubtless prove of value to all investigators whose work involves the electro-chemical properties of organic compounds. H. A. B.

Physics of the Household. By C. J. Lynde. New York: The Macmillan Company, 1914. Pp. xi + 313. Price, $1.25.

"This is an elementary text-book of physics, written for students of household science." It is so distinctly practical in its aim that the treatment of dynamics is reserved for the last chapter and occupies fourteen pages. There is practically no discussion of the theories of physics. The laws of physics are stated concisely and are illustrated by examples of their application in common instruments used about the house, ranging in intricacy of construction from the tack lifter to the vacuum cleaner. The book is one which may well serve the purpose for which it is designed but it would not do for use in a course of instruction in which physics is taught for its scientific content or as a medium for the inculcation of scientific method.

On page 59 Henry's law is ascribed to "The American physicist Henry." The Henry of the law was William Henry of Manchester, England. The American physicist Joseph Henry was a younger man by twenty-three years than the other, and could not very well, without extraordinary precocity, have announced a discovery in 1803, when he was six years old.

W. F. M.

Mechanics and Heat. By W. B. ANDERSON. New York: McGraw-Hill Book Company, 1914. Pp. xi + 349. Price, $2.00 net.

Professor Anderson has made the interesting attempt of writing a college text on mechanics and heat in which the mathematics is reduced to an amount not much greater than that found in the more advanced high school texts. On the other hand he has placed more emphasis on practical applications and elementary methods of measurement than is usual in college texts.

The book is evidently intended for the instruction of students in agriculture and doubtless serves its purpose for this type of students. The mathematical structure is, however, too light to serve as a foundation for a severe training in engineering. The treatment is very clear and interesting and the illustrations excellent.

K. E. G.

Allgemeine Ergebnisse und Probleme der Naturwissenschaft. By DR. BERNHARD BAVINK. Leipzig: S. Hirzel, 1914. Pp. xiii + 314. Price, Mk. 7.

This volume aims at giving a philosophical treatment in a condensed form of some of the more general results and problems of present day science. In the first section the subjects presented include the atomic theory of matter, the equivalence of different forms of energy, the kinetic theory of heat, the electromagnetic theory of light, the electromagnetic nature of mass, the emission and absorption of light, and the principle of relativity. The second section includes a discussion of the bearing of the law of entropy on the problem of the future of the world, and a detailed account of such matters as the configuration and motion of fixed star systems, world history, recent advances in our knowledge of meteorological phenomena, and problems relating to cosmogony generally. The third section is devoted to the problem of life, and deals with the physical and chemical basis of life, the simple living cell, vitalism, the origin of life, the principle of causality in biology, and problems arising from a study of multicellular organisms. In the concluding section the subjects dealt with are the theories of evolution and mutation, and problems dealing with the origin of man and his position in the Universe.

The book contains much that is of great interest to those wishing to ascertain the philosophical bearing of recent contributions to knowledge in different branches of science.

J. C. M.

Report on Radiation and the Quantum-Theory. By J. H. Jeans. London: The Electrician Printing and Publishing Co., Ltd., 1914. Pp. iv + 90. Price 6s.

In this brief but comprehensive report, Professor Jeans discusses the modern problem of radiation and its proposed solution by the quantum theory. For some time past Jeans has been reluctant to concede that the general principle of equipartition of energy, which can be demonstrated for dynamical systems, does not apply to the ether as well as to matter in systems which are in true equilibrium. But in this report he frankly accepts the necessity of assuming that the relations of the ether to matter are not subject to dynamical principles, and admits that some laws other than the Newtonian laws must determine the phenomena of radiation. He shows that the Newtonian laws lead to the Rayleigh-Jeans formula, which is inconsistent with the observed facts, and that the hypothesis of the energy quantum leads to Planck's formula. He then gives an exposition of Nicholson's theory of line-spectra, and an account of Bohr's theory of the atom and its relations to the emission of radiation. He also discusses the photoelectric effect and the recent work of Debye and others on the heat capacities of solids and shows that the quantum theory will explain the facts observed. In a final chapter on the physical basis of the quantum theory he discusses most acutely the course which speculation must take in the endeavor to construct a new mechanics which shall be comprehensive enough to contain the laws of radiation. He points out that Einstein's light-quanta are inconsistent with the undulatory theory of light, but gives no definite positive conception of what the quantum is, or of the way in which its theory can be developed. In fact nowhere, except in Planck's preface to his Theory of Radiation, is there given a more striking statement of the necessity for a new theory of the physical universe and of the immense distance which we seem to be from realizing such a theory.

The report is written with all the author's well known brilliancy and clearness, and will take rank with similar reports by other English masters as a physical classic.

W. F. M.

Elements of General Science. By O. W. Caldwell and W. L. Eikenberry. New York: Ginn and Co., 1914. Pp. xiv + 308. Price, $1.00.

This book represents a pedagogical effort to properly direct the science instruction in the first year of high school work. It emphasizes in particular the physics and chemistry of air and water. The topics discussed are grouped under five headings, the air, water and its uses, work and energy, the earth's crust, and life upon the earth. The treatment throughout is logical and connected, and the principles developed are illustrated with numerous examples drawn from nature.

The chapters dealing with weather, the relation of air to food production, the distribution of disease germs, the climatic influence of bodies of water, water supplies and sewage disposal, soil physics, plant physics, the world's food supply, nutrition, reproduction and the struggle for existence are particularly well written. A book of this character should be in the hands of every teacher of elementary science. J. C. M.

Photo-Electricity. By ARTHUR LLEWELYN HUGHES. Cambridge Press, 1914. Pp. i–viii + 1–142.

Within a single year three books have appeared on photo-electricity and all of them by men who have been actively engaged in pushing ahead the bounds of knowledge in this field. Nevertheless each book shows marked individuality and no two of them are at all alike. Allen's is the most compendious (220 pages) and Pohl and Pringsheim's the most special, dealing largely with work done at the Physical Institute in Berlin, while Hughes' book is remarkable for its combination of thoroughness with conciseness, for its scholarly fairness to all observers, and for the skill and insight with which the varied facts of photo-electricity are given a theoretical significance. Indeed it is by forsaking entirely the historical method and aiming immediately at the statement and explanation of photo-electric facts as they now stand that Hughes has succeeded so well in compressing a very complete treatment of a large field into 140 pages. In many respects the book is noteworthy. R. A. M.

Mathematics, from the Points of View of the Mathematician and of the Physicist. By E. W. HOBSON. Cambridge: University Press, 1914. Pp. 1 + 24. Price, 1s.

In this address, delivered originally before the Mathematical and Physical Society of University College, London, Professor Hobson has pointed out, with great clearness and entire impartiality, the reasons which underly the different attitudes of mathematicians and of physicists toward the study of mathematics. The subject is illuminated by the author's great learning and enlivened by characteristic touches of humor and by accounts of significant misunderstandings between some of the great physicists and mathematicians of the past. A student of either science can scarcely fail to derive both pleasure and profit from a perusal of the lecture. H. A. B.

Die Lichtbrechung in Gasen als physikalisches und chemisches Problem. By DR. ST. LORIA. Braunschweig: Vieweg und Sohn, 1914. Pp. vi + 92. Price, Mk. 3.

This monograph which is one of the Vieweg collection contains a concise summary of all the work which has been done on the refractivity and dispersion of gases. It contains chapters on the electromagnetic theory of dispersion, the methods of determining the refractivity of gases, the dependence of refractive power of gases on pressure and temperature, and the refractivity and dispersion of compound gases and of gaseous mixtures. The references inserted as footnotes are most complete, and altogether the work is one of the best prepared of those of the series which have been brought to the attention of the reviewer. J. C. M.

ERRATA AND ADDENDA.

Volume III. The index (June, 1914) failed to include reference to the article by Frances G. Wick entitled "A Spectrophotometric Study of the Absorption, Fluorescence, and Surface Color of Magnesium Platinum Cyanide," which was published in the May, 1914, issue, p. 382.

Page 177, Vol. IV. To the paper by David L. Webster entitled "The Effect of Pressure on the Absorption of Light by Bromine and Chlorine, and its Theoretical Significance," reference is added, at the request of the author, to the work of Prof. C. Schaefer, one of the first to investigate the deviations from Beer's Law in Gases, published in Annalen der Physik, 16, 1, pp. 93–105, January 26, 1905.

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